TARGET IIT-JEE

HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-I CLASS XI (DATE 20-09-09)

MATHEMATICS

Q.	1	2	3	4	5	6	7	8	9	10	11	[illegible]
A.	D	B	B	A	C	C	A	D	B,C,D	B,D	C,D	A,
Q.	13	14	15	16	17	18						
A.	A	C	C	C	B	A						
Q.	19						20					
A.	(A) → T ; (B) → R, S, T ; (C) → P, Q, R, S, T ; (D) → S						(A) – R, (B) – QRS, (C) – PS,					

PHYSICS

21. C 22. C 23. A 24. A 25. D 26. D 27. B

28. B 29. A,B,D 30. A,D 31. A,C,D 32. A,C,D 33. B 34. D

35. A 36. D 37. A 38. A

39. A → Q ; B → R ; C → S ; D → P

40. A → Q ; B → R ; C → S ; D → P

CHEMISTRY

41. A 42. A 43. A 44. C 45. A 46. B 47. D

48. A 49. A,B,D 50. B,C 51. C,D 52. B,D 53. B 54. A

55. A 56. B 57. B 58. D

59. A → R ; B → S ; C → P ; D → Q,R ; E → T

60. A → Q ; B → P ; C → S ; D → R

SOLUTIONS

MATHEMATICS

1. $\left|\frac{2}{1-r}-2\right| < \frac{1}{10} \Rightarrow \left|\frac{1}{1-r}-1\right| < \frac{1}{20}$

$\Rightarrow \left|\frac{1-1+r}{1-r}\right| < \frac{1}{20} \Rightarrow \left|\frac{r}{1-r}\right| < \frac{1}{20}$

2. Note that from theory of equations

$a + b + c = 0$

hence $a^3+b^3+c^3 = 3abc = 3(-4) = -12$ Ans.]

3. $D \geq 0 \Rightarrow 16x^2 - 16(x + 6) \geq 0$

$\Rightarrow (x-3)(x+2) \geq 0$]

4. $3 \tan A - 4 = 0$

$\Rightarrow \tan A = \frac{4}{3} \Rightarrow \sin A = -\frac{4}{5}$; $\cos = -\frac{3}{5}$

$\Rightarrow 5 \sin 2A + 3 \sin A + 4 \cos A$

$= 10 \sin A \cos A + 3 \sin A + 4 \cos A$

$= 10\left(\frac{12}{25}\right) - \frac{12}{5} - \frac{12}{5} = 0$

5. $m + n = a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha + 3a\cos^2\alpha\sin\alpha + a\sin^3\alpha$

$\Rightarrow a(\cos\alpha + \sin\alpha)^3 = (m+n)$ — (1)

Similarly, $a(\cos\alpha - \sin\alpha)^3 = (m-n)$ — (2)

from (1) & (2)

$a^{1/3}(\cos\alpha + \sin\alpha) = (m+n)^{1/3}$ — (3)

$a^{1/3}\cos\alpha - \sin\alpha) = (m-n)^{1/3}$ — (4)

square equation (3) & (4) and add.

$\therefore (m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$

6. 3x, 4y, 5z are in G.P.

$\therefore 16y^2 = 15xz$ ....(1)

and $y = \frac{2xz}{x+z}$ ....(2)

using (2) in (1)

$16 \times 4x^2z^2 = 15(x+z)^2xz$

$\frac{(x+z)^2}{xz} = \frac{64}{15}$; $\frac{x}{z} + \frac{z}{x} + 2 = \frac{64}{15}$;

$\therefore \frac{x}{z} + \frac{z}{x} = \frac{34}{15}$

$\therefore m = 34$ and $n = 15$

$\therefore m + n = 34 + 15 = 49$ Ans.

7. $T_n = \frac{1}{3}\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\ldots\ldots\right]$

$= \frac{2}{3}\left[\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\frac{1}{5\cdot6}+\ldots\ldots\right]$

hence T_n using method of diff;

$T_n = \frac{2}{3}\frac{1}{(n+1)(n+2)} = \frac{2}{3}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$

$\therefore S_\infty = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$ Ans.]

8. Consider ax, by, cz and (4d–ax–by – cz)

A.M. ≥ G.M.

