MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT-JEE

HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-II CLASS XII & XIII (DATE 28-11-09)

MATHEMATICS

1. C **2.** B **3.** A **4.** C **5.** A, B, C, D **6.** A, D **7.** A,B,C,D

8. A,B,D **9.** B

10. (A) → (S), (B) → (R), (C) → (P), (D) → (P)

11. (A) → (S), (B) → (R), (C) → (Q), (D) → (Q)

12. $\frac{\pi A\sqrt{3}}{9}$ sq. units **13.** 16 **14.** 8 **15.** 6 **16.** 0

17. 0 **18.** 0 **19.** 1

PHYSICS

20. D **21.** A **22.** D **23.** D **24.** A,B **25..** A,B,C **26.** A,B

27. B,D **28.** A,B,D **2**

9. (A) → P,R ; (B) → P, R ; (C) → Q,S ; (D) → P, Q, R, S

30.

31. $\theta = \tan^{-1}\frac{\pi H}{2L}$ **32.** $C_{eq} = \frac{\varepsilon_0 A}{d(2^n - 2)}$ **33.** $I = \frac{q}{\sqrt{2}\,a}$ **34.** $\theta' = 4°$ **35.** 30 N

36. 3 **37.** $i_2 = \frac{mg\pi}{\mu_0 i_1}$ **38.** 4

CHEMISTRY

39. B **40.** D **41.** C **42.** B **43.** A, B, C, D **44.** A, D **45.** A, C, D

46. B, C **47.** B, D

48. (A)–P, Q, R, S (B)–P, Q, S (C)–P, S (D)–S

49. (A)–Q, S (B)–R (C)–P (D)–P

50. 50.31% **51.** 64 **52.** 76.83% B and 23.17%C **53.** – 0.7406 volt. **54.** 0.368 **55.** 0.92 M

56. 9.6 **57.** 0.18 M

1. C

Let ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & circle $x^2+y^2=a^2e^2$

radius of circle = ae

point of intersection of circle & ellipse

$x^2+y^2=a^2e^2$

$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=a^2e^2, \left(\frac{a}{e}\sqrt{e^4+e^2-1}, \frac{a}{e}(e^2-1)\right)$

Now area of triangle

$$=\frac{1}{2}\begin{vmatrix}\frac{a}{e}\sqrt{e^4+e^2-1} & \frac{a}{e}(1-e^2) & 1\\ ae & 0 & 1\\ -ae & 0 & 1\end{vmatrix}=\frac{1}{2}\left|\frac{a}{e}(1-e^2)(2ae)\right|=30$$

or $a^2(1-e^2)=30$ $\quad e=\sqrt{1-\frac{30}{a^2}}$ $\quad a=\frac{17}{2}$

then $2ae=2a\sqrt{\frac{a^2-30}{a^2}}=13$

2. B

$\cos(2\sin^{-1}(\tan(\sin^{-1}x)))=\frac{3}{4}$

$\sin^{-1}x=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$

$\tan\sin^{-1}x=\frac{x}{\sqrt{1-x^2}}$ $\quad\therefore \cos\left(2\sin^{-1}\frac{x}{\sqrt{1-x^2}}\right)$

$=1-2\sin^2\left(\sin^{-1}\frac{x}{\sqrt{1-x}}\right)=1-2\frac{x^2}{1-x^2}$

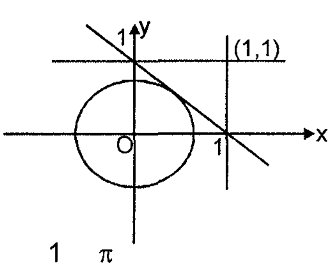
$\therefore \quad \frac{1-3x^2}{1-x^2}=\frac{3}{4}$ $\quad\therefore 4-12x^2=3(1-x^2)$

i.e. $\quad 9x^2=1$ $\qquad x=\pm\frac{1}{3}$

Number of solution = 2

3. A

Sample space $=1-\frac{\pi}{4}\left(\frac{1}{2}\right)^2$ $\quad =1-\frac{\pi}{16}$



$P(A)=\frac{\frac{1}{2}-\frac{\pi}{16}}{1-\frac{\pi}{16}}=\frac{8-\pi}{16-\pi}$

4. C

$\tan^2x-2\tan x+2=5^{\sec^2y}$

$(\tan x-1)^2+1=5^{\sec^2y}$

$\therefore \quad y\neq(2n+1)\frac{\pi}{2}$

Since $\quad \sec^2y\geq 1$

$\therefore \quad (\tan x-1)^2+1\geq 5$

$(\tan x-1-2)(\tan x-1+2)\geq 0$

$\tan x\leq -1$ $\quad$ or $\quad \tan x\geq 3$

$\tan x\leq -1$ $\quad$ or $\quad \tan x\geq 3$

$\therefore \quad \tan x\notin(-1, 3)$

$\therefore \quad x\notin\left(n\pi-\frac{\pi}{4}, n\pi+\tan^{-1}(3)\right)$

5. A, B, C, D

$S_n = \int_0^{\pi/2}\frac{\sin(2n-1)x}{\sin x}dx$ ....(i)

