MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT-JEE
HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-II CLASS XII & XIII (DATE 26-01-10)

MATHEMATICS

Section I

1. B **2.** B **3.** B **4.** A

Section II

1. A,B **2.** A,D **3.** A, B **4.** A, D **5.** B,D

Section III

1. (A) $\rightarrow$ (Q), (B) $\rightarrow$ (P), (C) $\rightarrow$ (S), (D) $\rightarrow$ (R) **2.** (A) $\rightarrow$ (R), (B) $\rightarrow$ (S), (C) $\rightarrow$ (Q), (D) $\rightarrow$ (P)

Section IV

1. 10 **2.** 10 **3.** 8 **4.** 16 **5.** 6 **6.** 16 **7.** 2 **8.** 3

PHYSICS

Section I

1. C **2.** A **3.** B **4.** D

Section II

1. B,C **2.** A,C **3.** B **4.** B,C **5.** A,C

Section III

1. (A) $\rightarrow$ P,S ; (B) $\rightarrow$ R,S ; (C) $\rightarrow$ R,S ; (D) $\rightarrow$ P, S **2.** (A) $\rightarrow$ Q,S ; (B) $\rightarrow$ P ; (C) $\rightarrow$ P,R ; (D) $\rightarrow$ P, R

Section IV

1. 1 **2.** 8 **3.** 1 **4.** 5 **5.** 0.5 **6.** 5 **7.** 2 **8.** 1

CHEMISTRY

Section I

1. A **2.** A **3.** D **4.** B

Section II

1. A, B, D **2.** B, D **3.** B, C, D **4.** A,B **5.** B,C,D

Section III

1. (A – P) ; (B – P,R,S) ; (C – P,S) ; (D – P) **2.** (A – Q,R,S) ; (B – P) ; (C – Q,R,S) ; (D – Q,R,S)

Section IV

1. 49.53 **2.** 0.0025 **3.** 1 **4.** 4157 **5.** 3.87 BM **6.** – 48.13 kJ **7.** 3.57 Å **8.** 50%

Section I

1. **B**

$T_n = (n + 1)(n\omega + 1)(n\omega^2 + 1)$

$T_n = (n + 1)(n^2 + 1 - n)$, $\quad T_n = n^3 + 1$

$S_n = \Sigma T_r = \Sigma(r^3 + 1) = \left[\frac{n(n+1)}{2}\right]^2 + n$

2. **B**

$$I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x\, f(x(2-x))dx = \int_{\sin^2 t}^{1+\cos^2 t} (2-x) f((2-x).x)dx$$

$$\left(\text{Use} \int_a^b f(x) = \int_a^b f(a+b-x)dx\right)$$

$I_1 = 2.\, I_2 - I_1 \;\Rightarrow \frac{I_1}{I_2} = 1$ Hence (B)

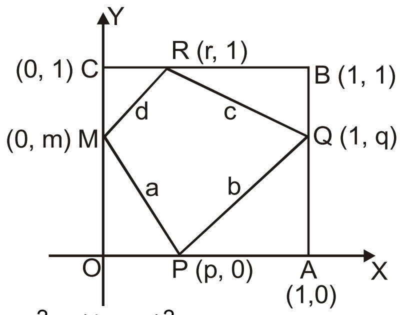
3. **B**

Given S be the area of square with vertices
O (0, 0), A (1, 0), B (1, 1), C(0, 1).
Let PQRM be the quadrilateral with vertices
P(p, 0), Q(1, q), R(r, 1) and M(0, m)
and sides MP = a, PQ = b, QR = c, RM = d

Then $a^2 = p^2 + m^2$

$b^2 = (1 - p)^2 + q^2$

$c^2 = (1 - q)^2 + (1 - r)^2$



$d^2 = r^2 + (1 - m)^2$

$\therefore \quad a^2 + b^2 + c^2 + d^2 = p^2 + (1 - p)^2 + q^2 + (1 - q)^2 + r^2 + (1 - r)^2 + m^2 + (1 - m)^2$

$= 2\,[p^2 + q^2 + r^2 + m^2 - p - q - r - m + 2]$

$$= 2\left[\left(p-\frac{1}{2}\right)^2 + \left(q-\frac{1}{2}\right)^2 + \left(r-\frac{1}{2}\right)^2 + \left(m-\frac{1}{2}\right)^2 + 1\right] \ge 2$$

$\Rightarrow \quad a^2 + b^2 + c^2 + d^2 \ge 2$ ....(1)

also, since $0 \le x \le 1$

$\therefore \quad x^2 \le 1$

$\therefore \quad a^2 \le 1,\ b^2 \le 1,\ c^2 \le 1,\ d^2 \le 1$

$\Rightarrow \quad a^2 + b^2 + c^2 + d^2 \le 4$ ....(2)

from (1) & (2) we get

$2 \le a^2 + b^2 + c^2 + d^2 \le 4$

4. **A**

$$f(x) = \begin{cases} [x] + \sqrt{\{x\}} & x < 1 \\ \dfrac{1}{[x] + \{x\}^2} & x \ge 1 \end{cases}$$

Consider the function f(x) is the interval (0, 2)

$$f(x) = \begin{cases} \sqrt{x} & 0 < x < 1 \\ \dfrac{1}{1 + (x-1)^2} & 1 \le x < 2 \end{cases}$$