$\Rightarrow \frac{4d}{4} \geq [(ax)(by)(cz)(4d - ax - by - cz)]^{1/4}$

$\Rightarrow d^4 \geq abc\ xyz(4d - ax - by - cz)$

$\Rightarrow xyz(4d - ax - by - cz) \leq \frac{d^4}{abc} \Rightarrow$ D

9. Let $f(x) = (x-a)(x-b)(x-c)$

$\therefore g(x) = k(x-a^2)(x-b^2)(x-c^2)$

Since $abc = -f(0) = -1$, $k = ka^2b^2c^2 = -g(0) = 1$

$\therefore g(x^2) = (x^2-a^2)(x^2-b^2)(x^2-c^2)$

$= -f(x)\,f(-x)$.

put x = 3, $g(9) = -f(3) \cdot f(-3) = 899$

put x = 1, $g(1) = -f(1)\,f(-1) = -f(1)\cdot(-1) = f(1)$

10. log a, log b, log c are in A.P.

$\Rightarrow 2\log b = \log a + \log c$

$\therefore b^2 = ac$ ...(1)

$\Rightarrow$ a, b, c are in G.P. $\Rightarrow$ (B)

also given (log a – log 2b), (log 2b – log 3c), (log 3c – log a) are in A.P.

$\Rightarrow 2(\log 2b - \log 3c) = \log 3c - \log 2b$

$\Rightarrow 3\log 2b = 3\log 3c$

$\Rightarrow 2b = 3c$ ....(2)

$\Rightarrow 4b^2 = 9c^2$ ...(3)

from (1) and (3)

$4ac = 9c^2 \Rightarrow a = \frac{9c}{4}$ and $b = \frac{3c}{2}$

$a = \frac{9c}{4}$; $b = \frac{3c}{2}$ and $c = c$

$\therefore$ a, b, c form the sides of triangle $\Rightarrow$ (D)

a, 2b and 3c are not in H.P.

|||y verify (A)

11. $\Sigma \cos 3A = 1 + 4 \sin\frac{3A}{2}\ \sin\frac{3B}{2}\ \sin\frac{3C}{2} = 1$

$\Rightarrow\ A = \frac{2\pi}{3}$ or $B = \frac{2\pi}{3}$ or $C = \frac{2\pi}{3}$

12. The roots of a cubic polynomial $p(x)=x^3+ax^2+bx + c$ satisfy $x_1 + x_2 + x_3 = -a$. Therefore, they form an arithmetic progression if and only if $-a/3$ is a root, i.e. $P(-a/3) = 0$. This gives the condition
$(-a/3)^3 + a(-a/3)^2 + b(-a/3) + c = 0$
or $2(a/3)^3 = (a/3)b - c$
Only the polynomial in (D) $x^3 + 3x^2 - 2x - 4$ satisfies this condition and other do not. Hence roots in (A), (B), (C) are not in A.P.

13. $f(x) = (\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) + k [(\sin^2 + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x]$

$f(x) = 1 - 3 \sin^2 x (1 - \sin^2 x) + k [1 - 2 \sin^2 x(1 - \sin^2 x)]$

$f(x) = (2k + 3) \sin^4 x - \sin^2 x(2k + 3) + k + 1$

$f(x) = (2k + 3)\left(\sin^2 x - \frac{1}{2}\right)^2 - \frac{2k+3}{4} + k + 1$

$f(x) = (2k + 3)\left(\sin^2 x - \frac{1}{2}\right)^2 + \frac{2k+1}{4}$

$f(x)$ is constant when $2k + 3 = 0$

$k = -\frac{3}{2}$

Number of values of $k = 1$

14. At $k = -0.7$, $f(x) = 0$

$0 = 1.6\left(\sin^2 c - \frac{1}{2}\right)^2 - 0.1$

$\Rightarrow \left(\sin^2 c - \frac{1}{2}\right)^2 = \frac{1}{16}$

$\sin^2 c = \frac{3}{4}$ or $\frac{1}{4}$ or $\sin c = \pm\frac{\sqrt{3}}{2}, \pm\frac{1}{2}$

so no. of different values = 4

15. $f(c) = 0$

$(2k + 3)\left(\sin^2 c - \frac{1}{2}\right)^2 + \frac{2k+1}{4} = 0$

$\Rightarrow \left(\sin^2 c - \frac{1}{2}\right)^2 = -\frac{(2k+1)}{4(2k+3)}$

Since $0 \le \left(\sin^2 c - \frac{1}{2}\right)^2 \le \frac{1}{4}$

$\Rightarrow -\frac{(2k+1)}{4(2k+3)} \ge 0 \Rightarrow \frac{2k+1}{2k+3} \le 0$

$-\frac{3}{2} < k \le -\frac{1}{2}$ ...(1)