$\therefore S_{n+1}=\int_0^{\pi/2}\frac{\sin(2n+1)}{\sin x}dx$ ....(i)

$\therefore S_{n+1}-S_n=$

$\int_0^{\pi/2}\frac{\sin(2n+1)-\sin(2n-1)x}{\sin x}dx=2\int_0^{\pi/2}\cos 2nx=0$

$\therefore S_{n+1}=S_n$ ....(iii)

$\Rightarrow$ similarly $S_{n+2}=S_{n+1}$ $\Rightarrow$ $S_n=S_{n+1}=S_{n+2}$

$\therefore$ Now Given $V_n=\int_0^{\pi/2}\left(\frac{\sin nx}{\sin x}\right)^2dx$ ....(iv)

$\therefore \quad V_{n+1}-V_n=S_{n+1}$ ....(v) (from (ii)

$\therefore \quad V_{n+1}-V_n=S_n$ (from (iii)

$V_{n+1}=V_n+S_n$

similarly $V_{n+2}=V_{n+1}+S_{n+2}$

$\therefore \quad V_{n+2}=V_{n+1}+S_n$

6. A, D

$a\ell=2\Delta\Rightarrow\ell=\frac{2\Delta}{a}=\frac{abc}{2Ra}\Rightarrow\frac{b\ell}{c}=\frac{bc}{2R}.\frac{b}{c}=\frac{b^2}{2R}$

Hence LHS $=\frac{1}{2R}(a^2+b^2+c^2)$

$\Rightarrow 2R(\sin^2A+\sin^2B+\sin^2C)$

$=2R(2+2\cos A\cos B\cos C)=4R(1+\cos A\cos B\cos C)$

7. A,B,C,D

$|z_1|=1, |z_2|=2$

(A) $\quad ||z_1|-2|z_2||\leq|z_1-2z_2|\leq|z_1|+2|-z_2|$

$|1-2(2)|\leq|z_1-2z_2|\leq 1+2(2)$

$3\leq|z_1-2z_2|\leq 5$

(B) $\quad |z_1|-|z_2|\leq|z_1+z_2|\leq|z_1|+|z_2|$

$|1-2z|\leq|z_1+z_2|\leq 1+2$

$1\leq|z_1+3z_2|<3$

(C) $\quad ||z_1||-3|z_2||\leq|z_1\ 3z_2|\leq|z_1|+3|-z_2|$

$5\leq|z_1-3z_2|\leq 7$

(D) $\quad ||z_1|-|z_2||\leq|z_1-z_2|\leq|z|+|z_2|$

$1\leq|z_1-z_2|\leq 3$

8. A,B,D

$\frac{dx}{dy}=xy+x^2y^2\Rightarrow\frac{1}{x^2}\frac{dy}{dx}-\frac{y}{x}=y^3$

Let $-\frac{1}{x} = t$

or $\frac{1}{x^2}\,dx = dt$ or $\frac{dt}{dy} + yt = y^3$

I.F. $= e^{\int y\,dy} = e^{y^2/2}$ or $te^{y^2/2} = \int y^3 e^{y^2/2}\,dy + c$

or $te^{y^2/2} = y^2e^{y^2/2} - 2e^{y^2/2} + c$

or $-\frac{1}{x} = y^2 - 2 + ce^{-y^2/2}$ or $\frac{1}{x} = 2 - y^2 + ce^{-y^2/2}$

9. B

$$\int_0^1 \frac{\sin t}{1+t}\,dt = \alpha \Rightarrow \int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi+2-t}\,dt \quad t = 4\pi - 2 + 2u$$

$$= \int_0^1 \frac{\sin\frac{4\pi-2+2u}{2}}{4\pi+2-(4\pi-2+2u)}\,2du$$

$$= \int_0^1 \frac{2\sin(2\pi-1+u)}{4-2u}\,du = \int_0^1 \frac{\sin(u-1)}{(2-u)}\,du$$

Now let $u - 1 = -t$

$$= \int_1^0 \frac{\sin(-t)}{(2-(-t+1))}(-dt) = -\int_0^1 \frac{\sin t\,dt}{(1+t)} = -\alpha$$

10. (A) → (S), (B) → (R), (C) → (P), (D) → (P)

(A) Both the conic are confocal, so they will be orthogonal

(B) $\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6$

(C) Let $z = \cos\frac{\pi}{7} + i\sin\frac{\pi}{7}$

$z^7 + 1 = 0$

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}\left(z + \frac{1}{2}\right) + \frac{1}{2}\left(z^3 + \frac{1}{z^3}\right) + \frac{1}{2}\left(z^5 + \frac{1}{z^5}\right)$$

$$= \frac{1 + z^2 + z^4 + z^6 + z^8 + z^{10}}{2z^5}$$

$1 + z^2 + z^4 + z^6 + z^8 + z^{10} = z^6 + z^4 - z^3 + z^2 - z + 1$

$= z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 + z^5$

$= \frac{z^7 + 1}{z + 1} + z^5 = z^5$

(D) Let $a = 1 - x$, $b = x - y$, $c = y - z$ and $d = z$ then

$a + b + c + d = 1$ and $a^2 + b^2 + c^2 + d^2 = \frac{1}{4}$

$\Rightarrow (a^2 - b^2) + (a - c)^2 + (a - d)^2 + (b - c)^2 + (b - d)^2 + (c - d)^2 = 0$

