MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT-JEE

HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-I CLASS XII & XIII (DATE 26-01-10)

MATHEMATICS

Section I

1. B **2.** B **3.** C **4.** B **5.** D **6.** D **7.** B **8.** A

Section II

1. A, C, D **2.** C **3.** B, C, D **4.** A,B

Section III

1. D **2.** D **3.** B **4.** D **5.** D **6.** D

Section IV

1. (A) $\rightarrow$ P; (B) $\rightarrow$ (P,Q,R) ; (C) $\rightarrow$ P,Q,R,S ; (D) $\rightarrow$ P **2.** (A) $\rightarrow$ (R), (B) $\rightarrow$ (S), (C) $\rightarrow$ (P), (D) $\rightarrow$ (Q)

PHYSICS

Section I

1. B **2.** C **3.** B **4.** C **5.** C **6.** B **7.** **8.** B

Section II

1. A **2.** B,D **3.** A,C **4.** A,C,D

Section III

1. A **2.** C **3.** A **4.** A **5.** A **6.** C

Section IV

1. (A) $\rightarrow$ P,Q,R,S,T ; (B) $\rightarrow$ P,R,S, ; (C) $\rightarrow$ P,Q ; (D) $\rightarrow$ Q ; **2.** (A) $\rightarrow$ Q,S ; (B) $\rightarrow$ P,R ; (C) $\rightarrow$ P, R ; (D) $\rightarrow$ (Q,S)

CHEMISTRY

Section I

1. B **2.** A **3.** A **4.** B **5.** A **6.** C **7.** C **8.** D

Section II

1. A, B, D **2.** B,D **3.** A,B,D **4.** A,C,D

Section III

1. B **2.** C **3.** A **4.** B **5.** A **6.** B

Section IV

1. (A) $\rightarrow$ P,Q,S ; (B) $\rightarrow$ P ; (C) $\rightarrow$ P,Q,S; (D) $\rightarrow$ P,Q,S **2.** (A) $\rightarrow$ P,; (B) $\rightarrow$ S ; (C) $\rightarrow$ R,T ; (D) $\rightarrow$

SOLUTIONS

MATHEMATICS

Section I

1. **B**

Let the co-ordinate of C is (x_1, y_1) and let the co-ordinates of A & B are (0, 0) and (a, 0)

given $k = \frac{\sin A}{\sin B} = \frac{BC}{AC}$

$\Rightarrow$ $(BC)^2 = k^2 (AC)^2$

$\Rightarrow$ $(x_1 - a)^2 + y_1^2 = k^2(x_1^2 + y_1^2)$

$\Rightarrow$ $(1-k^2)\, x_1^2 + (1-k^2)\, y_1^2 - 2ax_1 + a^2 = 0$

$(\because\ k \neq 1)$

$\Rightarrow$ $x_1^2 + y_1^2 - \frac{2ax_1}{1-k^2} + \frac{a^2}{1-k^2} = 0$

Hence locus of C is

$x^2 + y^2 - \frac{2ax}{1-k^2} + \frac{a^2}{1-k^2} = 0$. This is a circle.

2. **B**

The centres of the given circles are $(-\lambda_i, 0)$ (i = 1, 2, 3). The distances from the origin of the centres are λ_i (i = 1, 2, 3). It is given that $\lambda_2^2 = \lambda_1 \lambda_3$. Let P (h, k) be any point on the circle $x^2 + y^2 = c^2$. Then $h^2 + k^2 = c^2$

Now, L_i = length of the tangent from (h, k) to $x^2 + y^2 + 2\lambda_i x - c^2 = 0$

$= \sqrt{h^2 + k^2 + 2\lambda_i h - c^2} = \sqrt{c^2 + 2\lambda_i h - c^2}$

$[\because\ h^2 + k^2 = c^2]$

$= \sqrt{2\lambda_i h}$, i = 1, 2, 3.