Also $-\frac{1}{4}\left(\frac{2k+1}{2k+3}\right) \le \frac{1}{4}$

$\Rightarrow 1 + \left(\frac{2k+1}{2k+3}\right) \ge 0 \Rightarrow \frac{4k+4}{2k+3} \ge 0$

$\frac{k+1}{2k+3} \ge 0 \Rightarrow k \ge -1$

or $k < -\frac{3}{2}$ ...(2)

(1) & (2) $\Rightarrow -1 \le k \le -\frac{1}{2}$

16. Let $\frac{a}{r}$, a, ar be the roots

$\therefore\ a^3 = -\frac{k}{2}$ ...(1)

now $\frac{a}{r} + a + ar = \frac{19}{2}$ ....(2)

and $\frac{a^2}{r} + a^2 r + a^2 = \frac{57}{2}$

$a\left(\frac{a}{r} + ar + a\right) = \frac{57}{2}$

$a.\frac{19}{2} = \frac{57}{2} \Rightarrow a = 3$

from (1) $k = -2a^3 = -54$ Ans

17. $a = 3$

now substituting in (2),

$r = 3/2$ or $2/3$

hence the GP's are 2, 3, 9/2,.....

or 9/2, 3, 2,......

hence $S_n = \frac{2\left(\left(\frac{3}{2}\right)^n - 1\right)}{\frac{3}{2} - 1} = 4\left(\left(\frac{3}{2}\right)^n - 1\right)$ Ans

18. $S_\infty = \frac{\frac{9}{2}}{1 - \frac{2}{3}} = \frac{9}{2}.\frac{3}{1} = \frac{27}{2}$ **Ans.**

19. (A) $f(\theta) = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$
$= \sin^2\theta + \cos^2\theta + \operatorname{cosec}^2\theta + \sec^2\theta + 4$
$= 5 + 1 + \cot^2\theta + 1 + \tan^2\theta$
$= 9 + (\tan\theta - \cot\theta)^2 \ge 9$

(B) $\sin\alpha - \sin\beta = a$, $\cos\alpha + \cos\beta = b$

$\Rightarrow a^2 + b^2 = 2 + 2\cos(\alpha + \beta) = 4\cos^2\frac{\alpha+\beta}{2} \le 4$

(C) $\frac{1}{\sqrt{2}}(\sin A + \sin B) = \frac{1}{\sqrt{2}}\left(\sin A + \sin\left(\frac{\pi}{2} - A\right)\right)$

$= \frac{1}{\sqrt{2}}$ $(\sin A + \cos A)$ $= \sin\left(A + \frac{\pi}{4}\right) \le 1.$

(D) Let $A = 7 \cos x + 6 \sin x = 6(2 \cos x + \sin x) - 5 \cos x = 6 - 5 \cos x$

Now, $2 \cos x + \sin x = 1 \Rightarrow \sin = 1 - 2 \cos x$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - 4\cos x + 4\cos^2 x$

$\therefore \cos x = 0$ or $\frac{4}{5}$. But $\cos x \ne \frac{4}{5} \Rightarrow \cos x = 0$

$\therefore$ $A = 6$

20. (A) Given $a^2 - 4a + 1 = 4$

$\Rightarrow$ $a^2 + 1 = 4(1 + a)$

$y = \frac{(a-1)(1+a^2)}{a^2 - 1} = \frac{a^2+1}{a+1} = \frac{4(a+1)}{a+1} = 4$ **Ans.**

$\Rightarrow$ (R)

(B) $\sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$

taking log on both the sides

$\frac{x+1}{4} \log |x - 3| = \frac{x-2}{3} \log |x - 3|$

$\Rightarrow \log |x-3| \left[\left(\frac{x+1}{4}\right) - \left(\frac{x-2}{3}\right)\right] = 0$

$\Rightarrow$ $\log |x - 3| = 0$ or $\left[\left(\frac{x+1}{4}\right) - \left(\frac{x-2}{3}\right)\right] = 0$

