# MOTION IIT-JEE
(Where Faith Counts the Success)

# TARGET IIT-JEE
## HINT & SOLUTIONS

## ANSWER KEY WITH SOLUTION

### PAPER-II CLASS XII & XIII (DATE 13-09-09)

### MATHEMATICS

**1.** C **2.** B **3.** B **4.** D **5.** B,C **6.** A,B,D
**7.** A,C **8.** A,C **9.** A,D
10. (A) – R, (B) – R, (C) – T, (D) – Q
11. (A) – PQR, (B) – PQRS, (C) – PQ, (D) – P
**12.** 0 **13.** 7 **14.** 1 **15.** **16.** 2 **17.** 2
**18.** 0 **19.** 4

### PHYSICS

**20.** D **21.** D **22.** A **23.** C **24.** B,C,D **25.** B **26.** A,C
**27.** A,B **28.** B,D
**29.** $A \to R$ ; $B \to S$ ; $C \to P$; $D \to Q$
**30.** $A \to R$ ; $B \to S$ ; $C \to Q$; $D \to P$
**31.** $\frac{1}{(2)^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$ **32.** 0.64 volts **33.** $\frac{100}{3}\mu C$ **34.** 2/3 **35.** 4R from O
**36.** 6.00 m **37.** 3.225 m **38.** $20 cm^2$

### CHEMISTRY

**39.** D **40.** B **41.** C **42.** B **43.** C,D **44.** B,D **45.** D
**46.** A,B,D **47.** B,D
**48.** $A \to P,Q,R,S$ ; $B \to Q$ ; $C \to R$ ; $D \to P,Q,S$
**49** $A \to P,Q$ ; $B \to P,Q,R,S$ ; $C \to R,S$ ; $D \to R,S$
**50.** 86.67 % **51.** 50% **52.** 16 **53.** 54% **54.** 8.4 **55.** 12.3 **56.** 6
**57.** 70

**1.** Given $f(x) = [\sin x + [\cos x + [\tan x + [\sec x]]]]$
$= [\sin + p]$, where $p = [\cos x + [\tan x + [\sec x]]]$
$= [\sin x] + p$, (as p is an integer)
$= [\sin x] + [\cos x + [\tan x + [\sec x]]]$
$= [\sin x] + [\cos x] + [\tan x] + [\sec x]$

Now, for $x \in (0, \pi/4)$, $\sin x \in \left(0, \frac{1}{\sqrt{2}}\right)$,

$\cos x \in \left(\frac{1}{\sqrt{2}}, 1\right)$, $\tan x \in (0, 1)$, $\sec x \in (1, \sqrt{2})$

$\Rightarrow$ $[\sin x] = 0$, $[\cos x] = 0$,
$[\tan x] = 0$ and $[\sec x] = 1$
$\Rightarrow$ The range of f(x) is 1.

**2.** Applying LMVT in [0, 1] and then in [1, 2]

$c_1 \in (0, 1)$, $\quad \frac{f(2)-f(1)}{2-1} = f'(c_1) \le 3$

$\Rightarrow \quad f(1) \le 5$

$c_2 \in (1, 2)$, $\quad \frac{f(2)-f(1)}{2-1} = f'(c_2) \le 3$

$\Rightarrow \quad f(1) \ge 5$

Hence, $f(1) = 5$ Ans.

**3.** Clearly $\vec{r} - \vec{a}$, $\vec{r} - \vec{b}$ and $\vec{r} - \vec{c}$ are coplanar

$\Rightarrow \quad (\vec{r} - \vec{a}) . ((\vec{r} - \vec{b}) \times (\vec{r} - \vec{c})) = 0$

$\Rightarrow \quad (\vec{r} - \vec{a}) . (\vec{r} \times (\vec{r} - \vec{c}) - \vec{b} \times (\vec{r} - \vec{c})) = 0$

$\Rightarrow \quad (\vec{r} - \vec{a}) . (-\vec{r} \times \vec{c} - \vec{b} \times \vec{r} + \vec{b} \times \vec{c}) = 0$

$\Rightarrow \quad -\vec{r} . (\vec{r} \times \vec{c}) - \vec{r} . (\vec{b} \times \vec{r}) + \vec{r} . (\vec{b} \times \vec{c})$

$+ \vec{a} . (\vec{r} \times \vec{c}) + \vec{a} . (\vec{b} \times \vec{r}) - \vec{a} . (\vec{b} \times \vec{c}) = 0$

$\Rightarrow \vec{r} . (\vec{b} \times \vec{c}) - \vec{r} . (\vec{a} \times \vec{c}) + \vec{r} . (\vec{a} \times \vec{b}) = [\vec{a}\, \vec{b}\, \vec{c}]$

$\Rightarrow \vec{r} . (\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}) = [\vec{a}\vec{b}\vec{c}]$