Therefore, $L_2^2 = 2\lambda_2 h = 2h\left(\sqrt{\lambda_1 \lambda_3}\right)$ $[\because\ \lambda_2^2 = \lambda_1\lambda_3]$

$= \sqrt{2\lambda_1 h}\,\sqrt{2\lambda_3 h} = L_1 L_3$

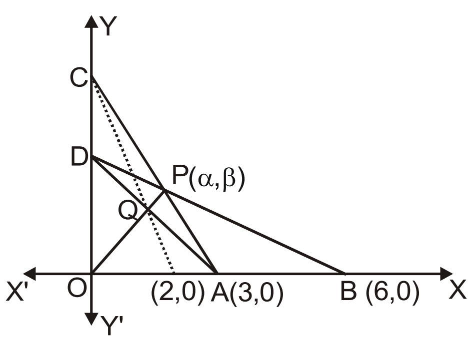
Hence, L_1, L_2, L_3 are in G.P.

3. **C**

Numbr of quadrilaterals = (total number of quadrilaterals) – (choosing 3 points from m and 1 from remaining points) – (choosing 4 points from m points)

$= {}^nC_4 - {}^mC_3\,({}^{n-m}C_1) - {}^mC_4$

4. **B**



The equation of BP is

$$y - \beta = \frac{0-\beta}{6-\alpha}(x - \alpha)$$

so that the coordinates of D are $\left(0, \frac{6\beta}{6-\alpha}\right)$

Similarly the coordinates of C are $\left(0, \frac{3\beta}{3-\alpha}\right)$

Now, the equation of AD is

$\frac{x}{3} + \frac{(6-\alpha)}{6\beta} y = 1$ ....(1)

and the equation of OP is $\beta x = \alpha y$ ....(2)

Solving (1) and (2), we get

$$x = \frac{6\alpha}{6+\alpha},\ y = \frac{6\beta}{6+\alpha}$$

Hence coordinates of Q are $\left(\frac{6\alpha}{6+\alpha}, \frac{6\beta}{6+\alpha}\right)$

Then the equation of CQ is

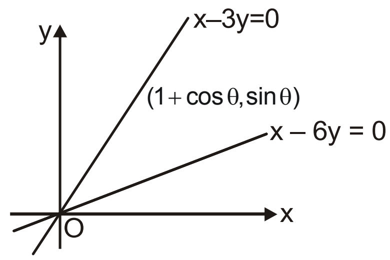
$$y - \frac{3\beta}{3-\alpha} = \frac{\frac{6\beta}{6+\alpha} - \frac{3\beta}{3-\alpha}}{\frac{6\alpha}{6+\alpha} - 0}(x - 0)$$

$\Rightarrow$ $y - \frac{3\beta}{3-\alpha} = \frac{-9\alpha\beta}{6\alpha(3-\alpha)} x$

$\Rightarrow$ $y = \frac{3\beta}{(3-\alpha)}\left(1 - \frac{x}{2}\right)$

which pass through the point (2, 0) for all values of (α, β).

5. **D**



We have

$\frac{1+\cos\theta - 3\sin\theta}{1+\cos\theta - 6\sin\theta} < 0$

$\therefore$ $\frac{2\cos^2\theta/2 - 6\sin\theta/2\cos\theta/2}{2\cos^2\theta - 12\sin\theta/2\cos\theta/2} < 0$

$\Rightarrow$ $\frac{1 - 3\tan\theta/2}{1 - 6\tan\theta/2} < 0$

$\Rightarrow$ $\frac{\tan\theta/2 - 1/3}{\tan\theta/2 - 1/6} < 0$

$\Rightarrow \frac{1}{6} < \tan\theta/2 < \frac{1}{3}$

$\Rightarrow n\pi + \tan^{-1}\frac{1}{6} < \theta/2 < n\pi + \tan^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow 2n\pi + 2\tan^{-1}\left(\frac{1}{6}\right) < \theta < 2n\pi + 2\tan^{-1}\left(\frac{1}{3}\right)$