$\Rightarrow a = b = c = d$

$\therefore\ x = \frac{3}{4},\ y = \frac{1}{2},\ z = \frac{1}{4}$

The distance of the plane $4x + 2y + 4z + 7 = 0$ from

the point $\left(\frac{3}{4}, \frac{1}{2}, \frac{1}{4}\right) = \frac{3+1+1+7}{6} = 2$ units.

11. (A) → (S), (B) → (R), (C) → (Q), (D) → (Q)

(A) $\alpha, \beta, \gamma, \delta$ are in G.P. $(r > 1)$

$\alpha + \beta = -p$; $\gamma + \delta = -q$

$\alpha\beta = 2$; $\gamma\delta = 32$

$\alpha(1 + r) = -p$; $\alpha(r^2 + r^3) = -q$

$\alpha^2 r = 2$; $\alpha r^2.\alpha r^3 = 32$

$\alpha = \pm 1$; $\frac{r^5}{r} = 16 \Rightarrow r = 2$

$p = \pm 3$, $q = \pm 12$

Roots are ±1, ±2 and ±4, ±8. ⇒ 16

(B) coefficient of T_2, T_3, T_4 in AP.

i.e. ${}^{2n}C_1$, ${}^{2n}C_2$, ${}^{2n}C_3$ in A.P.

$2\,.\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2} \quad \Rightarrow 2 = \frac{2}{2n-1} + \frac{2n-2}{3}$

$\Rightarrow 6n - 3 = 2n^2 - 3n + 4 \Rightarrow (2n - 7)(n - 1) = 0$

$\Rightarrow n = \frac{7}{2}$ or $n = 1$ ($n = 1$ is rejected)

(C) $\Delta > 0$ and $D = b^2 - 4ac$ a, b, c ∈ I

for $b^2x^2 - \Delta x - 4ac = 0$

$D = \Delta^2 + 4(4ac)\,b^2 = (b^2 + 4ac)^2$

Since D is perfect square so the roots will be rational

and equal to $\frac{\Delta \pm (b^2 + 4ac)}{2b^2}$

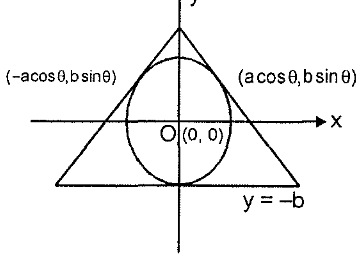
$= \frac{b^2 - 4ac \pm (b^2 + 4ac)}{2b^2} = 1,\ \frac{-4ac}{b^2}$

(D) $\int \frac{dx}{1 + \cos x} = \int \frac{dx}{2\cos^2 x/2} = \int \sec^2\frac{x}{2}.\frac{1}{2}dx = \tan\frac{x}{2} + c$

∴ f is a polynomial function ⇒ a = 1.

12. Let the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Equation of tangent $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$



when $y = -b$

$\frac{x\cos\theta}{a} - \sin\theta = 1 \quad \Rightarrow \quad x = \frac{a(1 + \sin\theta)}{\cos\theta}$

Area of triangle $A = \frac{1}{2} \times \frac{2a(1+\sin\theta)}{\cos\theta}$

$\times \left(b + \frac{b}{\sin\theta}\right) = \frac{a(1+\sin\theta).b(1+\sin\theta)}{\sin\theta\cos\theta} = \frac{ab(1+\sin\theta)^2}{\sin\theta\cos\theta}$

Area of ellipse $(\delta) = \pi ab = \frac{A\sin\theta\cos\theta}{1 + \sin^2\theta + 2\sin\theta}$

$$\delta = \pi A \frac{\sin\theta\cos\theta}{1+\sin^2\theta+2\sin\theta}$$

For maximum or minimum $\frac{d\delta}{d\theta} = 0 \Rightarrow \sin\theta = \frac{1}{2}$

So $S = \pi A \frac{\frac{1}{2}\times\frac{\sqrt{3}}{2}}{1+\frac{1}{4}+1}$. So maximum area of ellipse = $\frac{\pi A\sqrt{3}}{9}$ sq. units

13. $\underset{x\to 0}{\text{Limit}} \frac{\log_{\sec(x/2)}\cos x}{\log_{\sec x}\cos(x/2)} = \underset{x\to 0}{\text{Limit}} \frac{\ell n\cos x\,\ell n\sec x}{\ell n\sec\frac{x}{2}\,\ell n\cos\frac{x}{2}}$

