# TARGET IIT-JEE

## HINT & SOLUTIONS

### ANSWER KEY WITH SOLUTION

### PAPER-I CLASS XII & XIII (DATE 13-09-09)

#### MATHEMATICS

| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A. | D | C | D | A | D | B | B | B | B,C,D | B,C | B,C,D | All |
| Q. | 13 | 14 | 15 | 16 | 17 | 18 | | | | | | |
| A. | C | A | B | D | B | D | | | | | | |

| Q. | 19 | 20 |
|---|---|---|
| A. | (A) – Q, (B) –R , (C) – P, (D) – S | (A)–(S), (B)–(P), (C)–(P), (R), (D)–(Q) |

#### PHYSICS

21. A 22. B 23. C 24. B 25. A 26. B

27. B 28. A 29. A,C,D 30. A,C,D 31. A,B 32. B

33. A 34. D 35. C 36. A 37. D 38. B

39. A $\rightarrow$ P,Q,R,S ; B $\rightarrow$ P,Q ; C $\rightarrow$ Q,R ; D $\rightarrow$ Q,R,S

40. A $\rightarrow$ P,R,S ; B $\rightarrow$ P,Q,R,S ; C $\rightarrow$ P ; D $\rightarrow$ P,Q

#### CHEMISTRY

41. B 42. B 43. B 44. B 45. D 46. C

47. A 48. A 49. B,C,D 50. A,B,D 51. C,D 52. D

53. B 54. C 55. B 56. C 57. C 58. B

59. A $\rightarrow$ P,Q ; B $\rightarrow$ R,S ; C $\rightarrow$ P,Q ; D $\rightarrow$ R

60. A $\rightarrow$ Q, R, S ; B $\rightarrow$ P, R,S ; C $\rightarrow$ Q,R,S ; D $\rightarrow$ P,R,S

## MATHEMATICS

**1.** $f(2) = f((3)^{1/4}) \Rightarrow$ many-to-one function and

$f(x) \neq \sqrt{3}\ \forall\ x \in R \Rightarrow$ into function.

**2.** The period of f(x) is 7

$\Rightarrow$ The period of $f\left(\frac{x}{3}\right)$ is $\frac{7}{1/3} = 21$

The period of g(x) is 11

$\Rightarrow$ The period of $g\left(\frac{x}{5}\right)$ is $\frac{11}{1/5} = 55$

Hence, $T_1$ = period of $f(x)\,g\left(\frac{x}{5}\right) = 7 \times 55 = 385$

and $T_2$ = period of $g(x)\,f\left(\frac{x}{3}\right) = 11 \times 21 = 231$.

$\therefore$ Period of $F(x) = \text{LCM}\{T_1, T_2\}$
$= \text{LCM}\{385, 231\} = 7 \times 11 \times 3 \times 5 = 1155.$

**3.** $f(-1) = 0;\quad g(-1) = 0;\ f'(-1) = g'(-1)$
$\Rightarrow a = -1;\quad b = -1;\quad c = 1;$
hence $(a + b + c^2) = -1$ Ans.

**4.** $F'(\theta) = \theta\cos(\theta + \alpha) = 0$
$\Rightarrow$ $\theta = 0$ or $\alpha + \theta = \pi/2,\ \ \theta = (\pi/2) - \alpha$

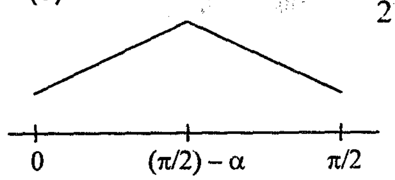
if $\theta \in \left(0, \frac{\pi}{2} - \alpha\right)$ then $F'(\theta) > 0$

$\Rightarrow$ F is increasing

if $\theta \in \left(\frac{\pi}{2} - \alpha, \frac{\pi}{2}\right)$ then $F'(\theta) < 0$

$\Rightarrow$ F is decreasing

Hence $F(\theta)$ is maximum when $\theta = \frac{\pi}{2} - \alpha$




$$F(\theta) = \int_0^{\theta} x\cos(x+\alpha)\,dx = x\sin(x+\alpha)\Big|_0^{\theta} - \int_0^{\theta}\sin(x+\alpha)\,dx$$

$= \theta\sin(\theta + \alpha) + \cos(\theta+\alpha)\Big|_0^{\theta}$
$= \theta\sin(\theta + \alpha) + \cos(\theta + \alpha) - \cos\alpha$

$\therefore$ put $\theta = \frac{\pi}{2} - \alpha$

$$F\left(\frac{\pi}{2} - \alpha\right) = \left(\frac{\pi}{2} - \alpha\right) + (0) - \cos\alpha$$

$= \frac{\pi}{2} - \alpha - \cos\alpha$ which is the maximum value.]