6. **D**

Bell A ring after 2, 4, 6, 8, ... 480 min.
Bell B ring after 5, 10, 15, 20, ... 480 min.
Bell C ring after 6, 12, 18, 24, ... 480 min.
Bell D ring after 8, 16, 24, 32, ... 480 min.
Now LCM of 2, 5, 6, 8 = 120
So bells ring simultaneously after 120, 240, 360, 480 min.
No. of times = 4

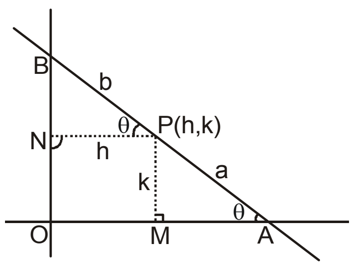
7. **B**

In ΔPMA, $\sin\theta = \frac{k}{a}$ ....(1)

and in ΔPBN, $\cos\theta = \frac{h}{b}$ ....(2)

Squaring and adding (1) and (2) then

$\frac{h^2}{b^2} + \frac{k^2}{a^2} = 1$



$\therefore$ Locus of P is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

which is an ellipse.

8. **A**

$\because \frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^m}{n!}$

$\frac{1}{10!}\left\{\frac{(2)10!}{1!9!} + \frac{(2)10!}{3!7!} + \frac{10!}{5!5!}\right\} = \frac{2^m}{n!}$

$\Rightarrow \frac{1}{10!}\{2\,{}^{10}C_1 + 2\,{}^{10}C_3 + {}^{10}C_5\} = \frac{2^m}{n!}$

$\frac{1}{10!}\{{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\} = \frac{2^m}{n!}$

$\Rightarrow \frac{1}{10!}(2)^{10-1} = \frac{2^m}{n!}$

$\therefore$ m = 9 and n = 10

Hence x – y + 1 = 0 and x + y + 3 = 0 are perpendicular to each other then orthocentre is the point of intersection which is (–2, –1)

$\therefore$ –2 = 2m – 2n

and –1 = m – n

point is (2m – 2n, m – n)

Section II

1. **A, C, D**

Vertices of the given triangle are (0, 0), $\left(\frac{a}{m_1}, a\right)$ and $\left(\frac{a}{m_2}, a\right)$ so that the area of the triangle is equal to $\frac{a^2}{2}\left|\frac{(m_2 - m_1)}{m_1 m_2}\right|$.

Since m_1, m_2 are the roots of $x^2 - ax - a - 1 = 0$, so

$m_1 + m_2 = a$, $m_1 m_2 = -(a + 1)$

$\Rightarrow (m_1 - m_2)^2 = a^2 + 4(a + 1) = (a + 2)^2$

$\Rightarrow |m_1 - m_2| = |(a + 2)|$

So, the required area is $\Delta = \frac{a^2}{2}\left|\frac{a+2}{-(a+1)}\right| = \frac{a^2}{2}\left|\frac{a+2}{a+1}\right|$

Since, the area of Δ is a positive quantity.

$\Delta = \frac{a^2(a+2)}{2(a+1)}$, if a > –1 or a < – 2

and $\Delta = -\frac{a^2(a+2)}{2(a+1)}$, if –2 < a < –1

2. **C**

3. **B, C, D**

If a be the side of the square, then diagonal d = $a\sqrt{2}$. By hypothesis

$a_n = \sqrt{2}\, a_{n+1}$

$\Rightarrow a_{n+1} = \frac{a_n}{\sqrt{2}} = \frac{a_{n-1}}{(\sqrt{2})^2} = \frac{a_{n-2}}{(\sqrt{2})^3} = ... = \frac{a_1}{(\sqrt{2})^n}$

$\therefore a_{n+1} = \frac{a_1}{(\sqrt{2})^n}$

$\Rightarrow a_n = \frac{a_1}{(\sqrt{2})^{n-1}} = \frac{10}{2^{(n-1)/2}}$ (Replacing n by n – 1)