$f(1) = 1 \qquad \lim_{x\to 1} f(x) = 1$

continuous at x = 1

$$f'(1^+) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{1+h^2} - 1}{h} = \lim_{h\to 0} \frac{-h}{1+h^2} = 0$$

$$f'(1^-) = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to 0} \frac{\sqrt{1-h} - 1}{-h}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1-h} + 1} = \frac{1}{2}$$

$\therefore \quad$ f(x) is not differentiable at x = 1

Section II

1. **A,B**

$$T_n = \tan^{-1}\left(\frac{4}{4n^2+3}\right) = \tan^{-1}\frac{1}{n^2 + \frac{3}{4}} = \tan^{-1}\left(\frac{1}{1 + n^2 - \frac{1}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\left(n+\frac{1}{2}\right) - \left(n-\frac{1}{2}\right)}{1 + \left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right) = \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$$

$T_1 = \tan^{-1}\frac{3}{2} - \tan^{-1}\frac{1}{2}$

$T_2 = \tan^{-1}\frac{5}{2} - \tan^{-1}\frac{3}{2}$

$T_3 = \tan^{-1}\frac{7}{2} - \tan^{-1}\frac{5}{2}$

$T_n = \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$

$S_n = \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\frac{1}{2}$

$S_\infty = \frac{\pi}{2} - \tan^{-1}\frac{1}{2}$

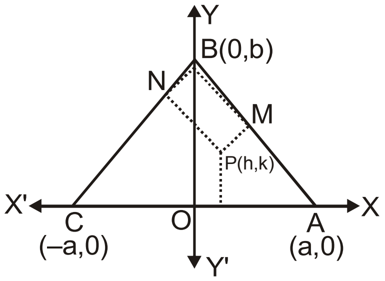
$$= \frac{\pi}{2} - \left(\frac{\pi}{4} - \tan^{-1}\frac{1}{3}\right) = \frac{\pi}{4} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4} + \cot^{-1}3$$

2. **A,D**

$\because$ AB = BC,

According to question

$k^2 = PM \times PN$



$$k = \frac{\left|\frac{h}{a}+\frac{k}{b}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} \times \frac{\left|-\frac{h}{a}+\frac{k}{b}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\Rightarrow \quad \frac{k^2}{a^2}+\frac{k^2}{b^2} = \left|\left(\frac{k}{b}-1\right)^2-\frac{h^2}{a^2}\right| \qquad ....(1)$$

$$\Rightarrow \quad \frac{k^2}{a^2}+\frac{k^2}{b^2} = \left(\frac{k}{b}-1\right)^2-\frac{h^2}{a^2}$$

$$\Rightarrow \quad \frac{k^2}{a^2}+\frac{k^2}{h^2} = \frac{k^2}{b^2}+1-\frac{2k}{b}-\frac{h}{a^2}$$

$$\frac{h^2}{a^2}+\frac{k^2}{a^2}+\frac{2k}{b}-1=0 \qquad \text{or} \qquad h^2 + k^2 + \frac{2a^2}{b}k - a^2 = 0$$

$\therefore$ Locus of P is $x^2 + y^2 + \frac{2a^2}{b}y - a^2 = 0$

and from (1),

$$\frac{k^2}{a^2}+\frac{k^2}{b^2} = \frac{h^2}{a^2}-\left(\frac{k}{b}-1\right)^2 = \frac{h^2}{a^2}-\frac{k^2}{b^2}-1+\frac{2k}{b}$$

$$\Rightarrow \quad \frac{h^2}{a^2} - k^2\left(\frac{1}{a^2}+\frac{2}{b^2}\right) + \frac{2k}{b} - 1 = 0$$

$$\therefore \text{ Locus } \frac{x^2}{a^2} - \left(\frac{1}{a^2}+\frac{2}{b^2}\right)y^2 + \frac{2y}{b} - 1 = 0$$

$\therefore$ Locus of P is circle or hyperbola.

3. **A, B**

$\because$ a > b > c and given equations are

$ax + by + cz = 0$

$bx + cy + az = 0$

$cx + ay + bz = 0$

For non trivial solution

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$3abc - (a^3 + b^3 + c^3) = 0$

$\therefore$ a + b + c = 0

one root of Q.E. is 1 & other is $\frac{c}{a}$

a > b > c & a + b + c = 0

a & c must have opposite sign $\frac{c}{a} < 0$

4. **A, D**

GIven, $(3\sqrt{3} + 5)^{2n+1} = \alpha + \beta$ ....(i)

$\because$ $0 < \beta < 1$

Let $\beta' = (3\sqrt{3} - 5)^{2n+1}$ ....(ii)

$\therefore$ $0 < \beta' < 1$

From Eqs. (i) and (ii)

$$\alpha + \beta - \beta' = (3\sqrt{3} + 5)^{2n+1} - (3\sqrt{3} - 5)^{2n+1}$$

$$\Rightarrow \quad = 2[{}^{2n+1}C_1 (3\sqrt{3})^{2n} 5 + {}^{2n+1}C_3 (3\sqrt{3})^{2n-2} (5)^3 + ... + {}^{2n+1}C_{2n+1} 5^{2n+1}]$$

$\alpha + \beta - \beta' = 10I$

But $-1 < \beta - \beta' < 1$

$\therefore$ $\beta - \beta'$ is an integer

$\therefore$ $\beta - \beta' = 0$

$\therefore$ $\alpha = 10\,I$

$\therefore$ α divisible by 10

$\Rightarrow$ α is an even integer.