**4.** I Use Rolle's theorem in [0, 1]
II Use LMVT in [0, 4]
III Use LMVT in [0, 2]

**5.** $\lim_{n\to\infty} \frac{A_n}{n} = \lim_{n\to\infty} \frac{3n+1}{2n} = \frac{3}{2}$

$L = \lim_{n\to\infty} \frac{G_n}{n} = \lim_{n\to\infty} \frac{1}{n}((n + 1)(n + 2) \ldots (2n))^{1/n}$

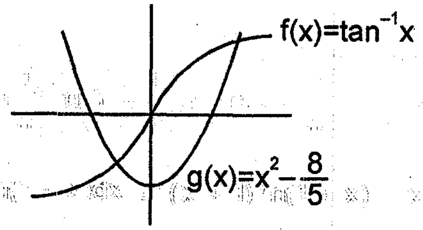
$\ell n\, L = \lim_{n\to\infty} \frac{1}{n}\left[\ell n\left(1 + \frac{1}{n}\right) + \ell n\left(1 + \frac{2}{n}\right) \ldots\ldots \ell n\left(1 + \frac{n}{n}\right)\right]$

$= \int_0^1 \ell n(1 + x)dx = (x + 1)\ell n(1 + x) - x\Big|_0^1$

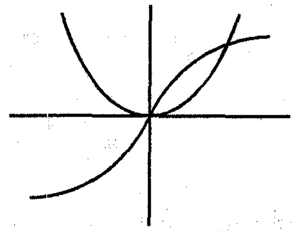
$= 2\,\ell n\, 2 - 1 = \ell n \frac{4}{e}$

$\therefore \quad L = \frac{4}{e}$

**6.** (A)

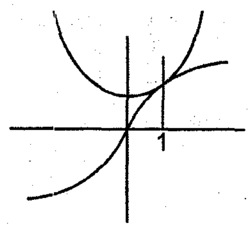



(B)



$f'(0) = 1$
$g'(0) = 0$

(C)




$f'(x) = \frac{1}{2}$.

$g(x) = \frac{1}{4}x^2 + \left(\frac{\pi - 1}{4}\right)$

$g(1) = \frac{\pi}{4} = f(1)$

$g'(1) = \frac{1}{2}$

f & g touch each other.

(D) If we pull down the graph of g(x) in (C) slightly down, there will be two solutions.

**7.** $I = \int_0^{\pi/2} f(\sin\theta)\cos\theta\, d\theta$

$I = \int_0^{\pi/2} f(\cos\theta)\sin\theta\, d\theta$

$2I = \int_0^{\pi/2} (f(\sin\theta)\cos\theta + f(\cos\theta)\sin\theta)\, d\theta$

$\le \int_0^{\pi/2} 1\, d\theta = \frac{\pi}{2}$

8. Taking log. LHS = $\log \dfrac{\sin x_1}{x_1} + \log \dfrac{\sin x_2}{x_2} + \log \dfrac{\sin x_3}{x_3}$

The function $f(x) = \log \dfrac{\sin x}{x}$ is concave down

$$\therefore \frac{1}{3}\left(\log\frac{\sin x_1}{x} + \log\frac{\sin x_2}{x_2} + \log\frac{\sin x_3}{x_3}\right) \le \frac{\log\sin\left(\frac{x_1+x_2+x_3}{3}\right)}{\frac{x_1+x_2+x_3}{3}}$$

$\Rightarrow$ option A

$f(x) = e^x$ is concave up.

$$\Rightarrow \frac{e^{x_1} + e^{x_2} + e^{x_3}}{3} \ge e^{\frac{x_1+x_2+x_3}{3}}$$

$\Rightarrow$ Option C.

9. Let $P(x) = ax^3 + bx^2 + cx + d$.

$$\int_{-1}^{1} P(x)\,dx = 2\left(\frac{b}{3} + d\right).$$

Since $P(\alpha) + P(\beta) = 2\left(\frac{b}{3} + d\right)$

which is independent of a & c

$\alpha = -\beta$ in which case

$$2(b\beta^2 + d) = 2\left(\frac{b}{3} + d\right)$$

$$\Rightarrow \quad \beta = \pm\frac{1}{\sqrt{3}}$$

$$\alpha = -\frac{1}{\sqrt{3}},\ \beta = \frac{1}{\sqrt{3}}.$$

10. (A) $\lim\limits_{x\to\infty} \dfrac{f(x)}{x}$ $\left(\dfrac{\infty}{\infty}\right)$ form

applying L–H Rule.

$$= \lim_{x\to 0}\frac{f'(x)}{1} = 0$$

(B) $f(x) = x^4 + ax + b.$

$$f'(x) = 4x^3 + a \Rightarrow x = -\left(\frac{9}{4}\right)^{\frac{1}{3}}.$$

Since there is only one turning point then cannot be four real roots.