Area of $S_n < 1 \Rightarrow a_n^2 < 1$

$\Rightarrow \frac{100}{2^{n-1}} < 1$

$\Rightarrow 2^n > 200 > 2^7$

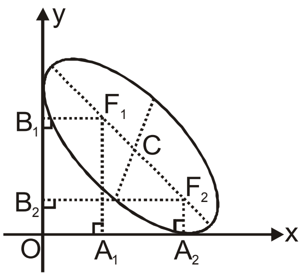
$\Rightarrow n > 7$

$\therefore$ n = 8, 9, 10

4. **A,B**

Section III

1. **D**



The coordinate axes are tangent to the ellipse at any position. So, origin lies on the director circle of the ellipse.

Let C be (α, β). The equation of director circle depends only on the location of centre of ellipse and its semi-major and minor axes, not on their orientation.

Hence, director circle is

$(x - \alpha)^2 + (y - \beta)^2 = a^2 + b^2 = 2^2 + 1^2 = 5$

Since it passes through (0, 0)

$\Rightarrow \alpha^2 + \beta^2 = 5 \quad \Rightarrow$ locus is $x^2 + y^2 = 5$

2. **D**

Let F_1 and F_2 be (h_1, k_1) and (h_2, k_2) respectively.

Distance between foci $F_1F_2 = 2ae$

$\Rightarrow (h_1 - h_2)^2 + (k_1 - k_2)^2 = 2 \times 2 \times \sqrt{1 - \frac{1}{4}} = 2\sqrt{3}$

using the given property $(F_1A_1)(F_2A_2)$

$= (F_1B_1)(F_2B_2) = (1)^2 = 1 \quad \Rightarrow \quad h_1h_2 = k_2k_2 = 1$

$\therefore \left(h_1 - \frac{1}{h_1}\right)^2 + \left(k_1 - \frac{1}{k_1}\right)^2 = 2\sqrt{3}$

$\Rightarrow x^2 + y^2 - \left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 2\sqrt{3} + 4$ is the required locus

3. **B**

You may use the fact that the product of perpendicular distances of any tangent on ellipse from its foci is always equal to square of semi-minor axis.

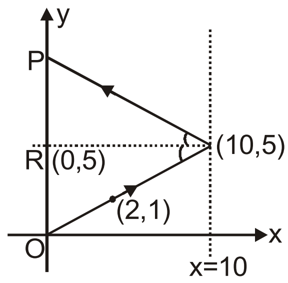
Since the path of centre is $x^2 + y^2 = 5$ when y = 1, x = 2.

The incident ray passes through origin and (2, 1)

$\Rightarrow$ its equation is $y = \frac{1}{2}x$.

Given parabola is $(y - 5)^2 = -4(x - 10)$

whose vertex (10, 5) lies on $y = \frac{1}{2}x$.



The tangent at the vertex is the line x = 10.

Figure shows that OP = 2 (OR) = 10

$\therefore$ reflected ray cuts y-axis at (0, 10)

4. **D**

$$\sqrt{x^2 + y^2} = \left|\frac{x + 1}{1.\cos\frac{\pi}{3} + 0.\sin\frac{\pi}{3}}\right|$$

$$\Rightarrow \sqrt{x^2 + y^2} = \left|\frac{x + 1}{\frac{1}{2} + 0}\right| \Rightarrow \sqrt{x^2 + y^2} = 2\left|\frac{x + 1}{\sqrt{1}}\right|$$

PS = 2 PM $\quad \Rightarrow \quad$ e = 2

$\therefore$ e > 1 hence a hyperbola whose focus S(0, 0) and directrix is x + 1 = 0.