$$\Rightarrow \quad (\alpha + \beta)^2 = [(3\sqrt{3} + 5)^2]^{2n+1}$$

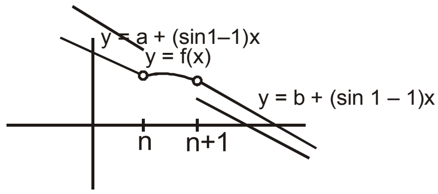
$$= (52 + 30\sqrt{3})^{2n+1}$$

$$= 2^{2n+1}(26 + 15\sqrt{3})^{2n+1}$$

$\therefore$ $(\alpha + \beta)^2$ is not divisible by 2^{2n+1}.

5. **B,D**

$\underset{x \to 0^+}{\text{Lt}}$



Clearly f(x) is periodic with period 1,

it is sufficient to consider f(x) in (0, 1)

$$f(x) = 1 : \underset{x \to 1^-}{\text{Lt}} f(x) = \sin 1$$

Also $f(x) \downarrow \Rightarrow f(x) < 1$ hence least value of M = 1

$$\underset{x \to n^+}{\text{Lt}} f(x) = 1 : \underset{x \to n^+}{\text{Lt}} h(x) = a + (\sin 1 - 1)\, n$$

$$\underset{x \to n+1^-}{\text{Lt}} f(x) = \frac{\sin 1}{1} : \underset{n \to n+1^+}{\text{Lt}} h(x) = b + (\sin 1 - 1)(n+1)$$

From graph it is clear that $b \le n + 1 - n \sin 1$

and $a \ge n + 1 - n \sin 1$

Section III

1. **(A) → (Q), (B) → (P), (C) → (S), (D) → (R)**

(A) Equation of the circle is $2x^2 + 2y^2 - 2\sqrt{2}\,x - y = 0$

Let $(\alpha, 0)$ be mid point of a chord. Then equation of the chord is

$$2\alpha x - \sqrt{2}(x+\alpha) - \frac{1}{2}(y+0) = 2\alpha^2 - 2\sqrt{2}\,\alpha$$

Since it passes through the point $\left(\sqrt{2}, \frac{1}{2}\right)$

$\therefore 2\sqrt{2}\,\alpha - \sqrt{2}\,(\sqrt{2} + \alpha) - \frac{1}{4} = 2\alpha^2 - 2\sqrt{2}\,\alpha$

i.e. $8\alpha^2 - 12\sqrt{2}\,\alpha + 9 = 0$ i.e. $(2\sqrt{2}\,\alpha - 3)^2 = 0$

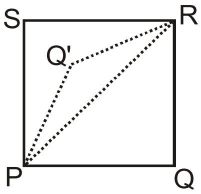
i.e. $\alpha = \frac{3}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}$ $\therefore$ number of chords is 1

(B) Let y = x + h, where $h \geq 1$ and z = x + h + k, where $k \geq 1$.
Then x + y + z = 20
$\Rightarrow x + x + h + x + h + k = 20 \Rightarrow 3x + 2y + k = 20$
when x = 1, then there are 8 ways ;
when x = 2, then there are 6 ways ;
when x = 3, then there are 5 ways ;
when x = 4, then there are 3 ways
when x = 4, then there are 2 ways ;
x > 5 is not possible
$\therefore$ total number of ways = 24

(C) $$\lim_{x\to 1} \int_1^{x^2} \frac{f(t)-t}{(x-1)^2}\,dt = \lim_{x\to 1} \frac{\int_1^{x^2} (f(t)-t)\,dt}{(x-1)^2}$$

$$= \lim_{x\to 1} \frac{2x(f(x^2)-x^2)}{2(x-1)} = \lim_{x\to 1} \frac{f(x^2)-x^2+2x^2f'(x^2)-2x^2}{1} = 4$$

(D)



taking P as origin position vector of Q, R and S

are

equation of PQ' and RS are $\vec{r} = t(\hat{i} + \hat{j} + \sqrt{2}\hat{k}), \vec{r} = P\hat{i} + P\hat{j} + \lambda\hat{i}$

$\therefore$ shortest distance = $\therefore$ k = 2

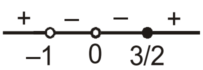
2. (A) → (R), (B) → (S), (C) → (Q), (D) → (P)

(A) $\log_{\sin x}(\log_3(\log_{0.2} x)) < 0 = \log_{\sin x} 1$

$\Rightarrow \log_3(\log_{0.2} x) > 1 \Rightarrow \log_{0.2} x > 3 = \log_{0.2}(0.2)^3$

$\Rightarrow 0 < x < (0.2)^3 \Rightarrow 0 < x < \frac{1}{125}$

(B) $\frac{(e^x-1)(2x-3)(x^2+x+2)}{(\sin x - 2)x(x+1)} \leq 0$



$\Rightarrow$ ≥ 0 $\Rightarrow x < -1$ or $x \geq \frac{3}{2}$

$\Rightarrow x \in (-\infty, -1) \cup \left[\frac{3}{2}, \infty\right)$

(C) $|2 - |[x] - 1|| \leq 2$

$\Rightarrow ||[x] - 1| - 2| \leq 2 \Rightarrow 0 \leq |[x] - 1| \leq 4$

$\Rightarrow -3 \leq [x] \leq 5 \Rightarrow x \in [-3, 6)$

(D) $|\sin^{-1}(3x - 4x^3)| \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \leq \sin^{-1}(3x - 4x^3) \leq \frac{\pi}{2}$