**5.** $$C_n = \int_{\frac{1}{n+1}}^{\frac{1}{n}} \frac{\tan^{-1}(nx)}{\sin^{-1}(nx)}dx$$

(put $nx = t$) $\Rightarrow$ $$C_n = \frac{1}{n}\int_{\frac{n}{n+1}}^{1}\frac{\tan^{-1}(t)}{\sin^{-1}(t)}dt$$

$$L = \lim_{n\to\infty} n^2 \cdot C_n = \lim_{n\to\infty} n \cdot \int_{\frac{n}{n+1}}^{1}\frac{\tan^{-1}t}{\sin^{-1}t}dt\ (\infty \times 0);$$

$$L = \lim_{n\to 0}\frac{\displaystyle\int_{\frac{n}{n+1}}^{1}\frac{\tan^{-1}t}{\sin^{-1}t}dt}{\frac{1}{n}}$$

applying Leibnitz rule

$$L = \lim_{n\to\infty}\frac{-\dfrac{\tan^{-1}\frac{n}{n+1}}{\sin^{-1}\frac{n}{n+1}}\left(\dfrac{1}{(n+1)^2}\right)}{-\dfrac{1}{n^2}} = \frac{\pi}{4}\cdot\frac{2}{\pi} = \frac{1}{2}\ \text{Ans.}$$

**6.** $(\vec{a} \times \hat{i}) \cdot (\vec{b} \times \hat{i}) = \begin{vmatrix} \vec{a}\cdot\vec{b} & \vec{a}\cdot\hat{i} \\ \vec{b}\cdot\hat{i} & \hat{i}\cdot\hat{i} \end{vmatrix} = (\vec{a}\cdot\vec{b}) - (\vec{a}\cdot\hat{i})(\vec{b}\cdot\hat{i})$

Similarly, $(\vec{a} \times \hat{j}) \cdot (\vec{b} \times \hat{j}) = (\vec{a}\cdot\vec{b}) - (\vec{a}\cdot\hat{j})(\vec{b}\cdot\hat{j})$

and, $(\vec{a} \times \hat{k}) \cdot (\vec{b} \times \hat{k}) = \vec{a}\cdot\vec{b} - (\vec{a}\cdot\hat{k})(\vec{b}\cdot\hat{k})$

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\Rightarrow$ $(\vec{a}\cdot\hat{i}) = a_1,\ (\vec{a}\cdot\hat{j}) = a_2,\ (\vec{a}\cdot\hat{k}) = a_3$

$(\vec{b}\cdot\hat{i}) = b_1,\ (\vec{b}\cdot\hat{j}) = b_2,\ (\vec{b}\cdot\hat{k}) = b_3$

$\Rightarrow (\vec{a} \times \hat{i}) \cdot (\vec{b} \times \hat{j}) + (\vec{a} \times \hat{j}) \cdot (\vec{b} \times \hat{j}) + (\vec{a} \times \hat{k}) \cdot (\vec{b} \cdot \hat{k})$

$= 3\vec{a}\cdot\vec{b} - (a_1b_1 + a_2b_2 + a_3b_3)$

$= 3\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b} = 2\vec{a}\cdot\vec{b}$

7. consider $g(x) = (f(x) + f'(x))e^{-x}$
From the given information, $g(a) = g(b)$.
By Rolle's Theorem, there exists $c \in (a, b)$ such that $g'(c) = 0$.
Here $g'(x) = (f''(x) - f(x))e^{-x}$
$g'(c) = 0 \Rightarrow f''(c) = f(c)$

8. Let $I = \int_0^1 f(x)\, x^{2002} dx$.
Using Cauchy inequality,
$$I^2 \le \int_0^1 (f(x))^2 dx \,.\, \int_0^1 x^{4004}\, dx = \frac{1}{4005}$$
$$\Rightarrow \quad I \le \frac{1}{\sqrt{4005}}$$
To show that this maximum value is achieved, take $f(x) = c\, x^{2002}$ with $c = \sqrt{4005}$.

