MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT - JEE
UNIQUE TEST – 2

MATHEMATICS CLASS – XII

Date :– 13– 06 - 2010 **Duration : 1 Hours** **Max. Marks : 77**

Name : ..... **Roll. No. :** .....

INSTRUCTIONS

Do not break the seal of the question paper booklet before instructed to do so by the invigilator

Section A contains 15 questions, Section-B contains 1 questions and Section-C contains 2 questions. Total number of pages are **8**. Please ensure that the Questions paper you have received contains ALL THE QUESTIONS in each section and PAGES.

SECTION - A

1. Question **1** to Quesion **8** has four choices (A), (B), (C), (D) out of which **only one is correct** & carry **4 marks** each. **1 mark** will be deducted for each wrong answer.

2. Question **9** to Question **12** has four choices (A), (B), (C), (D) out of which **one or more than one is/are correct** and carry **5 marks** each. **2 mark** will be deducted for each wrong answer.

3. Question **13** to Question **15** are based upon a **paragraph**. Each Question has 4 choices (A), (B), (C), (D) out of which **only one is correct** & carry **3 marks** each. **1 mark** will be deducted for each wrong answer.

SECTION - B

1. Questions **1** is **Matrix match type** questions. **Column-I** contains Four (A,B,C,D) entries and **Column-II** contains Five (P,Q,R,S,T) entries. Entry of **Column-I** are to be matched with **one or more than one entries** of **Column-II or vice versa. 2 mark** will be awarded for each part of **Column-I**. **NO NEGATIVE** marking for this section.

SECTION - C

1. Questions **1** to Questions **2** are **Integer answer type questions** (whose answer are upto 4 digits) & carry **4 marks** each. **NO NEGATIVE** marking for this section.

NOTE : GENERAL INSTRUCTION FOR FILLING THE OMR ARE GIVEN BELOW.

1. Use only **HB pencil** or **blue/black pen (avoid gel pen)** for darkening the bubble.

2. Indicate the correct answer for each question by filling appropriate bubble in your OMR answer sheet.

3. The Answer sheet will be checked through computer hence, the answer of the question must be marked by shading the circles against the question by dark **HB pencil or blue/black pen**.

4. While filling the bubbles please be careful about SECTIONS [i.e. Section-A include single correct answers, multi correct answers, paragraph type), Section-B (include match the column), Section-C (include integer answer type)].

SECTION-A	SECTION-B	SECTION-C
For example if only 'A' choice is correct then, the correct method for filling the bubble is **A B C D E** ● ○ ○ ○ ○ For example if only 'A & C' choices are correct then, the correct method for filling the bubble is **A B C D E** ● ○ ● ○ ○ the wrong method for filling the bubble are ⊘ ⊗ ◉ ◔ ◕ The answer of the questions in wrong or any other manner will be treated as wrong.	For example If Correct match for (A) is P; for (B) is R, S; for (C) is Q; for (D) is P, Q, S then the correct method for filling the bubble is **P Q R S T** **A** ● ○ ○ ○ ○ **B** ○ ○ ● ● ○ **C** ○ ● ○ ○ ○ **D** ● ● ○ ● ○	**Ensure that all columns are filled**. Answers, having blank column will be treated as incorrect. Insert leading zero(s) if required : '6' should be filled as 0006 '86' should be filled as 0086 '1857' should be filled as 1857

SECTION – A
[STRAIGHT OBJECTIVE TYPE]

Q.1 to **Q.8** has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct

1. If $f(9) = 9$, $f'(9) = 4$ then $\lim\limits_{x\to 9} \dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(A) 3 (B) 9 (C) 4 (D) Does not exist

2. $\lim\limits_{x\to 0} \dfrac{\left[(1+x)^{1/x} - e + \dfrac{ex}{2}\right]}{\sin^2 x}$ is

(A) $\dfrac{e}{8}$ (B) $\dfrac{11e}{24}$ (C) 2 (D) $e^{2\sqrt{2}}$

3. $\lim\limits_{n\to\infty} \{(1.5)^n + ([(1 + 0.0001)^{10000}])^n\}^{1/n}$ is equal to

(where $[*] \to$ denotes greatest integer function)

(A) 1.5 (B) 2 (C) 3 (D) Does not exist

4. $\lim\limits_{x\to 0} \left\{\sin^2\left(\dfrac{\pi}{2-ax}\right)\right\}^{\sec^2\left(\frac{\pi}{2-bx}\right)}$ is equal to

(A) e^{a^2/b^2} (B) e^{-a^2/b^2} (C) 1 (D) 0

(SPACE FOR ROUGH WORK)

5. What is the value of a & b respectively if $\lim_{x\to\infty}\left[\tan^{-1}\left(\frac{x+1}{x+2}\right)-\tan^{-1}\left(\frac{ax+b}{x+2}\right)\right]=\frac{\pi}{6}$

(A) R, $\sqrt{2}-1$ (B) R, $2-\sqrt{3}$ (C) $2-\sqrt{3}$, R (D) $\sqrt{2}-1$, R

6. If $f(x)=\begin{cases}[x] & \text{if } -3\le x<0\\ 2x+1 & \text{if } 0\le x\le 3\end{cases}$ and $g(x)=f(|x|)+|f(x)|\ \forall\ x\in[-3,3]$, then

(where $[*]\to$ denotes greatest integer function)

(A) g(x) is discontinuous at 1 point

(B) g(x) is discontinuous for more than 1 point

(C) g(x) is non-differentiable at 1 point

(D) g(x) is non-differentiable for every integer in [–3, 3]