$\Rightarrow$ $x = 4, 2$ or $x = 11$

Ans. $\Rightarrow$ Q,R,S

(C) critical points $x = 0, 6$

Case 1 : $x \ge 6$

$3^x + 1 - (3^x - 1) = 2\log_5(x - 6)$

$\Rightarrow x = 11$ Ans. $\Rightarrow$ (S)

Case 2 : $0 \le x \le 6$

$3^x + 1 - (3^x - 1) = 2 \log_5(6 - x) \Rightarrow x = 1$ Ans. $\Rightarrow$ (P)

Case 3 : $x < 0$

$3^x + 1 + 3^x - 1 = 2 \log_5(6 - x)$

$3^x = \log_5(6 - x)$

for $x < 0$ L.H.S. is less than one and R.H.S. is greater than one $\Rightarrow$ no solution

(D) $\frac{2n}{2}(4 + (2n - 1)3) = \frac{n}{2}(114 + (n - 1)2)$

$2(1 + 6n) = 112 + 2n \Rightarrow 110 = 10n \Rightarrow n = 11$ Ans. $\Rightarrow$ (S)

21. **C**

Given $a = kt + c$

at $t = 0$, $a = \tan 37° = 3/4$

$\Rightarrow c = \frac{3}{4}$

at $t = 3$ sec $a = 0$ (slope zero)

$\Rightarrow$ $k = -1/4$

$\Rightarrow$ $a = -\frac{t}{4} + \frac{3}{4}$

at velocity zero, $a = \tan 135° = -1$

$\Rightarrow$ $-1 = -\frac{t}{4} + \frac{3}{4}$ $\Rightarrow$ $t = 7$ sec.

22. **C**

Velocity of man with respect to ground

$V_{MG} = V_{MT} + V_{TG}$

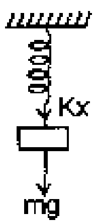
$= 1.5\,v + v = 2.5\,v$

Now distance $= \frac{L}{1.5v} \times 2.5v = \frac{5L}{3}$

23. **A**

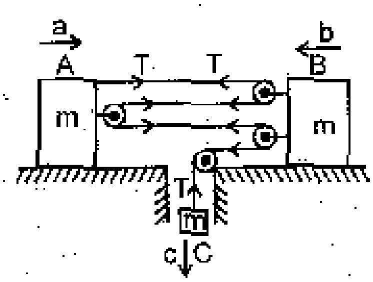
Before cutting $kx = mg$

After cutting



downward force = 2mg

24. **A**



from constrain

$-3a - 4b = -c$ ...(1)

$a = \frac{3T}{m}$ ...(2)

$b = \frac{4T}{m}$ ...(3)

$mc = mg - T$

$-3\left(\frac{3T}{m}\right) - 4\left(\frac{4T}{m}\right) = \frac{T - mg}{m}$ $\Rightarrow -9T - 16T = T - mg$

$-26.T = -mg$ $\Rightarrow$ $T = \frac{mg}{26}$

$b = \frac{4}{m} \cdot \frac{mg}{26} = \frac{2}{13} g$

25. **D**

26. **D**

27. **B**

Condition for collision in mid air.

$\alpha_{AB} = 0$ and $\vec{v}_{AB}$ should be directed from A to B.

$\therefore$ $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

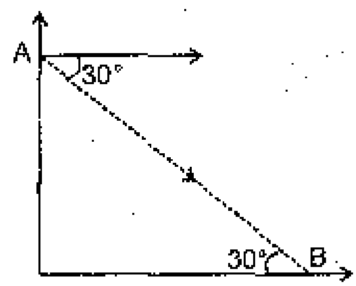
$= 20\,\hat{i} - [-20\cos(\theta + 30)]\,\hat{i} + 20\sin(\theta + 30)\,\hat{j}]$

$= [20 + 20\cos(\theta + 30)]\,\hat{i} - 20\sin(\theta + 30)\,\hat{j}]$

$\tan 30° = \frac{h_y}{h_x} = \frac{20\sin(\theta + 30°)}{20 + 20\cos(\theta + 30°)}$

$1 + \cos(\theta + 30°) = \sqrt{3}\,\sin(\theta + 30°)$

$\frac{1}{2} = \frac{\sqrt{3}}{2}\sin(\theta + 30°) - \frac{1}{2}\cos(\theta + 30°)$