$\Rightarrow -1 \leq 3x - 4x^3 \leq 1 \Rightarrow -1 \leq x \leq 1$

Section IV

1. Ans. 10

The given circle

$x^2 + y^2 = 1$ is ....(1)

with centre at O (0, 0) and radius 1. It cuts x-axis at the points when y = 0 then x = ± 1

i.e., at P (–1, 0) and Q(1, 0)

Equation of circle with centre at Q (1, 0) and radius r is

$(x-1)^2 + (y-0)^2 = r^2$ ....(2) (0 < r < 2)

Solving (1) & (2), we get

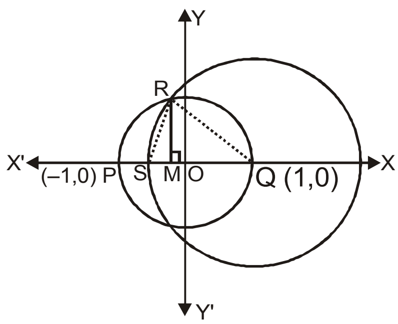
$x = \frac{2-r^2}{2}$ and $y = \pm \frac{r\sqrt{(4-r^2)}}{2}$

but R above the x-axis,

$\therefore R \equiv \left(\frac{2-r^2}{2}, \frac{r\sqrt{4-r^2}}{2}\right)$

So, SQ = r and MR = $\frac{r\sqrt{4-r^2}}{2}$

$\frac{2P\hat{i} + (R\hat{j}, P\hat{j}/2)}{\sqrt{6}\ \ x(x+1)}$ Area of ΔQSR = $\frac{1}{2}$. QS . MR



$\Delta = \frac{1}{2} \cdot r \cdot \frac{r\sqrt{4-r^2}}{2}$, Δ (say) = $\frac{r^2\sqrt{(4-r^2)}}{4}$

$\Delta^2 = \frac{r^4}{16}(4 - r^2) = A$ (say)

$\frac{dA}{dr} = 0 \Rightarrow r = \sqrt{\frac{8}{3}} \Rightarrow \left(\frac{d^2A}{dr^2}\right) < 0$ at $r = \sqrt{\frac{8}{3}}$

Max A $\Rightarrow$ max $\Delta = \frac{1}{4} \times \frac{8}{3}\sqrt{4 - \frac{8}{3}} = \frac{4}{3\sqrt{3}}$

$\Rightarrow 4 + 3 + 3 = 10$

2. **Ans.10** Let $p(x) = ax^2 + bx + c$

given $p(1) = 0 \Rightarrow a + b + c = 0$ ....(i)

$P(2) = 2 \Rightarrow 4a + 2b + c = 2$ ...(ii)

Now, given $p(x) \geq 0$, so $d \leq 0$ and $a > 0$

$b^2 - 4ac \leq 0$

$(a + c)^2 - 4ac \leq 0$ $(\because b = -(a + c))$

$(a - c)^2 \leq 0$

$a = c$ ...(iii)

putting (iii) in (i) and (ii), we get

$a = 2, c = 2$ and $b = -4$

$\Rightarrow P(0) + p(3) = 10$

3. **Ans. 8**

$\cos 2\theta = \frac{1}{3} \Rightarrow \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1}{3} \Rightarrow 3 - 3\tan^2\theta$

$= 1 + \tan^2\theta \Rightarrow \tan^2\theta = \frac{1}{2}$

Now $2\cos^2\alpha - 3\cos\alpha = 32.\ \frac{1}{2^4} = 2$

$\Rightarrow 2\cos^2\alpha - 4\cos\alpha + \cos\alpha - 2 = 0$

$\Rightarrow \cos\alpha = -\frac{1}{2}$

$\frac{2\pi}{3} + \frac{4\pi}{3} + \frac{8\pi}{3} + \frac{10\pi}{3} = \frac{24\pi}{3} = 8\pi \Rightarrow K = 8$ **Ans.**

4. **Ans. 16**

Let $Y = y - 4$ and $X = x + 3$

$Y^2 = 15X$

Let $P(t_1)$, $Q(t_2)$ and $R(t_3)$

$\because \quad t_3 = -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$

$\therefore \quad t_1t_2 = 2$ ....(i)

$t_1 + t_2 + t_3 = 0$ ....(ii)

let (h, k) be centroid under new co-ordinate system

$h = \frac{a(t_1^2 + t_2^2 + t_3^2)}{3} = \frac{15}{12}(t_1^2 + t_2^2 + (t_1 + t_2)^2)$