7. Let $I_1=\lim_{x\to0}\frac{\ln(\cos3x)}{2x^2}$; $I_2=\lim_{x\to0}\frac{\sin^2 3x}{x(1-e^x)}$; $I_3=\lim_{x\to1}\frac{\sqrt{x}-x}{\ln x}$, then

(A) $I_1<I_2<I_3$ (B) $I_2<I_3<I_1$ (C) $I_1<I_3<I_2$ (D) $I_2<I_1<I_3$

8. If $f(x)=x^3\left(\sqrt{x^3}-\sqrt{x^3+1}\right)$ then

(A) function is non-differentiable at x = 0

(B) function is differentiable at x = 0

(C) $\lim_{x\to\infty}\frac{f(x)}{x^{3/2}}=\frac{1}{2}$

(D) None of these

(SPACE FOR ROUGH WORK)

[MULTIPLE OBJECTIVE TYPE]

Q.9 to **Q.12** has four choices (A), (B), (C), (D) out of which **ONE OR MORE THAN ONE** is/are correct.

9. Let $f(x) = \max(x^3, x^2)$ & $g(x) = \sqrt{[x]^2} + \{x\}^2$ then which of the following holds good
(where $[*] \to$ greatest integer function & $\{*\} \to$ Fractional part function)
(A) f(x) is continuous & differentiable function $\forall\ x \in [1, \infty)$ (B) f(x) is non differentiable for finite number of points
(C) g(x) is continuous & differentiable for all $x \in I$ (D) g(x) is continuous but non differentiable for all $x \in I^+$

10. If $f(x) = \min\left(\cos x, \frac{1}{2}, \{\sin x\}\right)$; $0 < x \le 2\pi$, then f(x) is non differentiable at
(where $\{*\} \to$ fractional part function)
(A) $\frac{3\pi}{2}$ (B) 0 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

11. $f(x) = \begin{cases} \dfrac{e^{[x]} - e^{\{x\}}}{e^x} & x < 0 \\ \dfrac{\sin\{x\}}{\{\tan x\}} & x > 0 \text{ Except those values of x where } \tan x \in I \\ 2 & x = 0 \end{cases}$
(where $[*] \to$ greatest integer function & $\{*\} \to$ Fractional part function)
(A) continuous at x = 0 (B) discontinuous at x = 0 (C) $f'\left(\frac{\pi^+}{6}\right) = -\frac{1}{2}$ (D) $\lim\limits_{x \to \frac{\pi^+}{4}} \frac{1}{f(x)} = 0$

(SPACE FOR ROUGH WORK)

12. Let $f(x) = \begin{cases} x\left[\frac{1}{x}\right] + x[x] & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ then the correct statement(s) is/are

(where [*] → greatest integer function)

(A) Limit exists for x = – 1.
(B) $f(x)$ has a removable discontinuity at x = 1.
(C) $f(x)$ has a non removable discontinuity at x = 2.
(D) $f(x)$ is discontinuous at all positive integers.

[COMPREHENSION TYPE]

Q.13 to **Q.15** are based upon a **paragraph**. Each questions has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct.

Let $f(x) = \begin{cases} ax(x-1)+b & x<1 \\ x-1 & 1 \le x \le 3 \\ px^2+qx+2 & x>3 \end{cases}$

(i) f(x) is continuous for all x
(ii) f(x) is not differentiable at x = 1
(iii) $f'(x)$ is continuous at x = 3

13. Find the true relation
(A) 9p + 3q = 1 (B) 3p + q = 0 (C) 3p + q = – 1 (D) None of these

14. Find the value of p
(A) 1/3 (B) 2/3 (C) 1 (D) None of these

15. Which of the statement is correct
(A) $a \neq 0$ (B) $a \neq 1$ (C) $a \neq 2$ (D) None of these

(SPACE FOR ROUGH WORK)

SECTION – B
[MATRIX MATCH TYPE]

Q.1 has four choices (A), (B), (C), (D) out of which **ONE OR MORE THAN ONE** is/are correct

1. Match the column

	Column-I		**Column-II**
(A)	Let $L = \lim_{x\to a} \frac{x^x - a^a}{x-a}$ and $M = \lim_{x\to a} \frac{x^x - a^x}{x-a}$ $(a > 0)$.	(P)	½
	If L = 2M then the value of 'a' is equal to	(Q)	e
(B)	$\lim_{x\to 0} \frac{x(1-\cos 2x)^2 - a(\sin x - \tan x)^2}{\tan^5 x + a\sin^8 x}$	(R)	e^2
	is non-zero finite then 'a' may be		
(C)	$\lim_{n\to\infty} \frac{n^a \sin^2(n!)}{n+1} = 0$, $n \in N$ then 'a' may be	(S)	1/e
(D)	If $\lim_{x\to\infty} \left(\frac{x+a}{x-a}\right)^x = e$ then 'a' equals to	(T)	does not exists

(SPACE FOR ROUGH WORK)

SECTION – C
[INTEGER ANSWER TYPE]

Q.1 to **Q.2** are **INTEGER ANSWER TYPE** questions

1. No. of points in [0, 2] at which f(x) is not continuous where $f(x) = \begin{cases} |4x-5|[x] & ; x > 1 \\ [\cos \pi x] & ; x \le 1 \end{cases}$

(where [*] $\rightarrow$ greatest integer function)

2. If $f(x) = \begin{cases} (\cos x + ax)^{\frac{1}{x}} & ; x < 0 \\ b & ; x = 0 \\ \dfrac{(x+c)^{\frac{1}{2}} - 2}{\ell n\,(1+2x)} & ; x > 0 \end{cases}$

is continuous at x = 0 then find the value of $\dfrac{\sqrt{c}}{e^a b}$

(SPACE FOR ROUGH WORK)

Rough Space