5. **D**

$$\left|\frac{y - 1}{0.\cos\frac{\pi}{6} + 1.\sin\frac{\pi}{6}}\right| = \left|\frac{3x - 4y + 5}{3.\cos\frac{\pi}{6} - 4\sin\frac{\pi}{6}}\right|$$

$$\Rightarrow \quad 2\,|y - 1| = \left|\frac{3x - 4y + 5}{3.\frac{\sqrt{3}}{2} - 4.\frac{1}{2}}\right|$$

$$\Rightarrow \quad \left(3\sqrt{3} - 4\right)(y - 1) = \pm(3x - 4y + 5)$$

6. **D**

$$\sqrt{(x - 1)^2 + (y - 0)^2} = k.\left|\frac{x - 3}{1.\cos\frac{\pi}{6} + 0.\sin\frac{\pi}{6}}\right|$$

$$= \frac{2k}{\sqrt{3}}\frac{|x - 3|}{\sqrt{1^2 + 0^2}}$$

for parabola e = 1 $\qquad \therefore \qquad \frac{2k}{\sqrt{3}} = 1$

i.e. $\qquad k = \frac{\sqrt{3}}{2}$

Section IV

1. (A) → (P) ; (B) → (P, Q, R) ; (C) → (P, Q, R,S) ; (D) → (P)

(A) Required number of ways = (2 + 1) (3 + 1) (4 + 1) – 1 = 59

(B) The number of ways of selecting 3 points out of 12 points is $^{12}C_3$

Three points out of 7 collinear points can be selected in $^{7}C_3$ ways.

Hence, the number of triangles formed is

$$^{12}C_3 - {^7}C_3 = 185$$

(C) $\underbrace{0000000000}_{10}\ \underbrace{\phi\phi\phi}_{3}$

total ways = $^{13}C_3$ = 286

(D) Factorizing the given number, we have

$$38808 = 2^3 . 3^2 . 7^2 . 11$$

The total number of divisors of this number is same as the number of ways of selecting some or all of two 2's, two 3's, two 7's and one 11. Therefore, the total number of divisors

= (3 + 1) (2 + 1) (2 + 1) (1 + 1) = 72

Hence, the required number of divisors = 72 – 2 = 70

2. (A) → (R), (B) → (S), (C) → (P), (D) → (Q)

(A) Point of contact of tangent drawn from (–2, 2) on $y^2 = 4(x + y)$ are (0, 4) and (0, 0)

∴ Area = 4

(B) The conic is a parabola having focus is (2, 3) & Directrix

3x + 4 y – 6 = 0

∴ Latus rectum = 2 (⊥ distance of focus from the directrix)

$$= 2\left(\frac{6+12-6}{45}\right) = \frac{24}{5}$$

(C) $y + 4 = x^2$

$x^2 = 4.\frac{1}{4}(y + 4)$ focal distance = $\frac{25}{4}$

∴ distance from directrix $\left(y = \frac{-17}{4}\right)$

⇒ ordinate of points on the parabola whose focal distance is $\frac{25}{4} = \frac{-17}{4} + \frac{25}{4} = 2$

⇒ Point are $\left(\pm\sqrt{6}, 2\right)$ ⇒ a + b = 2

(D) Length of side = $8\sqrt{3}$ a = $8\sqrt{3}\ \frac{1}{2} = 4\sqrt{3}$

Section I

1. **B**

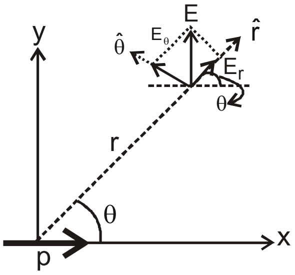
COM gives mv sin 30° = (2m) $v_\perp \Rightarrow v_\perp = v/4$

2. **C**

3. **B**

4. **C**

$E_r = \dfrac{2kp\cos\theta}{r^3}$



$E_\theta = \dfrac{kp\sin\theta}{r^3}$

$E_x = E_r\cos\theta - E_\theta\sin\theta$

$= \dfrac{kp}{r^3}[2\cos^2\theta - \sin^2\theta]$

$E_x = \dfrac{kp}{r^3}[2 - 3\sin^2\theta]$

For the point = (1, $\sqrt{2}$,0) ; sin θ = $\dfrac{\sqrt{2}}{\sqrt{3}}$ Hence $E_x = 0$]

5. **C**

6. **B**

7. **D**

8. **B**

Section II

1. **A** **2.** **B,D**

3. **A,C**

(B) $\dfrac{I_{min}}{I_{max}} = \left(\dfrac{\sqrt{I} - \sqrt{0.36I}}{\sqrt{I} + \sqrt{0.36I}}\right)^2 = \left(\dfrac{0.4}{1.6}\right)^2 = \dfrac{1}{16}$

(C) For maximum at P, path difference = nλ.

