MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT-JEE
HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-I CLASS XII & XIII (DATE 22-11-09)

MATHEMATICS

1.	B	**2.**	A	**3.**	B	**4**	A	**5.**	D	**6.**	B	**7.**	B
8.	B	**9.**	B,D	**10.**	B,C,D	**11.**	A,D	**12.**	A,C,D	**13.**	A	**14.**	C
15.	D	**16.**	C	**17.**	A	**18.**	C						

19. (A) → (P, Q, R) ; (B) → (P, Q, R) ; (C) → (P, Q, R, S); (D) → (P, R, S)

20. (A) → Q; (B) → (S) ; (C) → P ; (D) → R

PHYSICS

21.	A	**22.**	A	**23.**	B	**24.**	A	**25.**	C	**26.**	C	**27.**	D
28.	A	**29.**	A,C	**30.**	C,D	**31.**	C,D	**32.**	A,B,C	**33.**	D	**34.**	D
35.	C	**36.**	B	**37.**	C	**38.**	C						

39. A → Q; B → P ; C → S ; D → R

40. A → P; B → Q ; C → R ; D → S

CHEMISTRY

41.	A	**42.**	A	**43.**	D	**44.**	A	**45.**	C	**46.**	C	**47.**	B
48.	A	**49.**	C,D	**50.**	A	**51.**	B,C,D	**52.**	A,C	**53.**	D	**54.**	B
55.	C	**56.**	B	**57.**	C	**58.**	A						

59. A → P,S; B → R; C → Q,S; D → Q,S

60. A → Q,R,S; B → P,S; C → R ; D → Q,S

SOLUTIONS

MATHEMATICS

1. **B**

$$\frac{c+a}{b}+\frac{c+b}{a}=\frac{a^2+b^2-c^2+c(a+b+c)}{ab}$$

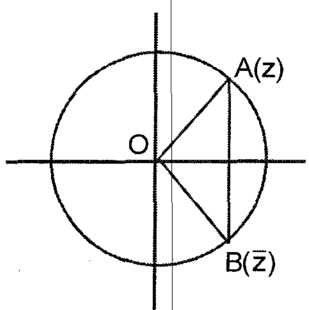
$$=\frac{a^2+b^2-c^2+2cs}{ab}$$

also $\frac{c}{r}=\frac{c.s}{\Delta}=\frac{c.s\,4R}{abc}=\frac{2Rc.2s}{abc}=\frac{2cs}{ab\sin C}$

Thus we finally, have

$$\frac{a^2+b^2-c^2}{ab}+\frac{2cs}{ab}=\frac{2cs}{ab\sin C}\Rightarrow \angle C=\frac{\pi}{2}$$ **Ans.**

2. **A**



$|z-\bar{z}|$ = straight line AB

while $|z|$ (arg z – arg $\bar{z}$) = Arc AB

$\therefore\ |z-\bar{z}|\le|z|(\arg z-\arg\bar{z})$ **Ans.**

3. **B**

Let $\vec{r}$ be the new position, then $\vec{r}=\lambda\vec{k}+\mu(\hat{i}+\hat{j})$

also $\vec{r}.\vec{k}=-\frac{1}{\sqrt{2}}$ $\Rightarrow$ $\lambda=-\frac{1}{\sqrt{2}}$

Also, $\lambda^2+2\mu^2=1$ $\Rightarrow 2\mu^2=\frac{1}{2}\Rightarrow\ \mu=\pm\frac{1}{2}$

$\therefore$ $\vec{r}=\pm\frac{1}{2}(\hat{i}+\hat{j})-\frac{\hat{k}}{\sqrt{2}}$ **Ans.**

4 **A**

$$\frac{\sqrt{3}}{2}\le x\le 1\ \Rightarrow\ \left(\frac{1}{2}\right)^2+x^2\ge 1$$

$$\therefore\ f(x)=\pi-\sin^{-1}\left(\frac{1}{2}\sqrt{1-x^2}+\frac{\sqrt{3}x}{2}\right)$$