(C) Consider $h(x) = e^{kx}f(x)$

$h(a) = h(b) = 0.$

$h'(x) = e^{kx}(f'(x) + kf(x))$

Using Rolle's Theorem there is atleast one point $c \in (a, b)$ such that $h'(c) = 0$

$\Rightarrow \quad e^{kc}(f'(c) + kf(c)) = 0$

$\Rightarrow \quad f'(c) + kf(c) = 0 \quad \Rightarrow \quad g(c) = 0.$

(D) Let $(f(x))^3 = \int_0^1 (f(t))^2 dt = k$

$\Rightarrow \quad f(x) = k^{1/3}$

Place back $(f(x))^3 = \int_0^1 k^{2/3} dt$

$\Rightarrow \quad k = k^{2/3}$

$k^3 = k^2$

$k^2(k - 1) = 0$

$k = 0, 1$

$f(x) = 0$ or $f(x) = 1$

11. (A) $\log_2 x + 2\log_2 y + 2\log_2 z = 4 = \log_2(xy^2z^2)$

$\Rightarrow \quad xy^2z^2 = 16$

$$\frac{x + \frac{y}{2} + \frac{y}{2} + \frac{z}{2} + \frac{z}{2}}{5} \ge \left(x\frac{y^2}{4}\frac{z^2}{4}\right)^{1/5} = 1 \quad (\text{AM} \ge \text{GM})$$

$$\Rightarrow \frac{x+y+z}{5} \ge 1,\ x + y + z \ge 5 \Rightarrow \quad P, Q, R$$

(B) $3^{|\sin x|} \in [1, 3]$, $2^{-|\sec y|} \in \left(0, \frac{1}{2}\right]$, $5\cos z \in [-5, 5]$

$\Rightarrow$ P, Q, R, S

$$a = 5\cos z + 3^{|\sin x|}\, 2^{-|\sec y|} \in \left(-5 + 0, \frac{3}{2} + 5\right] \in \left(5, 6\frac{1}{2}\right]$$

(C) cosec A, cosec B, cosec C are in H.P.

$\dfrac{1}{\sin A}, \dfrac{1}{\sin B}, \dfrac{1}{\sin C}$ are in A.P.

sin A, sin B, sin C are in A.P.

$\therefore \quad 2b = a + c$

$a + c > b \Rightarrow \quad 2b > b$ is true (No conclusion)

$a + b > c \Rightarrow \underbrace{2b - c}_{a} + b > c \Rightarrow$

$$b > \frac{2c}{3} \Rightarrow \frac{b}{c} > \frac{2}{3} \Rightarrow \frac{2b}{c} > \frac{4}{3}$$

$b + c > a \quad \Rightarrow \quad b + c > 2b - c$

$\Rightarrow \quad b < 2c \Rightarrow \quad \frac{b}{c} < 2$

$\frac{2b}{c} \in \left(\frac{4}{3}, 4\right) \quad \Rightarrow \quad P, Q$

(D) $a + b = 3$

HM ≤ AM for 3 numbers $\frac{a}{2}, \frac{a}{2}$, b we have

$$\frac{3}{\frac{2}{a}+\frac{2}{a}+\frac{1}{b}} \le \frac{\frac{a}{2}+\frac{a}{2}+b}{3} = 1;$$

$$\therefore \quad 1 \ge \frac{3}{\frac{2}{a}+\frac{2}{a}+\frac{1}{b}} \Rightarrow \quad \frac{2}{a}+\frac{2}{a}+\frac{1}{b} \ge 3$$

$$\frac{4}{a}+\frac{1}{b} \ge 3 \quad \Rightarrow \quad P$$

**12.** $(r - a)(r - b)(r - c)(r - d) = 9$
All of the factor on the LHS are distinct integer.
$9 = (-3)(-1)(1)(3)$
Assuming $a < b < c < d$,
$r - a = -3 \quad r - d = 3.$
$r - b = -1$
$r - c = 1$
Hence on addition $4r - (a + b + c + d) = 0.$

**13.** $f(1) = 1$
$f(2) = f(f(1)) + f(2 - f(1))$
$= f(1) + f(1) = 2$
$f(3) = f(f(2)) + f(3 - f(2))$
$= f(2) + f(3 - 2) = 3$
Let us assume $f(n) = n$.
$f(n + 1) = f(f(n)) + f(n + 1 - f(n))$
$= f(n) + f(n + 1 - n) = f(n) + f(1) = n + 1$

$$\therefore \quad \frac{1}{30}\sum_{r=1}^{20} f(r) = \frac{1}{30}\sum_{r=1}^{20} r$$

$$= \frac{1}{30}\sum_{r=1}^{20} f(r) = \frac{1}{30}\sum_{r=1}^{20} r = \frac{1}{30}.\frac{20\times 21}{2} = 7.$$

**14.** From the first equation,
$(1 + x)(1 - x^{10}) - x = 0$
$\Rightarrow \quad x^{11} + x^{10} = 1 \quad ....(1)$
The second equation gives
$(1 + x)(1 - x^{11}) - k$
$= 1 + x - x(x^{11} + x^{10}) - k$
$= 1 + x - x - k$ using (1)
$= 1 - k$
$= 0$ if $k = 1$.