$= \underset{x\to 0}{\text{Limit}} \left(\frac{\ell n\cos x}{\ell n\cos\frac{x}{2}}\right)^2$ Using L-H rule

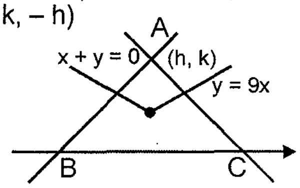
$= 4\underset{x\to 0}{\text{Lim}} \frac{\cos^2\frac{x}{2}}{\cos x}\cdot\frac{\ell n\cos x}{\ell n\cos\frac{x}{2}}$ Again applying L-H rule

$= 8\underset{x\to 0}{\text{Lim}} \frac{\sin x}{\cos x}\ \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = 16$

14. Let $B(x_1, y_1)$ we know $\frac{k-y_1}{h-x_1} = 1$ and

$h + x_1 + k + y_1 = 0$, then $x_1 = -k$; $y_1 = -h$

So $B(-k, -h)$



Let $C(x_2, y_2)$ and we know $\frac{y_2-k}{x_2-h} = -\frac{1}{9}$ and $y_2 + k = 9(x_2 + g)$ by solving

$x_2 = \frac{9k-40h}{41}$, $y_2 = \frac{9h+40k}{41}$

Line through BC $y + h = \left(\frac{50h+40k}{50k-40h}\right)(x + k)$

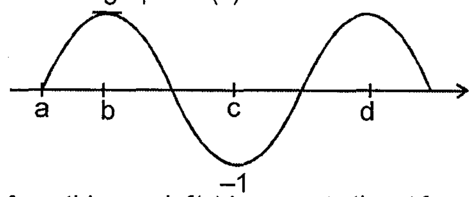
(f, g) lies on BC so $g + h = \frac{5h+4k}{5k-4h}(f + k) \Rightarrow (g + h)(5k - 4h) = (f + k)(5h + 4k)$

So locus of A, $(g + x)(5y - 4x) = (f + y)(5x + 4y) \Rightarrow$
$4x^2 + 4y^2 + x(5f + 4g) + y(4f - fg) = 0$

$\Rightarrow$ Sum of coefficient of x^2 and y^2 is = 4 + 4 = 8

15. Let $h(x) = f(x)\,f'(x)$

graph of f(x)



from this graph f(x) is zero at atleast four places and f'(x) is zero at atleast three places

hence h(x) is zero in atleast 7 places

hence h'(x) is zero in atleast 6 places.

$\therefore$ $h'(x) = g(x) = 0$ has minimum 6 solutions.

16. 0

$f(a - x) = f(x)$, $g(a - x) = -g(x)$

$3h(x) - 4h(a - x) = 5$

$$\int_0^a f(x)g(x)h(x)dx = \int_0^a f(a-x)g(a-x)h(a-x)dx$$

$$= \int_0^a f(x)(-g(x))\left(\frac{3h(x)-5}{4}\right)dx$$

$$= -\frac{3}{4}\int_0^a f(x)g(x)h(x)dx + \frac{5}{4}\int_0^a f(x)g(x)dx$$

$$\therefore \frac{7}{4}\int_0^a f(x)g(x)h(x)dx = \frac{5}{4}\int_0^a f(x)g(x)dx \quad ...(i)$$

$$\int_0^a f(x)g(x)h(x)dx = \int_0^a f(a-x)g(a-x)dx = \int_0^a f(x)(-g(x))dx$$

$$= -\int_0^a f(x)g(x)dx \quad \therefore \int_0^a f(x)g(x)dx = 0 \quad ....(ii)$$

from (i) to (ii)

$$\int_0^a f(x)g(x)h(x)dx = 0$$

17. $1 - {}^nC_1\frac{1+x}{1+nx} + {}^nC_2\frac{1+2x}{(1-nx)^2} - {}^nC_3\frac{1+3x}{(1+nx)^3} + ...(n+1)\text{terms}$

$= 1 - \frac{{}^nC_1}{1+nx} + \frac{{}^nC_2}{(1+nx)^2} - \frac{{}^nC_3}{(1+nx)^3}...... + \frac{{}^nC_n}{(1+nx)^n} - \frac{x}{1+nx}\left[{}^nC_1 - \frac{2.{}^nC_2}{1+nx} + \frac{3.{}^nC_3}{(1+nx)^2} - ......\text{n terms}\right] =$

$\left(1-\frac{1}{1+nx}\right)^n - \frac{nx}{1+nx}\left[{}^{n-1}C_0 - {}^{n-1}C_1.\frac{1}{1+nx} + {}^{n-1}C_2.\frac{1}{(1+nx)^2} - ......\right]$

$$= \left(1-\frac{1}{1+nx}\right)^n - \frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1} = 0$$