5. **D**

$${}^{k}C_k+{}^{k+1}C_k+{}^{k+2}C_k+\dots\dots\dots+{}^{n}C_k$$

$$\underbrace{{}^{k+1}C_{k+1}+{}^{k+1}C_k}+{}^{k+2}C_k+\dots\dots\dots+{}^{n}C_k$$

$$\underbrace{{}^{k+2}C_{k+1}+{}^{k+2}C_k}+{}^{k+3}C_k+\dots\dots\dots+{}^{n}C_k$$

$\underbrace{{}^{k+3}C_{k+1}}$ and so on

adding upto the last term given ${}^{n+1}C_{k+1}$

6. **B**

x y z y x

First digit = x can be from 1 to 8

where as z = can be any one from 0 to 9

when x = 1, y = 2, 3..........9   8 digit

x = 2, y = 3, 4.........9   7 digit

and so on

Thus the total = (8 + 7 + 6 + ...+ 2 + 1)×10

= 360 **Ans.**

7. **B**

Clearly each bag contains (n – 1) balls and

$a_i+b_i=n-1$ also $\sum_{i=1}^{n}a_i=\sum_{i=1}^{n}b_i=\frac{n(n-1)}{2}$

Now $a_i=(n-1)-b_i\Rightarrow a_i^2=(n-1)^2+b_i^2-2(n-1)b_i$

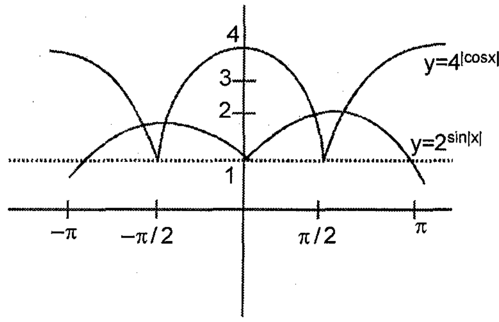
$\sum a_i^2=n(n-1)^2+\sum b_i^2-2(n-1)\frac{n(n-1)}{2}=\sum b_i^2$.

8. **B**

Figure clearly indicates that the graph of $y=4^{|\cos x|}$

and $y=2^{\sin(|x|)}$ meets four times in $[-\pi,\pi]$.

Hence these are four solutions.



9. B,D

10. B,C,D

(A) 1^{st} arm can move in 6 ways and so on ....

$\therefore\ 6^5 - 1$

(D) $\{n(n-1)(n-2)..........(n-r+1)(n-r)!\}/(n-r)!$

$$= \frac{n!}{(n-r)!} = {}^nC_r.r!$$

11. A,D

$$f(x) = -\frac{x^3}{3} + x^2\sin 1.5a - x\sin a.\sin 2a - 5\sin^{-1}(a^2 - 8a + 17)$$

$\therefore\quad f'(x) = -x^2 + 2x\sin 1.5a - \sin a\sin 2a \quad ...(1)$

$-1 \le a^2 - 8a + 17 \le 1$

$\Rightarrow\quad a^2 - 8a + 18 \ge 0$ and $a^2 - 8a + 16 \le 0 \quad \Rightarrow a = 4$

$\therefore\quad f'(x) = -x^2 + 2x\sin 6 - \sin 4\sin 8$

$f'(\sin 8) = -\sin^2 8 + 2\sin 8\sin 6 - \sin 4\sin 8$

$= \sin 8\,(2\sin 6 - \sin 4 - \sin 8)$

$= \sin 8\,[2\sin 6 - 2\sin 6\cos 2]$

$= 2\sin 8\sin 6\,(1 - \cos 2) < 0 \quad \begin{bmatrix} \because \sin 6 < 0 \\ \because \sin 8 > 0 \end{bmatrix}$

12. A,D

$f(x) = (x-1)^4(x-2)^n,\ n \in N \quad ...(1)$

$\therefore\ f'(x) = 4(x-1)^3(x-2)^n + (x-1)^4 n(x-2)^{n-1}$

$= (x-1)^3(x-2)^{n-1}(4x - 8 + nx - n)$

$= (x-1)^3(x-2)^{n-1}[(n+4)x - (n+8)]$

If n is odd, then $f'(x) > 0$ if $x < 1$ and sufficiently close to 1 and $f'(x) < 0$ if $x > 1$ and sufficiently close to 1