$\frac{1}{2} = \sin(\theta + 30 - 30)$

$\sin(\pi/6) = \sin\theta$

$\theta = \frac{\pi}{6} = 30$

$\therefore$ $\vec{v}_{AB} = (20 + 20\cos 60°)\,\hat{i} - 20\sin 60°\,\hat{j}$

$= 30\hat{i} - 10\sqrt{3}\,\hat{j}$

$$|\vec{v}_{AB}| = \sqrt{(30)^2 + (10\sqrt{3})^2} = 10\sqrt{9+3} = 20\sqrt{3}\ ms^{-1}$$

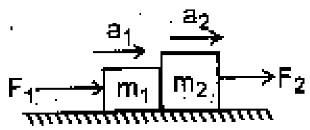
$$\text{time to collide} = \frac{\text{relative distance}}{\text{relative speed}}$$

$$\text{time to collide} = \frac{200}{20\sqrt{3}} = \frac{10}{\sqrt{3}}$$

28. **B**

Acceleration of 2 is more than acceleration of 1.

29. **A,B,D**



(A) when $a_1 = a_2$

or $\frac{F_1}{m_1} = \frac{F_2}{m_2} \Rightarrow N = 0$

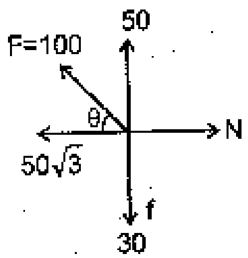
(B) when $a_1 < a_2$

or $\frac{F_1}{m_1} < \frac{F_2}{m_2} \Rightarrow N = 0$

30. **A,D**

31. **A,C,D**

32. **A,C,D**



$N = 50\sqrt{3}$

$\therefore F \sin\theta > mg$

Friction acts in downward direction.

$$f_{(s)(max.)} = \mu N = \frac{1}{4} \times 50\sqrt{3}$$

$$= \frac{25\sqrt{3}}{2} N = 21.625\ N$$

$\therefore f = 50 - 30 = 20\ N$

33. **B**

34. **D**

35. **A**

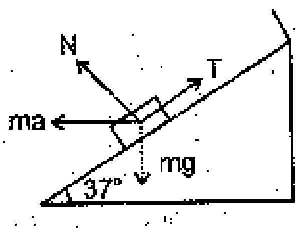
Relative to the bucket (which floats downstream) the boat's speed upstream and downstream must be the same. Thus the boatman also spends 40 min on returning to meet the bucket. Further, the speed of the boat downstream relative to the bank is 25 m/min (since it takes 40 min to travel 1 km to point A). After meeting the bucket, the boat spends 24 min on returning downstream, so the meeting place is 600 m above A, or 400 m from the place where the bucket is lowered into the water. Thus the bucket travels 400 m in the 80 min between being lowered and picked up, i.e. the speed of the current is 5 m/min and that of the boat relative to the water is 20 m/min.

36. **D**

37. **A**

38. **A**

Sol.



36. $N = mg\cos 37° - ma\sin 37° = \frac{3}{4}mg$

$a = \frac{5}{6} m/s^2$

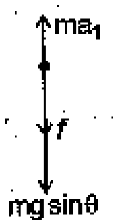
37. $T = mg\sin 37° + ma\cos 37°$

$T = 12\ N$

38. $mg\cos 37° = ma\sin 37°$

$a = \frac{40}{3}\ m/s^2$

39. $A \rightarrow Q$; $B \rightarrow R$; $C \rightarrow S$; $D \rightarrow P$



For a_1 = $ma_1 = mg\sin\theta + \mu N$

$= mg\sin\theta + \mu\, mg\cos\theta$

$= 2mg\sin\theta$

$a_1 = 2g\sin\theta$

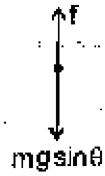
For a_2 $\quad a_2 \longleftrightarrow g\sin\theta$

$F = \mu N$

$= mg\sin\theta$

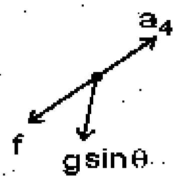
$ma_2 = mg\sin\theta \Rightarrow a_2 = g\sin\theta$