18. $z = \ell n\,|\tan x/2|$ $\frac{dz}{dx} = \frac{1}{2}\frac{\sec^2 x/2}{\tan x/2} = \frac{1}{\sin x}$

$$\frac{dy}{dx} = \frac{dy}{dz}.\frac{dz}{dx} = \frac{1}{\sin x}.\frac{dy}{dz}$$

$$\frac{d^2y}{dx^2} = -\text{cosec}\,x\cot x\frac{dy}{dz} + \text{cosec}\,x\frac{d^2y}{dz^2}.\frac{dz}{dx}$$

$$\frac{d^2y}{dx^2} = -\text{cosec}\,x\cot x\,\frac{dy}{dz} + \text{cosec}^2 x\,\frac{d^2y}{dz^2}$$

$$\therefore\ \frac{d^2y}{dx^2} + \cot x\,\frac{dy}{dz} + 4y\,\text{cosec}^2 x = 0 \text{ or}$$

$$-\text{cosec}\,x\cot x\,\frac{dy}{dz} - 4y\,\text{cosec}^2 x$$

$$= -\text{cosec}\cot x\,\frac{dy}{dz} + \text{cosec}^2x\,\frac{d^2y}{dz^2} \text{ or } \frac{d^2y}{dz^2} + 4y = 0$$

19. given $\alpha + \beta = p$; $\alpha\beta = -p - c$

$\alpha\beta = -(\alpha + \beta) - c \Rightarrow \quad \alpha + \beta + \alpha\beta + 1 = 1 - c$

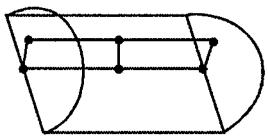
$$(\alpha + 1)(\beta + 1) = 1 - c \therefore \frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+c} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+c};$$

$$= \frac{(\alpha+1)^2}{(\alpha+1)^2-(\alpha+1)(\beta+1)} + \frac{(\beta+1)^2}{(\beta+1)^2-(\alpha+1)(\beta+1)}$$

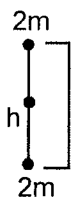
$$= \frac{\alpha+1}{(\alpha+1)-(\beta+1)} + \frac{\beta+1}{(\beta+1)-(\alpha+1)} = \frac{\alpha+1}{\alpha-\beta} + \frac{\beta+1}{\beta-\alpha} = 1$$

PHYSICS

20. D



COM of bath half ring portion will lie at a height $\frac{2R}{\pi}$ from ground and com of both rods will lie at ground, below the com of rings



$$\frac{2R}{\pi} \Rightarrow h_{cm} = \frac{2R}{\pi}/2,\ h_{cm} = \frac{R}{\pi}\ ]$$

21. A

When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

22. D

23. D

24. A,B

ω is just the angular velocity of circular motion and is equal to v/R.

At t = 0. positiion and velocity of particle is shown in figure.

$x = R\cos(\omega t + \phi)$

$v = -\omega R\sin(\omega t + \phi)$

$t = 0, x = 0$

$\therefore\ 0 = R\cos\phi$

or $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

Aslo, at t = 0, $v = -\omega R\sin\phi$

But from the figure it is clear that velocity is positive at t = 0

$\therefore\ \phi$ has to be $\frac{3\pi}{2}$

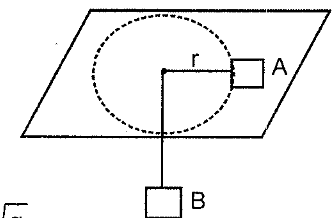
25. A,B,C

$$\phi = BA = \frac{B}{2}\frac{x}{\sqrt{2}}\frac{x}{\sqrt{2}}$$

$$\varepsilon = \frac{d\phi}{dt} = \frac{2x}{4}\frac{dx}{dt}B = \frac{x}{2}B.\frac{d(2y)}{dt}$$

$$i = \frac{xBv}{2\lambda(x+\sqrt{2}x)} = \frac{Bv}{x(1+\sqrt{2})}$$

26. A,B



For (A) T = mg

For (B) $T = \frac{mv^2}{r}$

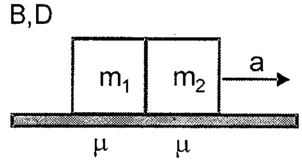
$= m\omega^2 r$

$m\omega^2 r = mg \qquad \omega = \sqrt{\frac{g}{r}}$ ...(1)

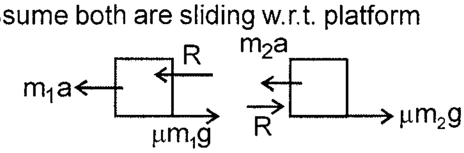
K.E. of system $= \frac{1}{2}m_A v_A^2 = \frac{1}{2}(m)\left(\sqrt{\frac{g}{r}}\right)^2 = \frac{mgr}{2}$

(C) Tension is cenral force on A, so angular momentum remains conserved.