9. Let $f(x) = (x - a)(x - b)(x - c)$
$\therefore \quad g(x) = k(x - a^2)(x - b^2)(x - c^2)$
Since $abc = -f(0) = -1$, $k = ka^2b^2c^2 = -g(0) = 1$
$\therefore \quad g(x^2) = (x^2 - a^2)(x^2 - b^2)(x^2 - c^2)$
$= -f(x)\, f(-x)$.
put $x = 3$, $g(9) = -f(3) \cdot f(-3) = 899$
put $x = 1$, $g(1) = -f(1)\, f(-1) = -f(1) \cdot (-1) = f(1)$

10. $f(I) = \left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$

If $I_1 = \left[0, \frac{1}{3}\right]$ $\quad f(I_1) = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right]$

If $I_2 = \left[\frac{2}{3}, 1\right]$ $\quad f(I_2) = \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{8}{9}, 1\right]$

hence $f(f(I)) = \left[0, \frac{1}{9}\right] \cup \left[\frac{2}{9}, \frac{1}{3}\right] \cup \left[\frac{2}{3}, \frac{7}{9}\right] \cup \left[\frac{8}{9}, 1\right]$

11. $f'(x) = \dfrac{(x+1)^n x^{n-1}(n - x)}{(x+1)^{2n+2}}$

sign of $f'(x)$: $\quad 0 \quad + \quad n \quad -$

f(x) has a local maximum at $x = n$.

$$\lim_{x \to \infty} \frac{x^n}{(x+1)^{n+1}} = 0$$

$f(x) \le f(n)$ since $f(n)$ is the global maximum.

12. $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

$f(x + k) = \begin{cases} 1, & x + k \text{ is rational} \\ 0, & x + k \text{ is irrational} \end{cases}$

where k is any rational number

$\Rightarrow \quad f(x + k) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

$\Rightarrow \quad f(x + k) = f(x)$
$\Rightarrow \quad$ f(x) is periodic function, but its fundamental period cannot be determined

$$f(x) = \begin{cases} x - [x], & 2n \le x < 2n + 1 \\ 1/2, & 2n + 1 \le x < 2n + 2 \end{cases}$$

Draw the graph from which it can be verified that period is 2.

$f(x) = (-1)^{\left[\frac{2x}{\pi}\right]} \Rightarrow f(x + \pi)$

$= (-1)^{\left[\frac{2(\pi + x)}{\pi}\right]} = (-1)^{\left[2 + \frac{2x}{\pi}\right]} = (-1)^{\left[\frac{2x}{\pi}\right] + 2} = (-1)^{\left[\frac{2x}{\pi}\right]}$

Hence, the period is $\pi$.

$$f(x) = ax + a - [ax + a] + \tan\left(\frac{\pi x}{2}\right) - a$$

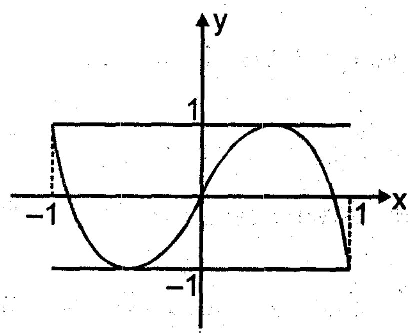
$$= \{ax + a\} + \tan\left(\frac{\pi x}{2}\right) - a$$

$\{ax+a\}$ is periodic with period $1/a$, $\tan\left(\frac{\pi x}{2}\right)$ is periodic with period 2.
Now, the LCM of $1/a$ and 2 exists.
Hence, f(x) is periodic.

13-15. $y = ax - bx^3$.

$\frac{dy}{dx} = a - 3b\, x^2 = 0 \quad \Rightarrow \quad x^2 = \frac{a}{3b}$

$x = \pm \sqrt{\frac{a}{3b}}$




Since $b > 0$ the possible graph of f(x) is

$f(1) = -1 \Rightarrow a - b = -1 \qquad f\left(\sqrt{\frac{a}{3b}}\right) = 1$

$\sqrt{\frac{a}{3b}}\left(a - b.\frac{a}{3b}\right) = 1 \quad \Rightarrow \quad 4a^3 = 27b.$

Now $\quad 4a^3 = 27(a + 1)$
$4a^3 - 27a - 27 = 0$
$(a - 3)(4a^2 + 12a + 9) = 0$

Since $a > 0$, $a = 3$, $b = 4$.

$f(x) = ax - bx^3 = 3x - 4x^3$.

$f(x) + p = 0 \Rightarrow 4x^3 - 3x = p$.

put $x = \cos\theta$.

$\cos 3\theta = p$.

$\theta = \frac{1}{3}\cos^{-1} p$.

$x = \cos\theta = \cos\left(\frac{1}{3}\cos^{-1} p\right)$.