$= \frac{15}{6}(t_1^2 + t_2^2 + 2)$

$= \frac{15}{6}(t_1^2 + \frac{4}{t_1^2} + 2) \qquad \because \quad t_1^2 + \frac{4}{t_1^2} \geq 4$

but equality is not taken

$\therefore \quad h > \frac{15}{6}(4 + 2)$

$h > 15$ and $k = 2a(t_1 + t_2 + t_3) = 0$

$\alpha + 3 = h > 15 \qquad \alpha > 12$

$\beta - 4 = k = 0$

$\beta = 4$

$\alpha + \beta > 16$

largest value of m = 16

5. **Ans. 6**

$[x]\,[y] = [x] + [y] + \{x\} + \{y\}$

let $x = m + \alpha$ and $y = n + \beta$ where $0 < \alpha, \beta < 1$

$mn = m + n + \alpha + \beta \qquad \alpha + \beta = 1$

$mn = m + n + 1$

$m = \frac{n+1}{n-1} \qquad n = 0, 2, 3$

(a) If $n = 0, m = -1$

$x + y = m + n + \alpha + \beta = 0$

(b) If $n = 2, m = 3$

$x + y = m + n + \alpha + \beta = 6$

(c) If $n = 3, m = 2$

$\therefore\ x + y = 6$

$\therefore$ Non integral solution lie either on $x + y = 0$

or on $x + y - 6 = 0$.

6. **Ans. 16,** For the points of intersection, we have

$$\frac{12 - y^2}{36} + \frac{y^2}{4} = 1$$

$\Rightarrow \qquad y = \pm\sqrt{3}$ and $x = 3$

Consider the point P $(3, \sqrt{3})$

Equation of the tangent at P to the circle is $3x + \sqrt{3}\,y = 12$

$\therefore$ slope of this tangent is $-\sqrt{3}$

Equationof the tangent at P to the ellipse is $\frac{x}{12} + \sqrt{\alpha^2 + a^2}\ \frac{\sqrt{3}}{4}\ y = 1$

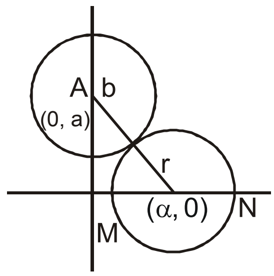
$\therefore$ slope of this tangent is $-\frac{1}{3\sqrt{3}}$

If α is angle between these tangents, then

$\tan\alpha = \frac{2}{\sqrt{3}}$

$\therefore \quad \alpha = \tan^{-1}\frac{2}{\sqrt{3}} \qquad \therefore k = 4$ and hence $k^2 = 16$

7. **Ans. 2**



Let radius = r

$\therefore$ from figure $\qquad = b + r$ ....(i)

Consider a point P (0, k) on the y-axis
M(α – r, 0) and N (α + r, 0)

Now, slope of MP = $\frac{-k}{\alpha - r}$, slope of NP = $\frac{-k}{\alpha + r}$

If ∠MPN = θ

$$\tan\theta = \left|\frac{\frac{k}{\alpha - r} - \frac{k}{\alpha + r}}{1 + \frac{k^2}{\alpha^2 - r^2}}\right| = \left|\frac{2kr}{\alpha^2 - r^2 + k^2}\right|$$

According to the given condition, θ is a constant for any choice α.

$$\frac{2kr}{\alpha^2 - r^2 + k^2} = \text{constant}$$

i.e. $$\frac{r}{\alpha^2 - r^2 + k^2} = \text{constant}$$

i.e. $$\frac{\sqrt{\alpha^2 + a^2} - b}{\alpha^2 - \left(\sqrt{\alpha^2 + a^2} - b\right)^2 + k^2} = \text{constant}$$

(from equation (i))

i.e. $$\frac{\sqrt{\alpha^2 + a^2} - b}{2b - \sqrt{\alpha^2 + a^2} - a^2 - b^2 + k^2}$$

$$\frac{\sqrt{\alpha^2 + a^2} - b}{\sqrt{\alpha^2 + a^2} - \lambda} = \text{constant} \left\{\text{putting } \frac{a^2 + b^2 - k^2}{2b} = \lambda\right\}$$

which is possible only if λ = b

$$\frac{a^2 + b^2 - k^2}{2b} = b \Rightarrow k = \pm\sqrt{a^2 - b^2}$$

$$\therefore \quad P \equiv \left(0, \pm\sqrt{a^2 - b^2}\right)$$

8. **Ans. 3** As SP × S'P' = b^2

$$\Rightarrow \sum_{i=1}^{10} (SP_i) \times (S'P'_i) = 2560$$

⇒ 10 × b^2 = 2560 ⇒ b^2 = 256
As a = 20 ⇒ a^2 = 400

From $b^2 = a^2(1 - e^2)$; e = $\frac{3}{5}$ ⇒ 5e = 3

PHYSICS

Section I

1. **C**

For delivering maximum power, the external resistance connected to a cell must be equal to its internal resistance. Observing that the points marked B are short-circuited, the equivalent (external) resistance between A and B is 4 Ω, 4 Ω, R is parallel

i.e. $\frac{2R}{2 + R} = 2$

(Since it is also equal to the internal resistance)

∴ R = ∞

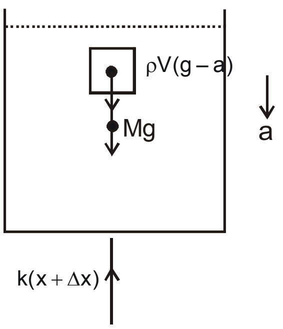
2. **A** **3.** **B**

4. **D**

Section II

1. **B,C** **2.** **A,C**

3. **B**



Mg – k (x + Δx) + ρ (V) (g – a) = Ma
where Mg + ρ Vg = k Δx
⇒ (M + ρV) a = – Kx

or $a = -\frac{kx}{(M + \rho V)}$ or $\omega = \sqrt{\frac{k}{M + \rho V}}$

⇒ $T = 2\pi\sqrt{\frac{(M + \rho V)}{k}}$

The amplitude of SHM will be x_0.