27. B,D



Assume both are sliding w.r.t. platform



$\mu m_2 g + R = m_2 a$

$\mu m_1 g - R = m_1 a$

$$\frac{\mu(m_1+m_2)g}{m_1+m_2} = a = \mu g$$

$R = (\mu m_2 g - \mu m_2 g) = 0$

28. A,B,D

ray (1) undergo T.I.R. at face AC at $\angle 30°$

$\therefore\ 30° > i_c$ since $\sin 30° > \sin i_c$

$\therefore \sin i_c = 1/\mu \Rightarrow 1/2 > 1/\mu \Rightarrow \mu = 2$

for ray (2) refracting angle is 30°

$\therefore\ 1 \sin i = \mu \sin r \Rightarrow i = 90°$

for ray (2) $i_1 + i_2 = A + \delta$

$90° + 0° = 30° + \delta$

$\Rightarrow \delta = 60°$

for ray (3) $i_1 + i_2 = A + \delta$

$i_1 = i_2 \Rightarrow 2i_1 = 60° + 120° \Rightarrow i_1 = 90°$

29. **(A) → P,R ; (B) → P, R ; (C) → Q,S ; (D) → P, Q, R, S**

(A) Speed of charged particle cannot be changed by magnetic force because magnetic force does no work on charged particle. Only electric field in case (P) and induced electric field in case (R) can change speed of charged particle.

(B) Magnetic field cannot exert a force on charged particle at rest. Only electric field in case (P) and induced electric field in case (R) can exert force on charge initially at rest.

In case (R)after the charge starts moving even the magnetic field can exert force on charge.

(C) A charged particle m\can move in a circle within uniform speed due to uniform and constant magnetic field in case. Even within a region of non uniform magnetic field, at all point on a circle field may be uniform for example on any circle coaxial with a current carrying riong.

(D) A moving charged particle is accelerated by electric field and also accelerated by magnetic field (provided v is not parallel to B.)

31. Component of velocity along the direction of B and perpendicular to B are

$V_x = V\cos\theta, \quad V_y = V\sin\theta$

Now the path of particle is helical. So for minimum time, particle should reach P while rotated by π radian.

So $t = \dfrac{\pi m}{qB}$ ...(i)

In the same time particle should cover a distance L along x-direction.

So $t = \dfrac{L}{v\cos\theta}$ ....(ii)

Here $\dfrac{H}{2} = \dfrac{mV\sin\theta}{qB}$ or $H = \dfrac{2mV\sin\theta}{qB}$

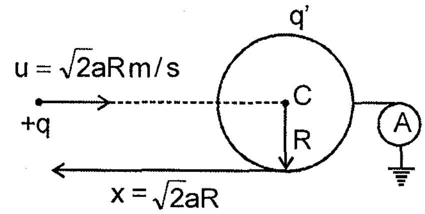
So $q = \dfrac{2mV\sin\theta}{HB}$ ...(iii)

from (i), (ii) and (iii) $\tan\theta = \dfrac{\pi H}{2L}$ or $\theta = \tan^{-1}\dfrac{\pi H}{2L}$

32. $$\frac{1}{C_{eq}} = \frac{1}{\frac{\varepsilon_0 A}{2d}} + \frac{1}{\frac{\varepsilon_0 A}{2^2 d}} + \frac{1}{\frac{\varepsilon_0 A}{2^3 d}} + \ldots + \frac{1}{\frac{\varepsilon_0 A}{2^{n-1} d}} = \frac{2(2^{n-1}-1)d}{\varepsilon_0 A}$$

$$C_{eq} = \frac{\varepsilon_0 A}{d(2^n - 2)}$$

33.



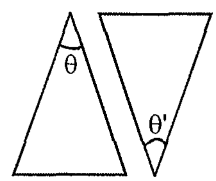
$$\frac{kq}{x} + \frac{kq'}{R} = 0$$

$$q' = -\frac{q}{x}R$$

$$\frac{dq'}{dt} = +\frac{qR}{x^2}v = \frac{qR}{2a^2R^2} \times \sqrt{2}aR$$

$$I = \frac{q}{\sqrt{2}a}$$

34. Prism should be kept as shown in figure



$\therefore\ \delta = A(\mu - 1)$

mean refractive index for crown glass

$$\mu_c = \frac{\mu_{CB} + \mu_{CR}}{2}$$

for flint glass $\mu_F = \dfrac{\mu_{FB} + \mu_{FR}}{2}$

$\therefore\ \theta(\mu_c - 1) = \theta'(\mu_F - 1)$

$\theta' = 4°$

30.