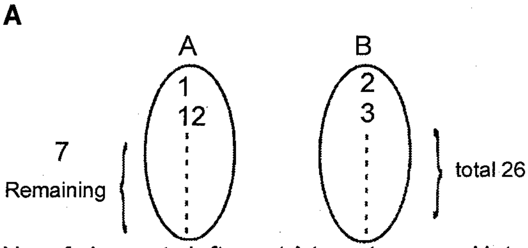
$\therefore\ x = 1$ is point of local maximum

Similarly if n is even, then x =1 is a point of local minimum

Further if n is even, then $f'(x) < 0$ for $x < 2$ and sufficiently close to 2 and $f'(x) > 0$ for $x > 2$ and sufficiently close to 2.

$\therefore\ x = 2$ is a point of local minimum.

13. A



No. of elements left n set A to get mapped into set B = 7

n (B) = 26

total no. of fns. = 26^7

14. C

No. of total functions $B \to C = 26^{24}$

onto functions = 24 !

Into functions = $26^{24} - (24)!$

15. D zero

$R_1 \cup R_2\ \{(1, 2), (2, 83), (3, 19), (12, 3)\}$

n (A) = 9

n (C) = 26

One-one onto mapping = zero

$\because\quad n(C) > n(A)$

16. C

Here for $f(x) = ax^2 + bx + c$ for given α.

$c > 0 \quad a < 0 \quad b < 0$

$-\frac{b}{2a} < 0$; $D > 0$; $-\frac{D}{4a} > 0$

$D = 16(\cot^{-1}\alpha)^2 - 16\tan^{-1}\alpha\,(\tan^{-1}\alpha - \pi)$

$= 16[(\cot^{-1}\alpha)^2 - (\tan^{-1}\alpha)^2 + \pi\tan^{-1}\alpha]$

$= 16[(\pi/2 - \tan^{-1}\alpha)^2 - (\tan^{-1}\alpha)^2 + \pi\tan^{-1}\alpha]$ =

$16[\pi^2/4 + (\tan^{-1}\alpha)^2 - \pi\tan^{-1}\alpha - (\tan^{-1}\alpha)^2 + \pi\tan^{-1}\alpha]$

$D = 4\pi^2 > 0$

17. A

$$x = \frac{4\cot^{-1}\alpha \pm 2\pi}{8(\tan^{-1}\alpha - \pi)}$$

taking (+) sign

$$x_1 = -\frac{2\cot^{-1}\alpha + \pi}{2(\pi + 2\cot^{-1}\alpha)} = \frac{-1}{2}$$

taking (–) sign

$$x_2 = \frac{\pi - 2\cot^{-1}\alpha}{2(\pi + 2\cot^{-1}\alpha)}$$

$$\cot^{-1}\alpha = \frac{\pi}{2}\left(\frac{1 - 2x_2}{1 + 2x_2}\right)$$

$$0 < \frac{\pi}{2}\left(\frac{1 - 2x_2}{1 + 2x_2}\right) < \pi$$

$$\Rightarrow x_2 \in \left(\frac{-1}{2}, \frac{1}{2}\right) \ \&\ x_2 \in \left(-\infty, \frac{-1}{2}\right) \cup \left(-\frac{1}{6}, \infty\right)$$

$$\Rightarrow \quad x_2 \in \left(-\frac{1}{6}, \frac{1}{2}\right)$$

sol^n set

$$x \in \left(-\frac{1}{6}, \frac{1}{2}\right) \cup \left\{-\frac{1}{2}\right\}$$

18. C

$$y_{max} = -\frac{D}{4a}$$

$$= \frac{-4\pi^2}{16(\tan^{-1}\alpha - \pi)} = \frac{\pi^2}{4\left(\frac{\pi}{2} + \cot^{-1}\alpha\right)}$$

$y_{max} \to \frac{\pi}{2}$ If $\cot^{-1}\alpha \to 0$

$y_{min} \to -\infty$ since $a < 0$

19. **(A)** → (P, Q, R) ; **(B)** → (P, Q, R) ;
(C) → (P, Q, R, S); **(D)** → (P, R, S)