**15.** Adding all the three equation
$2(x + y + z) = 12.8$ or $x + y + z = 6.4 \quad ...(1)$
Adding first two equations, we get
$x + y + z + [y] + \{x\} = 7.4 \quad ...(2)$
Adding 2nd and 3rd equations, we get
$x + y + z + [z] + \{y\} = 9.7 \quad ...(3)$
Adding Ist and 3rd equations, we get
$x + y + z + [x] + \{z\} = 8.5 \quad ...(4)$
From (1) and (2), $[y] + \{x\} = 1$
From (1) and (3), $[z] + \{y\} = 3.3$
From (1) and (4), $[x] + \{z\} = 2.1$
$\Rightarrow \quad [x] = 2, [y] = 1, [z] = 3, \{x\} = 0, \{y\} = 0.3$
and $\{z\} = 0.1$
$\Rightarrow \quad x = 2, y = 1.3, z = 3.1$
$\Rightarrow \quad x + 3y + z = 9$ **Ans.**

**16.** Let two linear functions be $f(x) = ax + b$ and $g(x) = cx + d$
They map $[-1, 1] \to [0, 2]$ and mapping is onto
$\Rightarrow \quad f(-1) = 0$ and $f(1) = 2$ and $g(-1) = 2$ and $g(1) = 0$
$\Rightarrow \quad -a + b = 0$ and $a + b = 2 \quad ...(1)$
and $-c + d = 2$ and $c + d = 0$
$\Rightarrow \quad a = b = 1$ and $c = -1, d = 1$
$\Rightarrow \quad f(x) = x + 1$ and $g(x) = -x + 1$

$$\Rightarrow \quad h(x) = \frac{x+1}{x-1} \quad \Rightarrow h(h(x))$$

$$= \frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1} = \frac{2x}{2} = x$$

$\Rightarrow \quad h(h(1/x)) = 1/x$
$\Rightarrow \quad h(h(x)) + h(h(1/x)) = x + 1/x > 2.$

**17.** $$\text{Let } P = \lim_{x\to\infty} \int_0^x \left(\frac{1}{\sqrt{(1+t^2)}} - \frac{1}{(1+t)}\right)dt$$

$$= \lim_{x\to\infty} \{\ln(t + \sqrt{(1+t^2)}) - \ln(1 + t)\}_0^x$$

$$= \lim_{x\to\infty} (\ln(x + \sqrt{(1+t^2)}) - \ln(1 + t))$$

$$= \lim_{x\to\infty} \ln\left(\frac{x+\sqrt{1+x^2}}{1+x}\right)$$

$$= \ln\left\{\lim_{x\to\infty}\left(\frac{x+\sqrt{1+x^2}}{1+x}\right)\right\} \quad \left(\text{form} \frac{\infty}{\infty}\right)$$

$$= \ln\left\{\lim_{x\to\infty}\left(\frac{1+\sqrt{1/x^2+1}}{(1/x)+1}\right)\right\} = \ln\left(\frac{1+1}{1}\right) = \ln 2.$$

**18.** $2f(x) + f(-x) = \frac{1}{x}\sin\left(x - \frac{1}{x}\right)$ ...(1)

Replacing x by – x then $2f(-x) + f(x)$

$$= -\frac{1}{x}\sin\left(-x + \frac{1}{x}\right)$$

$\Rightarrow$ $f(x) + 2f(-x) = \frac{1}{x}\sin\left(x - \frac{1}{x}\right)$ ...(2)

Subtracting (2) from (1) then $f(x) - f(-x) = 0$

or $f(x) = f(-x)$

From (1), $3f(x) = \frac{1}{x}\sin\left(x - \frac{1}{x}\right)$

or $f(x) = \frac{1}{3x}\sin\left(x - \frac{1}{x}\right)$

$\therefore$ $\int_{1/e}^{e} f(x)\,dx = \frac{1}{3}\int_{1/e}^{e}\frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx$

Put $x = \frac{1}{t}$ $\Rightarrow$ $dx = -\frac{1}{t^2}\,dt$

when $x = \frac{1}{e} \Rightarrow t = e;\quad x = e \Rightarrow t = \frac{1}{e}$

$$= \frac{1}{3}\int_{e}^{1/e}\frac{1}{1/t}\sin\left(\frac{1}{t} - t\right)\frac{-dt}{t^2}$$

$$= -\frac{1}{3}\int_{1/e}^{e}\frac{1}{t}\sin\left(t - \frac{1}{t}\right)dt = -\int_{1/e}^{e} f(x)\,dx$$