	Volume inside liquid	F	F_b	Weight of body	
(p)	$\frac{1}{3}\left[\pi H^2(2H) - \pi\left(\frac{\pi}{2}\right)^2 H\right]$	$\frac{5}{12}\pi H^3 \rho_0 g$	$\frac{7}{12}\pi H^3 \rho_0 g$	$\frac{2}{3}\pi H^3 \rho g$	$\rho = \frac{7}{8}\rho_0$
(q)	$\frac{2}{3}\pi H^3$	$\frac{\pi}{3} H^3 \rho_0 g$	$\frac{2}{3}\pi H^3 \rho_0 g$	$\frac{2}{3}\pi H^3 \rho g$	$\rho = \rho_0$
(r)	$4H^3$	0	$4H^3 \rho_0 g$	$8H^3 \rho_0 g$	$\rho = \frac{\rho_0}{2}$
(s)	πH^3	0	$\pi H^3 \rho_0 g$	$2H^3 \rho_0 g$	$\rho_0 = \frac{\rho_0}{2}$

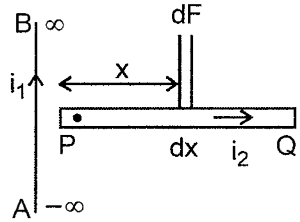
35. $W - V\rho_0 g = 40 - \frac{40}{g\rho_w} \times \rho_0 g$

$W - V\rho_0 g = 40 - 40\left(1 - \frac{\rho_0}{\rho_w}\right) + V\rho_0 g = 30N$

36. 3

$\frac{dN}{dt} = -\lambda N + \frac{9\lambda N_0^2}{N} \Rightarrow N = 3N_0$ it $t \to \infty$

37. $dF = i_2 dx . \frac{\mu_0}{2\pi}\frac{i_1}{x}$



$d\tau = \left(i_2 dx \frac{\mu_0}{2\pi}\frac{i_1}{x}\right)x$

$mg(\ell/2) = i_2 \frac{\mu_0}{2\pi} i_1 \int_0^\ell dx$

$mg(\ell/2) = i_2 \frac{\mu_0}{2\pi} i_1 \ell \Rightarrow i_2 = \frac{mg\pi}{\mu_0 i_1}$

38. Just before collision velocity of the particle is

$v^2 = (\sqrt{130g})^2 - 2g\ell = 128g\ell$

$v = \sqrt{64 \times 2g\ell} = 8\sqrt{2g\ell}$

e = 1/2, after 1st collision velocity become = $4\sqrt{2g\ell}$

after 2nd collision velocity become = $2\sqrt{2g\ell}$

after 3rd collision velocity becomes = $\sqrt{2g\ell}$

$\sqrt{2g\ell} < \sqrt{3g\ell}$

So, no circular motion.

CHEMISTRY

39. B

We have,

$\frac{t_{mix}}{t_{O_2}} = \sqrt{\frac{M_{mix}}{M_{O_2}}} \Rightarrow \frac{234}{224} = \sqrt{\frac{M_{mix}}{32}} \quad \therefore M_{mix} = 34.921.$

As the mixture contains 20% (mole %) of x, the molar ratio of O_2 and X may be represented as 0.8n : 0.2n, n being the total no. of moles.

$\therefore M_{mix} = \frac{32 \times 0.8n + m_x \times 0.2n}{n} = 34.921$

$\therefore M_x$ (mol. wt. of X) = 46.6.

40. D

$Q_{sp} \le K_{sp}$

$0.1 \times [OH^-]^2 \le 1.2 \times 10^{-11}$

$[OH^-]^2 \le 1.2 \times 10^{-10}, \quad [OH^-] \le 1.1 \times 10^{-5}$

$p^{OH} \ge 5 - \log 1.1, \quad p^{OH} \ge 4.95$

$14 - p^H \ge 4.95, \quad 14 - 4.95 \ge p^H$

$9.04 \ge p^H$ so maximum $p^H = 9.04$

41. C

42. B

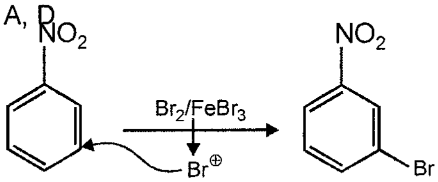
$P_s = 210 - 120\, X_A, \; P_A^o = 210 - 120 \times 1 = 90$

$P_B^o = 210 - 120 \times 0 = 210$

43. A, B, C, D

In benzyl amine lone ℓp localised but in aniline, benzamide, urea and formamide ℓp delocalised. So, these less basic than benzyl amine.

44. A, D

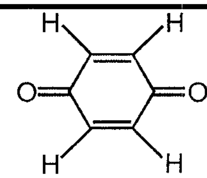


– M and – I of NO_2 group

– NO_2 group is meta directing and deactivating group *ed* is more at meta than ortho and pera positions.

45. A, C, D)

In Benzoquinone α-H present at sp^2 -C. So not show tautomerism.



46. B, C

The pressure of NH_3 will decrease due to addition of CO_2 (backward, shifting Le-chatelies's principle. The pressure of CO_2 will be more than 0.1 atm.