(A) Let $\frac{\pi}{\sqrt{3}}\sin x+\sqrt{\frac{2}{3}}\pi\cos x = t$

$t_{max} = \pi$; $t_{min} = -\pi$

f(x) = cos t $t \in [-\pi, \pi]$

$f(x) \in [-1, 1]$ ⇒ (P), (Q)

Trigonometric function is periodic

∴ f(x) is many-one ⇒ (R) ⇒ (A) ⇒ (P), (Q), (R)

(B) Let |sinx | + 1 = t

$t \in [1, 2]$

$f(x) = \log_2 t$ ⇒ $f(x) \in [0, 1]$

f(x) contain only one positive integer

domain is R ⇒ **(P, Q, R)**

(C) $[x] + [-x] = 0 \quad x \in I$

$= -1 \quad x \notin I$

∴ $\{[x] + [-x]\} = 0$

∴ domain is $(-\infty, \infty)$

Range contains only one integer and also constant function

f(x) is many-one obviously ⇒ **(P, Q, R, S)**

(D) We know $|e^x| \in (0, \infty) \quad \forall\ x \in R$

∴ $\{|e^x|\} \in [0, 1)$

∴ $[\{|e^x|\}] \in \{0\}$

f(x) is constant function

domain is R

obviously f(x) is many-one ⇒ **(P, R, S)**

20. **(A) → Q; (B) → (S) ; (C) → P ; (D) → R**

(A) $\boxed{R_1\ R_2\ R_3\ R_4}$ $\boxed{G_1\ G_2\ G_3\ G_4\ G_5}$

= 4 ! . 5 ! . 2 ! = 8 .6 ! ⇒ **(Q)**

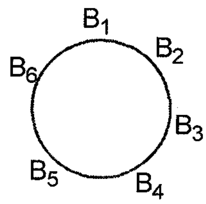
(B) $\boxed{R_1\ R_2\ R_3\ R_4}$ $G_1 \quad G_2 \quad G_3 \quad G_4 \quad G_5$

(6 !) (4 !) = (24) 6 ! ⇒ **(S)**

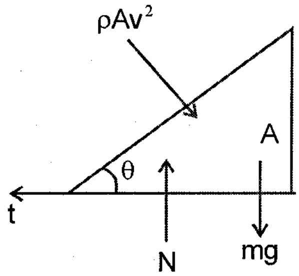
(C) $G_1 \times G_2 \times G_3 \times G_4 \times G_5$

= 5!. 4! = (4) 6! ⇒ **(P)**

(D) $5!.\ {}^6C_3.\ 3! = (20)\ 6!$ ⇒ **(R)**



21. A



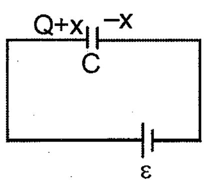
$N = mg + \rho Av^2 \cos\theta$

$F = \rho Av^2 \sin\theta = \mu N$

$$\Rightarrow \mu = \frac{\rho Av^2 \sin\theta}{mg + \rho Av^2 \cos\theta}$$

22. A

23. B



Electric field between the capacitor plates

$$= \frac{\sigma_1}{2\varepsilon_0} + \frac{(-\sigma_2)}{2\varepsilon_0}$$

$$E = \frac{Q+x}{2A\varepsilon_0} + \frac{x}{2A\varepsilon_0} = \frac{1}{2A\varepsilon_0}[Q+2x]$$

$$\Rightarrow \text{Potential different } E \times d = \frac{d}{2A\varepsilon_0}[Q+2x] = \varepsilon$$

$$\Rightarrow \varepsilon = \frac{Q+2x}{2C}$$

$$\Rightarrow -x = \frac{Q}{2} - C\varepsilon$$

24. A

$$a = -\frac{F}{m}$$

$$-F\left(\frac{L}{2}\right) = \frac{1}{12} mL^2 \alpha$$

$$a_B = a - \alpha\left(\frac{L}{2}\right) = 2 \text{ m/s}^2 \text{ to right}$$

25. C

Direction of impulse = $(\hat{j}/2) - \hat{i}$

$$\Rightarrow \text{Unit vector along impulse} = \frac{\hat{j} - 2\hat{i}}{\sqrt{5}}$$