If AB is shifted by a distance x, it will cause an additional path difference of 2x.

2x = λ (for minimum value o x)

$\Rightarrow$ x = $\dfrac{\lambda}{2}$ = 300 nm

4. **A,C,D**

Section III

1. **A**

In this situation, ma = qE

2. **C**

$V = \varepsilon L = \dfrac{ma}{q}L \Rightarrow \dfrac{q}{m} = \dfrac{La}{V}$

3. **A**

In this case mg = qE′

In previous case ma = qE

$\dfrac{E'}{E} = \dfrac{g}{a}$, $V' = \dfrac{Vg}{a}$

4. **A**

y-coordinate of the source at any time t is

y = (d/2) sin ωt

5. **A**

The amplitude A of the fringe is given by

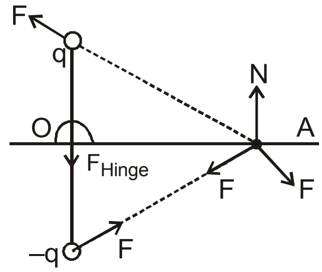
$\dfrac{A}{2d} = \dfrac{d/2}{D} \Rightarrow A = d$

6. **C**

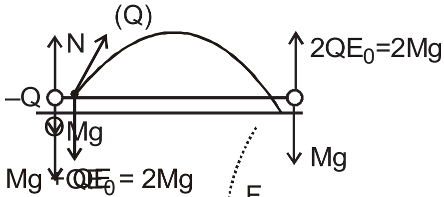
Section IV

1. (A) → P,Q,S, ; (B) → P,R,S, ; (C) → P,Q ; (D) → Q ;

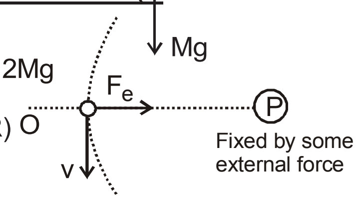
(P)



(Q)

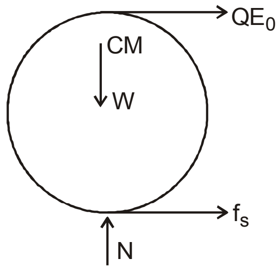


(R)



(S)

(T)



2. **(A) → Q,S ; (B) → P,R ; (C) → P, R ; (D) → (Q,S)**

Section I

1. **B**

In (I) and (III) the 3rd, 4th & 5th carbons are having same configurations hence they will form same osazone

2. **A**

Given $m_A = 2m_B$

$\therefore$ Mol. wt. of A = 2 × mol. wt. of B ....(i)

given u_{rms} of A = 2 × u_{rms} of B ...(ii)

Also number of molecules of A

= number of molecules of B ...(iii)

For gas A $P_A V_A = \frac{1}{3} M_A u^2_{rms\,A}$

For gas B $P_B V_B = \frac{1}{3} M_B u^2_{rms\,B}$

$\therefore \quad \frac{P_A V_A}{P_b V_B} = \frac{M_A}{M_B} \times \frac{u_A^2}{u_B^2}$ ...(iv)

Given $V_A = V_B$ ...(v)

$\therefore$ By equation (i), (ii), (iv) and (v)

$\frac{P_A}{P_B} = 2 \times (2)^2 = 8$

$\therefore \quad P_A = 8P_B$

3. **A**

4. **B**

$2CH_2 = CHCN + 2H^+ + 2e^- \rightarrow CN - (CH_2)_4CN$

$$m = \frac{\left(\frac{108}{2}\right) \times 10^{-3} \times 3000 \times 9.65 \times 3600}{96500} = 58.32 \text{ kg}$$

5. **A**

+M of –OH is > +M of $-OCH_3$

6. **C**

M.e. of $FeSO_4\ 7H_2O$ in 25 ml = m.e. of $KMnO_4$ used = 2 m.e.