4. **B,C** **5.** **A,C**

Section III

1. **(A) → P,S ; (B) → R,S ; (C) → R,S ; (D) → P, S**

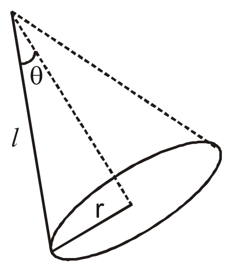
2. **(A) → Q,S ; (B) → P ; (C) → P,R ; (D) → P, R**

Section IV

1. It is conical pendulum in inclined plane

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g_{eff}}}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi l \sin\theta}{2\pi\sqrt{\frac{l\cos\theta}{g_{eff}}}}$$



$$v = \sin\theta\sqrt{\frac{lg_{eff}}{\cos\theta}}$$

$$\sin\theta = \frac{qE}{mg_{eff}} \qquad \cos\theta = \frac{q}{g_{eff}}$$

So $\quad v = \frac{qE}{m}\sqrt{\frac{l}{g}}$

2. $\frac{q_1}{C_1} = \frac{q_2}{C_2}; \quad q_1 + q_2 = 2Q_0$

$$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}; C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$$

$$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$$

$$q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$$

$$q_2\left(\frac{2d_0}{d_0 + vt}\right) = 2Q_0 \Rightarrow \quad q_2 = \frac{2Q_0}{2d_0}(d_0 + vt)$$

$$I = \frac{dq_2}{dt} = \frac{Q_0 v}{d_0} = 8\,\text{amp}$$

3. When switch is closed, then potential difference between the sphere should be V. Let q charge flows from sphere of radius R

Then $\frac{K\left(\frac{Q}{2}+q\right)}{2R} - \frac{K(Q-q)}{R} = V$

or $\frac{Q}{2} + q - 2Q + 2q = \frac{2RV}{K}$

or $3q = \frac{2RV}{K} + \frac{3Q}{2} \qquad$ or $q = \frac{8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$

So final charge on the sphere of radius R is

$Q_1 = Q - q = \frac{-8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$

and $Q_2 = \frac{Q}{2} + q = Q + \frac{8\pi\varepsilon_0 RV}{3}$

4. Power of equivalent mirror $P'_M = 2P_2 + P_M$

$$\frac{1}{f_2} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{1}{8} \times \frac{2}{R} = \frac{1}{4R}$$

$f_2 = 4R$ and $P_2 = \frac{1}{4R}$

$$-\frac{1}{f'_m} = 2 \times \frac{1}{4R} + \frac{2}{R} = \frac{5}{2R}$$

$$f'_m = -\frac{2}{5}R$$

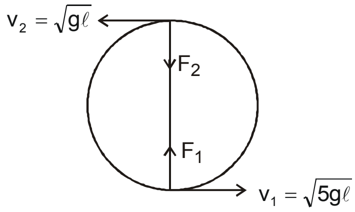
$|h| = 2\,|f'_m|$

$h = 2 \times \frac{2}{5}R = 4m$

$R = 5$ m

5. $\frac{1}{2}$

6. **5**

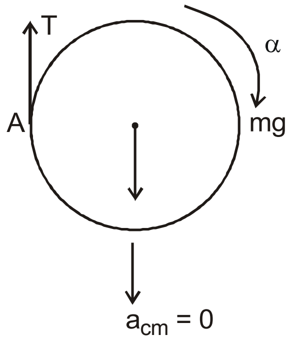


$F_1 = \frac{mv_1^2}{e}$

$F_2 = \frac{mv_2^2}{e}$

$\therefore \frac{F_1}{F_2} = 5$

7. **2**



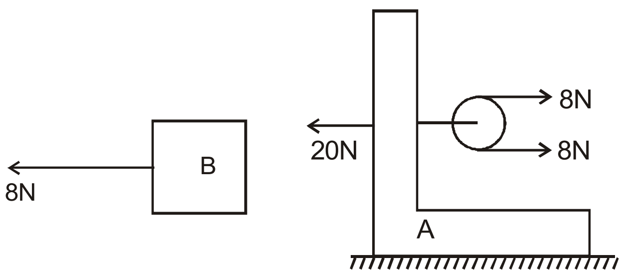
$a_A = a = \alpha.\,R$ ...(i)

$T - mg = 0$ ...(ii)

$T.R = \frac{mR^2}{2}.\alpha$ ...(iii)

$\therefore g = \frac{a}{2}$

8. A 1 kg block 'B' rests as shown on a bracket 'A' ....



$\therefore a_B = \frac{8}{1} = 8$

$\therefore a_A = \frac{20-16}{1} = 4$

$\therefore a_{rel} = 8 - 4 = 4$

Section I

1. A

+ M of – OH > + M of – OCH_3

2. A

For H_α line of Balmer series $n_1 = 2, n_2 = 3$
For H_β line of Balmer series $n_1 = 2, n_2 = 4$

$$\therefore \quad \frac{1}{\lambda_{H_\alpha}} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad ...(1)$$

and $$\frac{1}{\lambda_{H_\beta}} = R_H\left[\frac{1}{2^2} - \frac{1}{4^2}\right] \quad ...(2)$$

By Eqs. (1) and (2)

$$\frac{\lambda_\beta}{\lambda_\alpha} = \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4} - \frac{1}{16}}$$

$$\therefore \lambda_\beta = \lambda_\alpha \times \left[\frac{80}{108}\right] = 6500 \times \frac{80}{108} = 4814.8 \text{ Å}$$

3. D

Faraday of electricity passed

$$= \frac{1.25 \times 1.1 \times 3600}{96500} = 0.0513$$

$\Rightarrow$ meq of $KMnO_4$ required = 513

$\Rightarrow$ $M(KMnO_4) = \frac{51.3}{25} \times \frac{1}{5} = 0.41$ M

4. B

Example of pinacol - pinacolone rearrangement.