Hence, $2\int_{1/e}^{e} f(x)\,dx = 0 \Rightarrow \int_{1/e}^{e} f(x)\,dx = 0$

**19.** We have $(a\tan\beta - \sqrt{a^2-1}\tan\alpha)^2$

$+ (\sqrt{a^2+1}\tan\beta - \sqrt{a^2-1}\tan\gamma)^2$

$+ (a\tan\gamma - \sqrt{a^2+1}\tan\alpha)^2 \ge 0$

$\Rightarrow \{a^2 + a^2 - 1 + a^2 + 1\}(\tan^2\alpha + \tan^2\beta + \tan^2\gamma)$

$- \{a\tan\alpha + \sqrt{a^2-1}\tan\beta + \sqrt{a^2+1}\tan\gamma\}^2 \ge 0$

$\Rightarrow \tan^2\alpha + \tan^2\beta + \tan^2\gamma \ge \frac{4a^2}{3a^2} \Rightarrow 3\sum\tan^2\alpha \ge 4$

## PHYSICS

**20.** **D**

Apply COE

$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv'^2 \Rightarrow v' = (v_0^2 - 2gh)^{1/2}$

Thus v' is independent of θ.

**21.** **D**

Velocity of approach of the ball normal to the surface of collision $v = \sqrt{2gh}$

velocity of recoil = ev

change in momentum = mv + emv

= ( 1 + e) mv ≈ Impulse = N Δt

The normal reaction on the block ≈ N

∴ Frictional force on block = $\mu_k N = 0.2$ N

Change in momentum of block =0.2 × N Δt

= 0.2 × (1 + e) mv = 0.2 × 1.5 × mv

change in velocity of block = 0.2 × 1.5 × v = 0.3 $\sqrt{2gh}$

**22.** **A**

**23.** **C**

$$\Delta Q = -\frac{CV}{2} - CV - \left[-\frac{CV}{3} - \frac{CV}{3}\right] = \frac{-5CV}{6}$$

$$\Delta W_b = \frac{-5CV^2}{6}$$

**24.** **B,D** **25.** **B**

**26.** **A,C**

$\vec{v}_1 = u\hat{i} - gt\hat{j}$

$\vec{v}_2 = (g\sin\theta\cos\theta)t\hat{i} - (g\sin^2\theta)t\hat{j}$

$\vec{v} = \vec{v}_2 - \vec{v}_1 = (g\sin\theta\cos\theta t - u)\hat{i} - (g\sin^2\theta t - gt)\hat{j}$

$$\int_{\ell\cos\theta}^{x} dx = \int_0^t (g\sin\theta\cos\theta t - u)\,dt$$

$$\Rightarrow x = g\sin\theta\cos\theta\frac{t^2}{2} - ut + \ell\cos\theta$$

and $\int_{-\ell\sin\theta}^{y} dy = \int_0^t -(g\sin^2\theta t - gt)dt$

$$\Rightarrow y = \frac{g\cos^2\theta t^2}{2} - \ell\sin\theta$$

Distance = $\sqrt{x^2 + y^2}$

$$= \sqrt{\left(g\sin\theta\cos\theta\frac{t^2}{2} - ut + \ell\cos\theta\right)^2 + \left(\frac{g\cos^2\theta t^2}{2} - \ell\sin\theta\right)^2}$$

**27.** **A,B**

Let lower spring compresses maximum by x in metre. From conservation of mechanical energy, Decrease in potential energy of block = increase in elastic potential energy of both the springs

$(2)(10)(x + 1) = \frac{1}{2} \times 10 \times x^2 + \frac{1}{2} \times 10 \times (x+1)^2$

or $20x + 20 = 5x^2 + 5x^2 + 5 + 10x$

∴ $10x^2 - 10x - 15 = 0$

or $2x^2 - 2x - 3 = 0$

∴ $x = \frac{2 \pm \sqrt{4+24}}{4}$

or $x = 1.82$ m

$\therefore$ Maximum extension of upper spring = 1 + x = 2.82 m.

At equilibrium position net force on the block should be zero. So, let it is at distance y from where it was released.

Then, $mg = Ky + K(y-1)$

or $20 = 10y + 10(y-1)$

or $20y = 30$

$\therefore$ $y = 1.5$ m

**28.** B,D

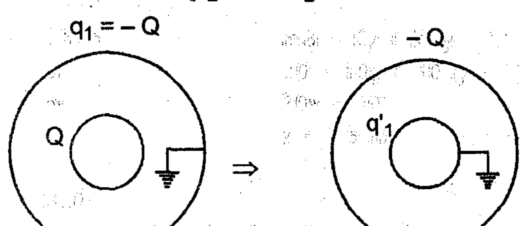
**29.** $A \to R$ ; $B \to S$ ; $C \to P$; $D \to Q$