For a_3



$a_3 = 0$

For a_4



$a_4 < 2g\sin\theta$

40. $A \to Q$; $B \to R$; $C \to S$; $D \to P$

Compare it with $v = u_x\hat{i} + u_y\hat{j}$

$u_x = a$, $u_y = b$

at $t = \frac{b}{2g}$, $v_y = b + (-g)\left(\frac{b}{2g}\right) = \frac{b}{2}$

$$\frac{v_x}{v_y} = \frac{2a}{b}$$

$$H_{max} = \frac{u_y^2}{2g} = \frac{b^2}{2g}$$

$$R = \frac{2u_xu_y}{g} = \frac{2.a.b}{g}$$

$$\frac{v_{av}}{v_0} = \frac{u_x}{\sqrt{a^2+b^2}} = \frac{a}{\sqrt{a^2+b^2}}$$

41. A

Wt. of one molecule of caffeine = 194 amu.

N present in one molecule

$$= \frac{28.9}{100} \times 194 = 56 \text{ amu} = 4\text{N atoms}$$

42. A

43. A

$$N_2O_4 \rightleftharpoons 2NO_2$$

Initial 4 moles

After decomposition $4 - \frac{20}{100} \times 4$ 2×0.8 mole

= 3.2 moles = 1.6 mole

Total no. of moles = 3.2 + 1.6 = 4.8

$\therefore$ Resultant pressure = $\frac{4.8}{4} \times 1 \text{ atm} = 1.2 \text{ atm}$

44. C

45. A

$$M + X_2 \longrightarrow MX_2$$

$$3MX_2 + X_2 \longrightarrow M_3X_8$$

$$M_3X_8 + N_2CO_3 \longrightarrow NX + CO_2 + M_3O_4$$

using POAC on X in equation (iii)

$\Rightarrow$ 8 × moles of M_3X_8 = 1 × moles of NX

$\Rightarrow$ moles of $M_3X_8 = \frac{206}{103 \times 8} = \frac{1}{4}$

moles of $MX_2 = 3 \times \frac{1}{4}$ $\Rightarrow$ moles of $M = \frac{3}{4}$

$\therefore$ mass of $M = \frac{3}{4} \times 56 = 42$ gm

46. B

Moles of CaO = $\frac{1.62}{56}$

$\Rightarrow$ moles of $CaCO_3 = \frac{1.62}{56}$

$\Rightarrow$ moles of $CaCl_2 = \frac{1.62}{56}$

wt. of $CaCl_2 = \frac{1.62}{56} \times 111$

= 3.21 gm

$\% CaCl_2 = \frac{3.21}{10} \times 100 = 32.1\%$

47. D

48. A

49. A,B,D

$$2KClO_3 \longrightarrow 2KCl + 3O_2$$

$$C_3H_8 + \frac{9}{2} O_2 \longrightarrow 3CO_2 + 3H_2O$$

moles of $H_2O = \frac{54}{18} = 3$

$\therefore$ moles of O_2 needed for the reaction = $\frac{9}{2}$

$\therefore$ moles of $KClO_3$ required = $\frac{2}{3} \times \frac{9}{2} = 3$ mole

volume of CO_2 at STP = 3 × 22.4 = 67. 2 Ltr.

mass of hydrocarbon combusted = 1 × 42 = 42 gm

50. B,C

moles of C_2H_6 = 3

moles of C_2H_6 mixed = $\frac{60}{30} = 2$

total mole of C_2H_6 = 5

moles removed = $\frac{2.4 \times 10^{24}}{6 \times 10^{23}} = 4$

$\therefore$ moles of C_2H_6 left = 1

Now, $C_2H_6 + 7/2\ O_2 \longrightarrow 2CO_2 + 3H_2O$

clearly 3 moles of H_2O or 54 gm H_2O will be formed

volume of H_2O = 54 ml

51. C,D

$$2X + 3Y \rightarrow X_2Y_3$$

moles : $\frac{W}{36}$ $\frac{W}{24}$

mole ratio is 2 : 3 hence no reactant will be left over.

mass of X_2Y_3 formed

$$= \frac{1}{3} \times \frac{W}{24} \times (36 \times 2 + 24 \times 3) = 2W$$

52. B,D

53. B

54. A

55. A

56. B

57. B

58. D

59. A → R ; B → S ; C → P ; D → Q,R ; E → T

60. A → Q ; B → P ; C → S ; D → R