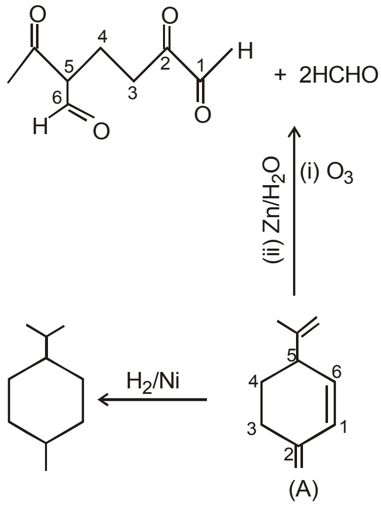
M.e. of $FeSO_4 . 7H_2O$ in 1000 ml = 80 m.e.

mass of $FeSO_4\ 7H_2O$ in solution

$= \frac{80}{1} \times 278 \times \frac{1}{1000} = 22.24$ gm

% of $FeSO_4 . 7H_2O = \frac{22.24}{25} \times 100 = 88.96 \simeq 89\%$

7. **C**



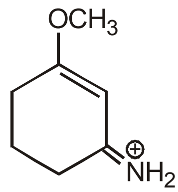
8. **D**

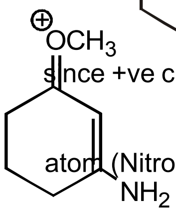
Section II

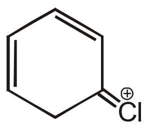
1. **A, B, D**

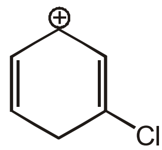
Since density of gold decreases after melting therefore it is favourable at low pressure and high temperature.

2. **B,D**

In (B), 

more stable than 

since +ve charge is located on less electronegative atom (Nitrogen) And in (D), 

is more stable than 

since octet of all atoms is complete.

3. **A,B,D**

4. **A,C,D**

$NH_3 \rightarrow N(g) + 3H(g)\ ;\ \Delta H_1 = x$ kcal mol^{-1}

$N_2H_4 \rightarrow 2N(g) + 4H(g)\ ;\ \Delta H_2 = y$ kcal mol^{-1}

$\Delta H_1 = 3 \times e_{N-H} = x$ ...(1)

$\Delta H_2 = 4 \times e_{N-H} + e_{N-N} = y$ ...(2)

From Eqs. (1) and (2)

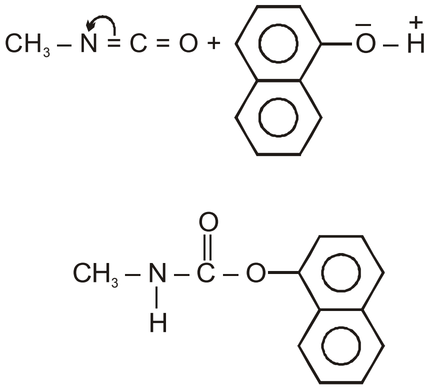
$\therefore \quad y = 4.\frac{x}{3} + e_{N-N}$

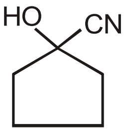
$\therefore \quad e_{N-N} = y - \frac{4x}{3} = \frac{3y-4x}{3}$ kcal mol^{-1}

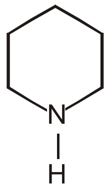
Section III

1. **B**

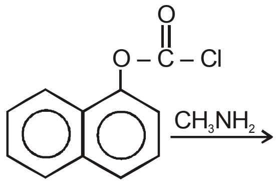
it is a simply an addition reaction

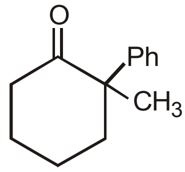


(A) (a)  (b) (c)