**30.** $A \to R$ ; $B \to S$ ; $C \to Q$; $D \to P$

**31.** Let charge $q_1$ comes from the earth on outer shell.

$V_{outer} = 0 \qquad q_1 = -Q$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{2r}+\frac{q_1}{2r}\right] = 0$$




or $q_1 = -Q$

When $S_2$ is closed and opened, $V_{inner} = 0$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q'_1}{r}-\frac{Q}{2r}\right] = 0 \quad \text{or} \quad q'_1 = \frac{Q}{2}$$

Proceeding in the similar manner after n such operations we get,

Charge on the inner shell, $q'_n = \dfrac{Q}{(2)^n}$

and the potential difference between the shells,

$$\Delta V = \frac{q'_n}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{2r}\right) \Rightarrow \Delta V = \frac{1}{(2)^{n+1}}\left[\frac{Q}{4\pi\varepsilon_0 r}\right]$$

**32.** $E = 2 \times \dfrac{60}{75} \times \left(\dfrac{1000}{100}\right) = 160$ volt ;

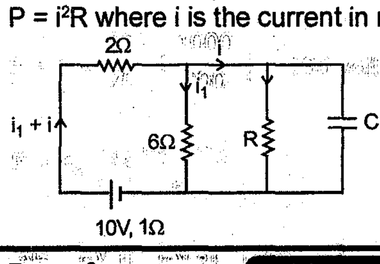
$$E = E_0\frac{\ell_1}{\ell_2}\left(\frac{R_1+R_2}{R_1}\right)$$

$$\frac{\Delta E}{E} = \left|\frac{\Delta \ell_1}{\ell_1}\right| + \left|\frac{\Delta \ell_2}{\ell_2}\right| + \left|\frac{\Delta(R_1+R_2)}{(R_1+R_2)}\right| + \left|\frac{\Delta R_1}{R_2}\right|$$

$\Rightarrow \Delta E = 0.64$ volts

**33.** At steady state no current will flow through capacitor. Power developed in resistor R

$P = i^2R$ where i is the current in resistor R.




Applying Kirchoff's law

$$(i_1 + i) = \frac{10}{3+\dfrac{6R}{6+R}} \qquad ...(i)$$

Where $i_1$ is current in resistor & $i_1 = \dfrac{i}{6}R$ ...(ii)

Putting the value of $i_1$ in (i) and by the relation

$\dfrac{dP}{dR} = 0$ for maximum power

$R = 2\Omega$

$i = \dfrac{5}{3}A$, $V = iR = \dfrac{10}{3}$ volt $\therefore q = C \times V = \dfrac{100}{3}\mu C$

**34.** 2/3

**35.** If the density of cone be ρ, then its mass will be

$m_1 = \dfrac{1}{3}\pi(2R)^2(4R)\rho = \dfrac{16}{3}\pi R^3\rho$ and its centre of mass $O_1$ will be at a height $(h/4) = (4R/4) = R$ from O on the line of symmetry, i.e., $y_1 = R$

Similarly the mass of the sphere

$m_2 = \dfrac{4}{3}\pi R^3(12\rho) = 16\pi R^3\rho = 3m_1$ and its centre of mass will be at its centre $O_2$, i.e., $y_2 = 5R$

Now treating sphere and cone as point masses with their masses concentrated at their centre of masses respectively and taking the line of symmetry as y-axis with origin at O, for the centre of mass of the toy

$$Y_{CM} = \frac{m_1y_1+m_2y_2}{m_1+m_2} = \frac{m_1\times R + 3m_1\times 5R}{m_1+3m_1} = 4R$$

i.e., centre of mass of the toy is at a distance 4R from O on the line of symmetry, i.e., at the apex of the cone.

**36.** Both the particles move under gravity. It means acceleration of two particles is identical or their relative acceleration is zero.

First consider horizontal motion of particle projected from B relative to the other particle.

$v_x = 14\cos 45° + 2\cos 45° = 8\sqrt{2}\ ms^{-1}$

$\therefore$ At time, t, horizontal separation between the particles,

$x = (22 - 8\sqrt{2}t)m$

Now considering vertically upward relative motion

$v_y = 14\sin 45° - 2\sin 45° = 6\sqrt{2}\ ms^{-1}$

$\therefore$ At time t, vertical separation, $y = (9 - 6\sqrt{2})m$

Hence, distance between the particle is,

$$r = \sqrt{x^2+y^2}$$

or $r^2 = (22 - 8\sqrt{2}t)^2 + (9 - 6\sqrt{2}t)^2$ ...(1)

For separation r to be minimum, $r^2$ must be minimum. Therefore,

$$\frac{d}{dt}(r^2) = 0$$

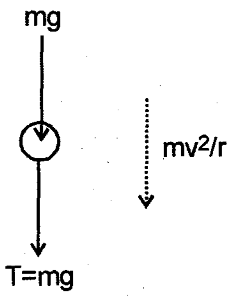
$\therefore \quad 2(22 - 8\sqrt{2}t)(-8\sqrt{2}) + 2(9 - 6\sqrt{2}t)(-6\sqrt{2}) = 0$