Section II

1. A, B, D

2. B, D

Number of particle from $K_4[Fe(CN)_6] = 5$
Number of particle from $FeSO_4(NH_4)_2SO_4.6H_2O = 5$
Number of particle from $KCl.\ MgCl_2.8H_2O = 5$

3. B, C, D

$N_2O_4(g) \rightleftharpoons 2NO_2(g)$

t = 0	a	0
t	$a(1-\alpha)$	$2a\alpha$

vapour density = $\frac{46}{1+\alpha} = 30.67$

so $1 + \alpha = 1.5 = 0.5 = 50\%$

Total pressure = $\frac{1.5 \times 1.5 \times 0.082 \times 300}{8.2} = 6.75$ atm

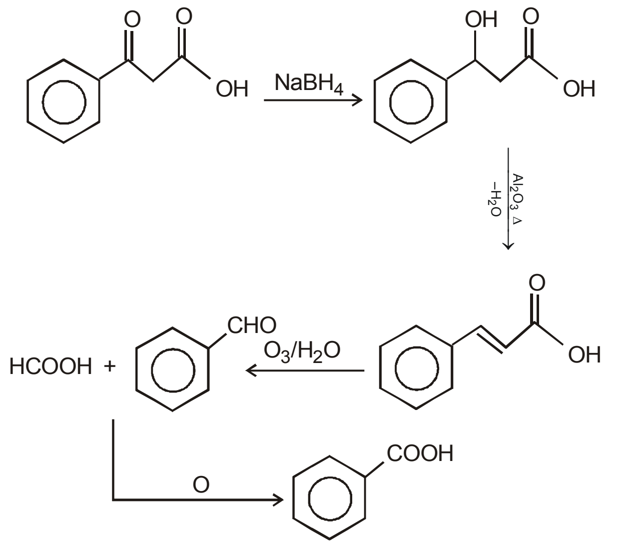
so $K_p = \frac{4\alpha^2}{1-\alpha^2}$ P = 9 atm

$\Rightarrow$ and for density of mixture = $\frac{138}{8.2}$ gm/L = 16.83 gm/L

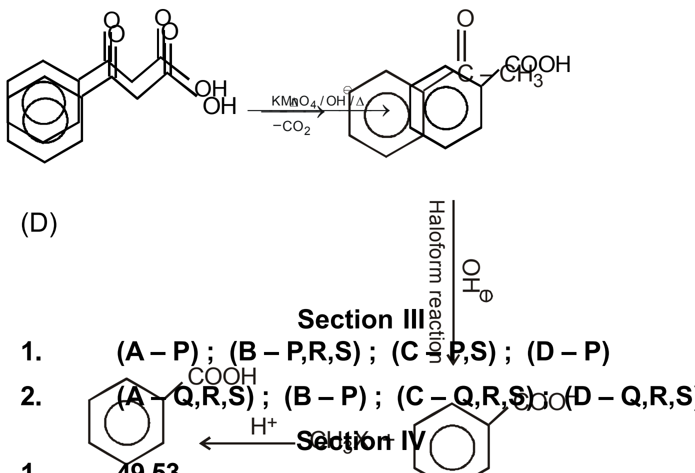
4. A,B

5. B,C,D

(B)



(C)



(D)

Section III

1. (A – P) ; (B – P,R,S) ; (C – P,S) ; (D – P)

2. (A – Q,R,S) ; (B – P) ; (C – Q,R,S) ; (D – Q,R,S)

Section IV

1. 49.53

	$N^{3+} \rightarrow N^{5+} + 2e$;	E° = – 0.94 V
	$2N^{3+} + 2e \rightarrow 2N^{2+}$	E° = 0.99 V
or	$3N^{3+} \rightarrow 2N^{2+} + N^{5+}$	

$$\therefore \quad E^0_{cell} = E^0_{OP_{HNO_2/NO_3^-}} + E^0_{RP_{HNO_2/NO}}$$

$$E^0_{cell} = -0.94 + 0.99 = 0.05$$

Also $$E^0_{cell} = \frac{0.059}{2}\log K_c$$

$$0.05 = \frac{0.059}{2}\log K_c \qquad \therefore \qquad K_c = 49.53$$

2. **2.50×10^{-3}**

$A \longrightarrow B$

$0.01 \qquad 0$

$0.01 - x_{eq} \qquad x_{eq}$

$$\frac{[B]_{eq}}{[A]_{eq}} = \frac{10^{-2}}{K_b} = 4 = \frac{[x]_{eq}}{0.01 - [x]_{eq}}$$