47. B, D

R — C̈l $\xrightarrow{Nu^{\ominus}}$ more reactive

sp^3C ℓ. localised

Ar — C̈l $\xrightarrow{Nu^{\ominus}}$ less reactive

sp^2C ℓ. delocalised

48. (A)–P, Q, R, S (B)–P, Q, S (C)–P, S (D)–S

49. (A)–Q, S (B)–R (C)–P (D)–P

50. 50.31%

Since HCl will react only with Na_2CO_3

$\therefore$ m.e. of HCl = m.e. of Na_2CO_3 = 8.96 × 0.1 = 0.896

$\therefore$ m.e. in 100 ml solution = 8.96

moles of $Na_2CO_3 = \frac{8.96}{2} \times 10^{-3} \times 106 = 0.475$

$\therefore$ % of Na_2CO_3 in the mixture $= \frac{0.475}{0.944} \times 100 = 50.31\%$

51. 64

22.4 L of gas = 1.00 mol of X_2

56.0 ml of gas $= \frac{56}{22400} = \frac{1}{400} = 2.5 \times 10^{-3}$ moles

$\therefore 2MX_2 \to 2MX + X_2$

One mole X_2 comes from = 2 moles of MX_2

$\therefore 2.5 \times 10^{-3}$ moles X_2 comes from

$= 2 \times 2.5 \times 10^{-3} = 5.0 \times 10^{-3}$ moles of MX_2

Molecular weight of $MX_2 = \frac{1.12}{5 \times 10^{-3}} = \frac{1120}{5} = 224$ g

Molecular weight of MX = $\frac{0.72}{5\times10^{-3}} = \frac{770}{5}$ = 144 g

Atomic weight of X = – 224 – 144 = 80

Atomic weight of X = 224 – 160 = 64

52. 76.83% B and 23.17%C

% of B = $\frac{K_1}{K_1+K_2}$ × 100

$= \frac{1.26\times10^{-4}}{(1.26+0.38)\times10^{-4}} \times 100 = 76.83$ %

% of C = $\frac{K_2}{K_1+K_2}$ × 100

$= \frac{3.80\times10^{-5}}{(1.26+0.38)\times10^{-4}} \times 100 = 23.17$ %

53. – 0.7406 volt.

The e.m.f. of the cell is given

$$E = \frac{2.303RT}{nF}\log\frac{C_2}{C_1} = \frac{0.0591}{1}\log\frac{C_2}{C_1}$$

where C_1 and C_2 are the concentration of H^+ ions in 0.5 N HCl and 0.1 N NaOH respectively.

C_2 = 0.5 × 0.87 = 0.435 mol/L

$$C_2 = \frac{K_W}{C_{OH^-}} = \frac{1.2\times10^{-14}}{0.1\times0.9} = 1.333\times10^{-13} \text{ g ion/L}$$

$$E = \frac{0.0591}{1}\log\frac{1.333\times10^{-13}}{0.435} = 0.0591\times(-12.5326)$$

E = – 0.7406 volt.

54. 0.368

Let the number of moles of CH_4 and C_2H_2 be x and y respectively.

$CH_4 + C_2H_2 + O_2 \longrightarrow CO_2 + H_2O$

x moles y moles

Applying POAC for C atoms,

1 × moles of CH_4 + 2 × moles of C_2H_2 = 1 × moles of CO_2

x + 2y = moles of CO_2

As no. of moles α pressure at const. temperature and volume.

$$\frac{\text{no. of moles of } CH_4 \text{ and } C_2H_2}{\text{no. of moles of } CO_2} = \frac{70.5}{96.4}$$

or $\frac{x+y}{x+2y} = \frac{70.5}{96.4}$

$\therefore$ $\frac{y}{x+y}$ = mole fraction of C_2H_2 = 0.368

55. 0.92 M

Q = 96.5 × 0.3 × 60 × 60 = 104220 C

104220 C deposite $\frac{104220}{96500}$ eq

= 1.08 equivalent of Ni^{2+}

n_{eq} of Ni^{2+} in solution = 0.5 × 2 × 2 = 2.0

n_{eq} of Ni^{2+} left = 2.0 – 1.08 = 0.92

Moles of Ni^{2+} left = 0.46

Hence 0.46 moles of Ni^{2+} are left in 0.5 L solution.

[$\because$ n_{eq} (NI^{2+}) = moles of Ni^{2+} × 2]

0.92 moles of Ni^{2+} are left in 1 L solution

So, molarity = 0.92 M

56. 9.6

n_{eq} of $NH_4OH = \frac{100\times0.75}{1000} = 0.075$

n_{eq} of HCl $= \frac{100\times0.25}{1000} = 0.025$

n_{eq} of NH_4 Cl formed = 0.025

n_{eq} of NH_4OH left = 0.05

Thus the solution now behaves like a buffer

$pOH = pK_b + \log\frac{[Salf]}{[Base]}$

$= 4.7 + \log\frac{0.025}{0.05} = 4.4$

pH = 14 – 4 .4 = 9.6

57. 0.18 M

$mg(OH)_2\,(s) \rightleftharpoons Mg^{2+}\,(aq) + 2\,OH^-\,(aq)$

$K_{sp} = 1.8\times10^{-11}$

$K_{sp} = [Mg^{2+}]\,[OH^-]^2 = 1.8\times10^{-11}$

$[Mg^{2+}]\times(1.0\times10^{-5})^2 = 1.8\times10^{-11}$

$[Mg^{2+}]$ = 0.18 M