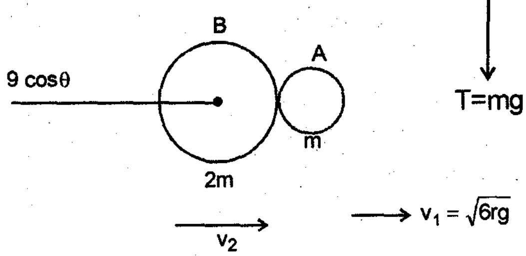
or $\quad t = \dfrac{23}{10\sqrt{2}}$ second

Substituting in equation (1)

$$r_{min} = 6.00 \text{ m}$$

**37.** Let velocity of ball A at highest point be v then first considering its FBD at this point figure shown.




$$mg + T = \frac{mv^2}{r} \qquad \text{where, } T = mg$$

or $\quad v^2 = 2rg$

If velocity of ball A just after the collision is $v_1$ then according to law of conservation of energy,

Its KE at A = (KE + PE) at highest point

or $\quad \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}mv^2 + mg.(2r)$

or $\quad v_1 = \sqrt{6rg} \quad$ or $\quad v_1 = 6\sqrt{2}\,ms^{-1}$

If angle of projection of ball B is θ, then horizontal component of its velocity will be 9 cos θ and it remains constant,

Now considering collision of balls B and A,

Let velocities of balls A and B, just after the collisions be $v_1$ and $v_2$ respectively as shown in figure shown.

Then according to law of conservation of momentum.

$$mv_1 + 2mv_2 = 2m.(9\cos\theta)$$

Since the collision is elastic, therefore, e= 1

or $\quad \dfrac{v_1 - v_2}{9.\cos\theta} = 1 \quad$ or

$v_1 - v_2 = 9\cos\theta$

Solving equations (1) and (2),

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \text{or} \quad \theta = 45°$$

and $\quad v_2 = \dfrac{3}{\sqrt{2}}ms^{-1}$

Height of initial position of ball A (lowest position) is equal to maximum height ascended by the ball B. It is equal to,

$$H = \frac{(9\sin\theta)^2}{2g} = \frac{81}{40}m$$

Height of point of suspension from the ground,

$= H + r = 3.225$ m

**38.** $\text{Area} = 0.2\left[\dfrac{90}{1.5} - \dfrac{90}{1.8}\right] = 2cm^2$

## CHEMISTRY

**39.** D

$NaI + AgNO_3 \rightarrow AgI + NaNO_3$

$AgI + Fe \rightarrow FeI_2 + Ag$

$FeI_2 + Cl_2 \rightarrow FeCl_3 + I_2$

wt. of $I_2$ = 381 kg

moles of $I_2 = \dfrac{381 \times 10^3}{254}$

moles of I $= \dfrac{2 \times 381 \times 10^3}{254}$

moles of $AgNO_3 = \dfrac{2 \times 381 \times 10^3}{254}$

wt. of $AgNO_3 = \dfrac{2 \times 381}{254} \times 170 = 510$ kg

| | | | | | |
|---|---|---|---|---|---|
| **40.** | B | **41.** | C | **42.** | B |
| **43.** | C,D | **44.** | B,D | **45.** | D |
| **46.** | A,B,D | **47.** | B,D | | |

**48.** A → P,Q,R,S ; B → Q ; C → R ; D → P,Q,S

**49** A → P,Q ; B → P,R,S ; C → R,S ; D → R,S

**50.** 86.67 %

$$(NH_4)_2CO_3 \longrightarrow NH_2-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 + 2H_2O$$

Moles of urea $= \dfrac{11.52}{96} = 0.12$

$$2MnO_4^- + 5\,(NH_4)_2C_2O_4 \rightarrow 2Mn^{2+} + 10\,CO_2 + 10\,NH_4^+$$

Moles of $(NH_4)_2C_2O_4 = 20 \times 1.6 \times \dfrac{5}{2} \times 10^{-3} = 0.08$

(i) $(CN)_2 + 4\,H_2O \rightarrow (NH_4)_2C_2O_4$

a $\qquad$ a

(ii) $(CN)_2 + 2H_2O \rightarrow NH_2CONH_2$

b $\qquad$ b

a = 0.08

b = 0.12

Total mass of $(CN)_2$ = 0.20

Weight of $(CN)_2$ = 0.2 × 52 = 10.4

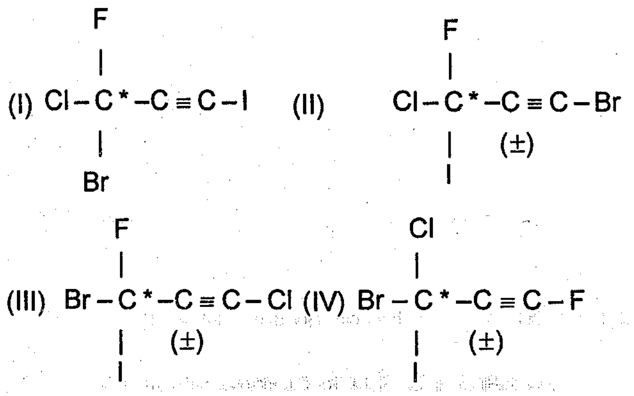
(a) % purity of $(CN)_2 = \dfrac{10.4}{12} \times 100 = 86.67\%$