(B) 

2. C
3. A

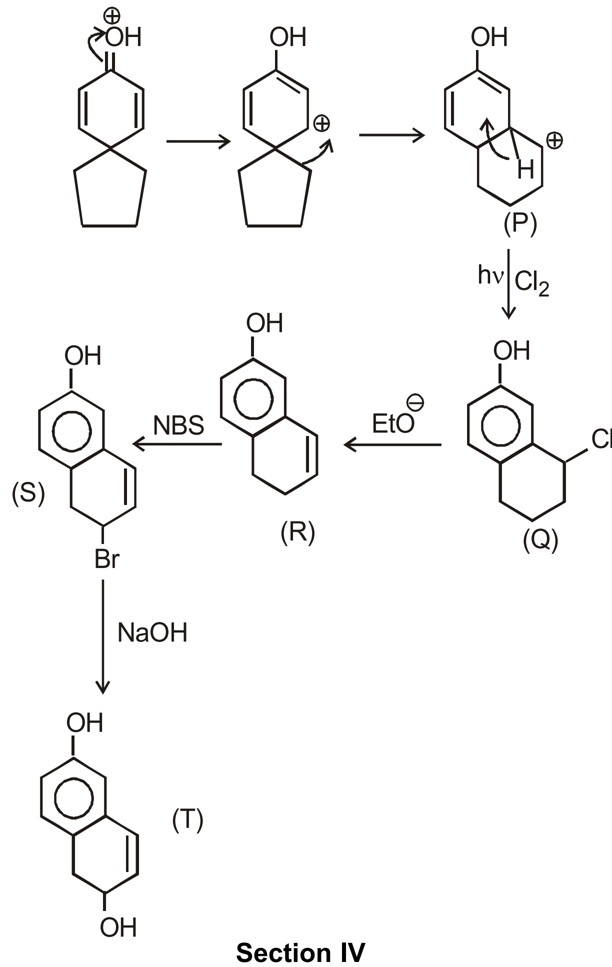


(C) (D) 

(Pinacol-Pinacolone rearrangement)

4. B
5. A
6. B

2. (A) → P ; (B) → S ; (C) → R,T ; (D) → S

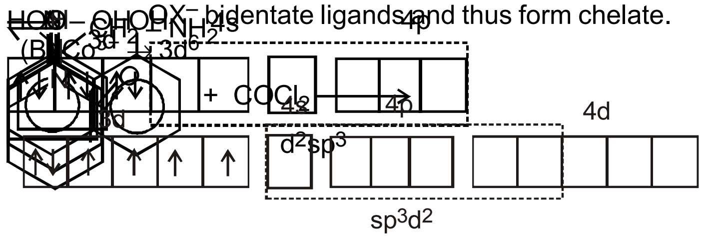


Section IV

1. (A) → P,Q,S ; (B) → P ; (C) → P,Q,S; (D) → P,Q,S

(A) $Co^{3+} \rightarrow$

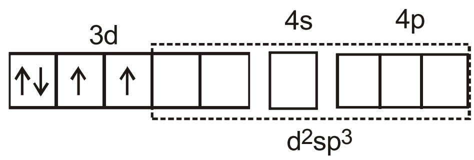
OX^- bidentate ligands and thus form chelate.



F^- weak ligands

(C) NO is NO^+. So oxidation state of Cr is (I) and Cr^+ has d^5 configuration. Hence hybridisation is d^2sp^3 as two empty d-orbitals are available with strong ligands and complex is paramagnetic.

(D) $Co^{3+} \rightarrow 3d^6$ configuration. with strong ligands pairing takes place and thus hybridisation is d^2sp^3 and diamagnetic.

(E) $Mn^{3+} \rightarrow 3d^4$ 

therefore, $\mu = \sqrt{2(2+2)} = 2.84$ B.M.