$$\Rightarrow v_a = \text{velocity of approach} = -\hat{i}.\left(\frac{\hat{j}-2\hat{i}}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}}$$

$$\Rightarrow v_s = \text{velocity of separation} = \frac{\hat{j}}{2}.\left(\frac{\hat{j}-2\hat{i}}{\sqrt{5}}\right) = \frac{1}{2\sqrt{5}}$$

$$\Rightarrow e = \frac{v_s}{v_a} = \frac{1}{4}$$

26. C

The effective emf due to batteries connected between A and C is zero while the effective resistance of the circuit is

$$\text{from } E_{eq.} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}} \text{ and } r_{eq.} = \frac{1}{\sum \frac{1}{r_i}}$$

$$\frac{2}{3} + \frac{2}{3} + 4 = \frac{16}{3}\Omega$$

$$\Rightarrow I = \frac{24}{16/3} = \frac{9}{2} = 4.5 \text{ A}$$

27. D

28. A

29. A,C

30. C,D

Refer to the theory of motion of charged particles in magnetic field and simultaneous electric and magnetic fields.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

31. C,D

32. A,B,C

At closed end node is there for displacement wave and antinode for pressure waves.

33. **D**

Since $\vec{M} \parallel \vec{B}$ $\therefore$ Torque is zero

34. **D**

The field must be in $+\hat{k}$ direction.

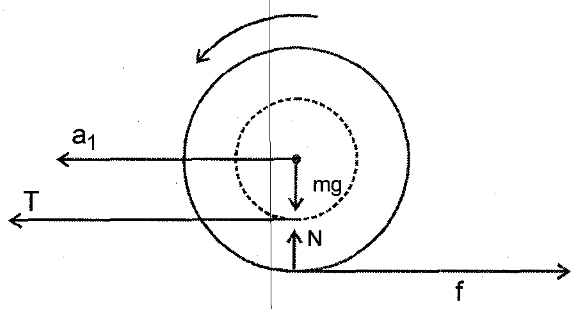
35. **C**

The emf is the difference between emf across straight segment OA and OC.

36. **B**

37. **C**

38. **C**



$ma_1 = T - f$

$N = mg$

$fr_2 - Tr_1 = mK^2 \alpha$, k = radius of gyration

$f = \mu N$

$a_1 = \alpha r_1$

$a = (r_2 - r_1)\alpha$

$a = (r_2 - r_1)(a_1/r_1)$

39. **A → Q; B → P ; C → S ; D → R**

$\Delta f_A = 0$

$$\Delta f_B = \frac{2v_0}{c - v_s} f_0 = \frac{2 \times 10}{300} \times 3000 = 200\,\text{Hz}$$

$$\Delta f_C = \frac{2cv_s}{c^2 - v_s} f_0 = \frac{2 \times 350 \times 50}{400 \times 300} \times 3000 = 875\,\text{Hz}$$

$$\Delta f_D = \frac{360 \times 2 \times 50}{400 \times 300} \times 3000 = 900\,\text{Hz}$$

40. **A → P; B → Q ; C → R ; D → S**

41. **A**

0.1 M Cl^- & 0.1 M CrO_4^{-2}

$[Ag^+]$ for AgCl $\Rightarrow Q_{sp} > K_{sp}$

$[Ag^+]\,[0.1] > 10^{-10}$

$[Ag^+] > 10^{-9}$

for Ag_2CrO_4

$[Ag^+]^2\,[CrO_4^{-2}] > 10^{-13}$

$\Rightarrow$ $[Ag^+]^2 > 10^{-12}$

$[Ag^+] > 10^{-6}$

PPt $10^{-9} < [Ag^+] \le 10^{-6}$

only AgCl will PPt

When CrO_4^{-2} ion starts ppt

$[Ag^+] = 10^{-6}$

$[Cl^-]\,[Ag^+] = Ksp = 10^{-10}$

$[Cl^-] = 10^{-4}$

% of Cl^- remains = $\frac{10^{-4}}{0.1} \times 100 = 0.1$

% of Cl^- precipitated = 100 – 0.1 = 99.9 %

42. **A**

43. **D**

$6C(s) + 3H_2(g) \longrightarrow C_6H_6$

$\Delta H = 6Q_2 + 3Q_3 - Q_1$

44. **C**

Sol. (Benzyne Mechanism)