$\therefore K_b = 0.25 \times 10^{-2}$ and $x_{eq} = \frac{0.04}{5} = 0.008$

$$t = \frac{2.303}{(K_f + K_b)} \log \frac{[x]_{eq}}{[x]_{eq} - x}$$

$$30 = \frac{2.303}{1.25 \times 10^{-2}} \log \frac{0.008}{(0.008 - x)}$$

$\therefore \frac{0.008}{0.008 - x} = 1.455 \qquad \therefore x = 2.50 \times 10^{-3}$

3. **1**

Given, $MBr_2 \rightleftharpoons MBr_2(aq.) \rightarrow M^{2+} + 2Br^-$

$MBr_2 + H_2S \longrightarrow MS + 2HBr$

K_{sp} of $MS = [M^{2+}][S^{2-}]$

$6 \times 10^{-21} = [0.05][S^{2-}]$

$\therefore \quad [S^{2-}] = 1.2 \times 10^{-19}$ M

Thus, MS will be precipitated if H_2S provides 1.2×10^{-19} M ions of S^{2-}.

Now for $H_2S \qquad H_2S \rightleftharpoons 2H^+ + S^{2-}$

$$K_1 \times K_2 = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$10^{-7} \times 1.2 \times 10^{-13} = \frac{[H^+]^2[1.2 \times 10^{-19}]}{[0.1]}$$

$\therefore \quad [H^+] = 10^{-1}$

and $\quad pH = 1$

4. **4157**

ΔH required for heating = $\underset{(boiler)}{mS\Delta T} + \underset{(water)}{mS\Delta T}$

= 0.11 × 900 × 90 + 1 × 400 × 90 = 44910 kcal

Since, only 70% of heat given is used upto do so.

Thus, Actual heat required = $\frac{44910 \times 100}{70}$ = 64157 kcal

5. **3.87 BM**

Complex $Cr(NH_3)_4ClBr$ has two isomers. Since co-ordination number of Cr is six and thus two forms may be

$[Cr(NH_3)_4Cl_2]Br \xrightarrow{AgNO_3} [Cr(NH_3)_4Cl_2]^+ + NO_3^-$

[B] $\qquad$ + AgBr

yellow ppt. partially soluble in conc. NH_3 i.e., $[Ag(NH_3)_2]Br$

$[Cr(NH_3)_4Br.Cl]Cl \xrightarrow{AgNO_3} [Cr(NH_3)_4ClBr]^+ + NO_3^-$

[A] $\qquad$ + AgCl

white ppt soluble in dil. NH_3 i.e., $[Ag(NH_3)_2Cl]$

Hybridisation of Cr in (A) and (B) is d^2sp^3 having 3 unpaired electrons ($3d^3$)

Magnetic moment = $\sqrt{[n(n+2)]}$ B.M.

$= \sqrt{[3(3+2)]} = 3.87$ B.M.

6. **– 48.13 kJ**

$C_2H_5OH(g) \rightarrow C_2H_4(g) + H_2O(g)$;

$\Delta H° = -45.54$ kJ ....(i)

$C_2H_5OH(g) \rightarrow CH_3CHO(g) + H_2(g)$;

$\Delta H° = -68.91$ kJ ....(ii)

Let a mole of ethanol be decomposed by Eq. (i) and b mole be decomposed by Eq (ii)

$\therefore \quad a + b = 1$ ...(iii)

Also the ratio of C_2H_4 and CH_3CHO formed is a/b

$\therefore \quad \frac{a}{b} = 8$ ...(iv)

By Eq. (iii) and (iv)

$\therefore \quad 8b + b = 1$

$b = \frac{1}{9}$ and $a = \frac{8}{9}$

$$\therefore \quad \Delta H^0_{Total} = -\left[45.54 \times \frac{8}{9} + 68.91 \times \frac{1}{9}\right] = -48.13 \text{ kJ}$$

7. In a unit cell, there are one Cs^+ and $\frac{1}{8} \times 8 = 1\ Cl^-$, i.e., one CsCl molecule

$$\therefore \quad \text{Density} = \frac{n \times \text{mol.wt.}}{V \times \text{Av.no.}} = \frac{n \times \text{mol.wt.}}{a^3 \times \text{Av.no.}}$$

$$\therefore \quad 3.97 = \frac{1 \times 168.36}{a^3 \times 6.023 \times 10^{23}}$$

$a = 4.13 \times 10^{-8}$ cm

$\therefore \quad a = 4.13$ Å

For a cube of side length 4.13 Å, diagonal

$= \sqrt{3} \times 4.13 = 7.15$ Å

As it is a b.c.c. with Cs^+ at centre (radius r^+) and Cl^- at corners (radius r^-) so,

$2r^+ + 2r^- = 7.15$ or $r^+ + r^- = 3.57$ Å

i.e., Distance between neighbouring Cs^+ and Cl^- = 3.57 Å

8. **50%**

$2AB_{3[g]} \rightleftharpoons N_2 + 3H_2$

$a \qquad 0 \qquad 0$

$a - 2x \qquad x \qquad 3x$

$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{15}{300} = \frac{P_2}{400} \Rightarrow P_2 = 20$ atm

Now $\frac{a + 2x}{a} = \frac{20}{20} \Rightarrow 2a + 4x = 3a \Rightarrow x = \frac{1}{4}a$

$\therefore$ % of $AB_{3[g]}$ decomposed = $\frac{2x}{a} \times 100 = 50\%$