**51.** $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{15}{300} = \frac{P_2}{400} \Rightarrow P_2 = 20$ atms

Now $\frac{a+2x}{a} = \frac{30}{20} \Rightarrow 2a + 4x = 3a \Rightarrow x = \frac{1}{4}a$

$\therefore$ % of $NH_3$ decomposed = $\frac{2x}{a} \times 100 = 50\%$

**52.** All alkyne isomers of M.F. $C_3F$ ClBrI are



Number of positional isomers (I, II, III, IV) = 4

Number of all optically active isomers = 8

Number of fractions obtained on fractional distillation = 4

No diastereomers possible with one asymmetric C* atom.

$\therefore$ sum of p + q + r + s = 16

**53.** $X_2Y_4$ (g) $\rightarrow$ $X_2$ (g) + $2Y_2$

$1-\alpha$    $2\alpha$

Total moles = $1 - \alpha + \alpha + 2\alpha = 1 + 2\alpha$

$PM = \rho RT \Rightarrow M_{mix} = \frac{\rho RT}{P} = 2 \times \frac{1}{12} \times 300$

$M_{mix} = 50$

$1 \times 100 = (1 + 2\alpha) \times 50 \Rightarrow 1 + 2\alpha = 2$

$\alpha = 1/2 = 50\%$

**54.** $NH_4OH + H_2SO_4 \rightarrow (NH_4)_2SO_4 + H_2O$

50 meq.   100×0.2×2

50 meq   40 meq.

meq. of $NH_4OH$ = 10

meq. of $NH_4^+$ = 40

$p^{OH} = p^{k_b} + \log\frac{40}{10} = 5 + 2\log 2$

$= 5 + 0.6020$

$P^H = 8.4$

**55.** meq. of $H_2SO_4 = 200 \times 0.2 \times 2 = 80$

meq. of NaOH = $100 \times 0.6 = 60$

meq. of $H_2SO_4$ left = 20

meq. of $Ca(OH)_2 = 100 \times 0.2 \times 2 = 40$

meq. of $Ca(OH)_2$ left = $40 - 20 = 20$

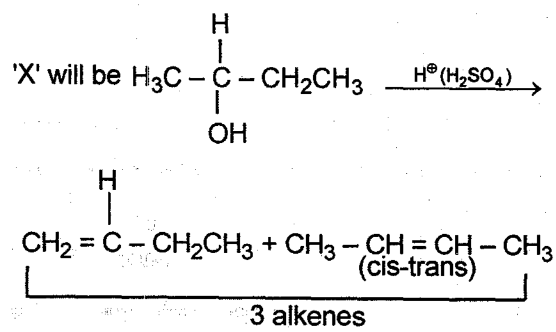
$(OH^-) = \frac{20}{1000} = 1/50$

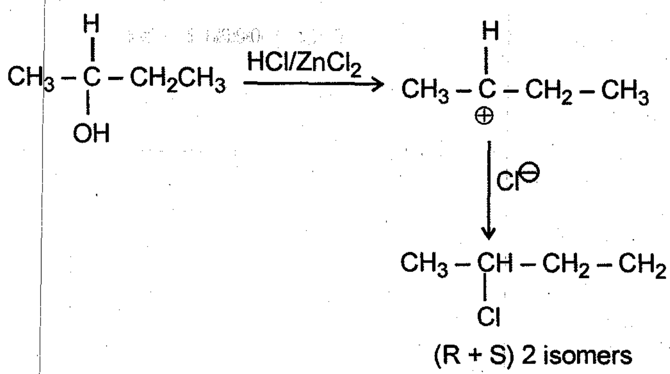
$p^{OH} = -\log 1/50 = \log 50$

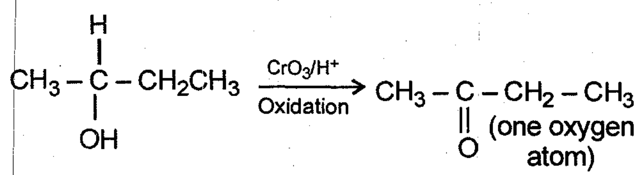
$p^H = 14 - \log 50$

$= 14 - 1.6990 = 12.3$

**56.** 'X' will be







$\therefore$ The sum of Y, Z and W is = 3 + 2 + 1 = 6

**57.** 70