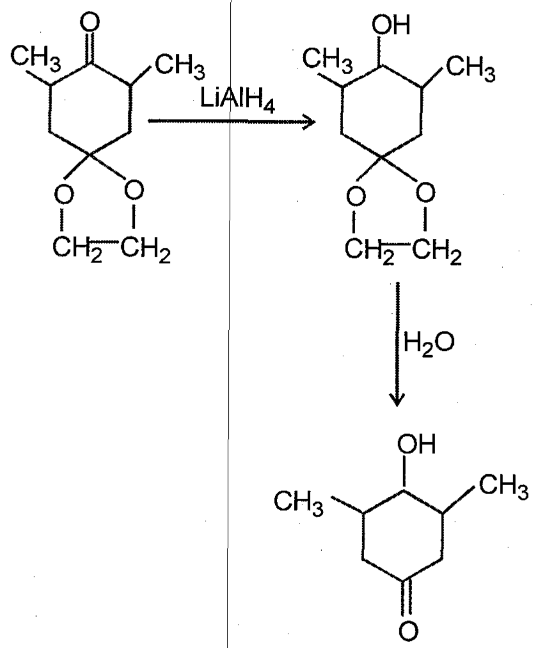
45. **C**

46. **C**

47. **B**

48. **A**

Sol.



49. C,D
50. A,D
51. B,C,D
52. A,C
53. D
54. B
55. C
56. B
57. C
58. A

Sol.

56. PV = nRT $\Rightarrow$ $\frac{114}{760} \times V = 0.01 \times 0.08 \times 300$

V = 1.6 lit

for He

$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{114}{300} = \frac{P_2}{600} \Rightarrow P_2 = 228$

P_2 = 228 mmHg

total pressure = 988 = 228 + 2 P (Pressure of NH_3)

$\Rightarrow$ P = 380 mmHg

57. moles of NH_3 = $\frac{PV}{RT} = \frac{380}{760} \times \frac{1.6}{0.08 \times 600} = \frac{1}{60}$ = 0.0166 mole

moles of NH_4Cl = $\frac{5.35}{53.5} = 0.1$

% decomposition = $\frac{0.0166}{0.1} \times 100$ = 16.6 %

58. Reaction proceeds in forward direction.

59. **$A \rightarrow P,S$; $B \rightarrow R$; $C \rightarrow Q,S$; $D \rightarrow Q,S$**

(A) $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

40 20

20 0 20

$P^H = P^{Ka} + \log 20/20 = P^{ka} = 5$

$[H^+] = 10^{-5}$

$0.1\text{ M } CH_3COONa \Rightarrow p^H = \frac{1}{2}[14 + 5 - 1] = 9$

$$E = -\frac{0.06}{1}\log C_1/C_2$$

$$= -0.06 \log \frac{10^{-5}}{10^{-9}} = -0.24\text{ V}$$

(B) $Ag^+ + e^- \longrightarrow Ag(s)$ $E^0 = 0.8\text{ V}$

$Ag\,I + e^- \longrightarrow Ag(s) + I^-$ $E^0 = x\text{ V}$

$Ag\,I \rightleftharpoons Ag^+ + I^-$ $E^0 = x - 0.8\text{ V}$

$$x - 0.8 = \frac{0.06}{1}\log Ksp = 0.059 \times (-16)$$

$x = -0.16\text{ V}$

(C) $$E = -\frac{0.06}{1}\log\frac{\sqrt{CKa}}{Kw}.\sqrt{K_b \times C}$$

$$= -0.06\log\frac{CKaKb}{Kw}$$

$= -0.06 \log 0.1 \times 10^{-10} \times 10^{14}$

$= -0.18\text{ V}$

(D) $$E = -\frac{0.06}{1}\log\frac{10^{-10}}{10^{-13}}$$

$= -0.06 \log 10^3$

$= -0.18\text{ V}$

60. **$A \rightarrow Q,R,S$; $B \rightarrow P,S$; $C \rightarrow R$; $D \rightarrow Q,S$**