**16-18.** $f^1(x) = g(f^0(x)) = \frac{x}{x-1}$

$$f^2(x) = g(f^1(x)) = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} = x$$

$$f^3(x) = \frac{x}{x-1}$$

Similarly $f^{2n}(x) = x$, $f^{2n+1}(x) = \frac{x}{x-1}$.

$$h(x) = \frac{x}{2^0} + \frac{x}{2(x-1)} + \frac{x}{2^2} + \frac{x}{2^3(x-1)} + \ldots\ldots$$

$$= x\left(1 + \frac{1}{4} + \frac{1}{16}\ldots\ldots\right) + \frac{x}{2(x-1)}\left(1 + \frac{1}{4} + \frac{1}{16}\ldots\ldots\right)$$

$$= \frac{4x}{5} + \frac{4x}{10(x-1)} = \frac{4x^2 - 2x}{5x - 5}$$

$$\lim_{x\to\infty}\frac{h(x)}{x} = \frac{4}{5}.$$

The domain of h(x) is $x \in (1, \infty)$

$$h'(x) = \frac{2x^2 - 4x + 1}{(x-2)^2} = 0$$

at $x = \frac{2 \pm \sqrt{2}}{2}$

sign scheme of $h'(x)$: $1 \quad - \quad \frac{2+\sqrt{2}}{2} \quad +$

h(x) has a local minimum at $x = \frac{2+\sqrt{2}}{2}$

$$\lim_{x\to 1^+} h(x) = \infty$$

$$h\left(\frac{(2+\sqrt{2})}{2}\right) = \frac{2}{5}$$

$$\lim_{x\to\infty} h(x) = \infty$$

$\therefore$ The range of h(x) is $\left[\frac{2}{5}, \infty\right)$.

**19.** (A) Let $I = \int_0^{\pi}(a\sin x + b\sin 2x)^2 dx$

$$I = \int_0^{\pi}(a\sin x - b\sin 2x)^2 dx \quad \text{(using P–5)}$$

$$\text{add } 2I = 2\int_0^{\pi}(a^2\sin^2 x + b^2\sin^2 2x)dx$$

$$I = 2\int_0^{\pi/2}(a^2\sin^2 x)dx + 2\int_0^{\pi/2}(b^2\sin^2 2x)dx$$

[Using P--6]

$$= 2a^2\frac{\pi}{4} + 2b^2\underbrace{\int_0^{\pi/2}\sin^2 2x\,dx}_{J}$$

Let $J = \int_0^{\pi/2}\sin^2 2x\,dx$ ; put $2x = t$

$$= \frac{1}{2}\int_0^{\pi}\sin^2 t\,dt = \int_0^{\pi/2}\sin^2 2t\,dt = \frac{\pi}{4}$$

hence $I = \frac{\pi a^2}{2} + \frac{\pi b^2}{2} = \frac{\pi}{2}(a^2 + b^2)$

$$(a) = \frac{\pi}{2}[a^2 + (1-a)^2] = \frac{\pi}{2}[2a^2 - 2a + 1]$$

$$= \pi\left[a^2 - a + \frac{1}{2}\right] = \pi\left[\left(a - \frac{1}{2}\right)^2 + \frac{1}{4}\right]$$

$\therefore$ I(a) is minimum when $a = \frac{1}{2}$ and minimum value

$= \frac{\pi}{4}$ **Ans.** $\Rightarrow$ (Q)

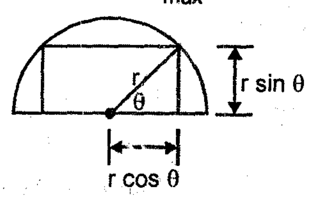
(B) $I = \frac{1}{2}\int_0^{\pi/2} x\,|\cos 2x|\,dx$ ; $\quad 2x = t \Rightarrow dx = \frac{dt}{2}$

$$I = \frac{1}{8}\int_0^{\pi} t\,|\cos t|\,dt \;;\; I = \frac{1}{8}\int_0^{\pi}(\pi - t)\,|\cos t\,dt|$$

$$2I = \frac{\pi}{8}\int_0^{\pi}|\cos t|\,dt = \frac{2\pi}{8} \Rightarrow \quad I = \frac{\pi}{8}$$

**Ans.** $\Rightarrow$ **(R)**

(C) Area of rectangle $= 2r\cos\theta \cdot r\sin\theta$
$= r^2 \sin 2\theta \quad \Rightarrow A_{max} = r^2$



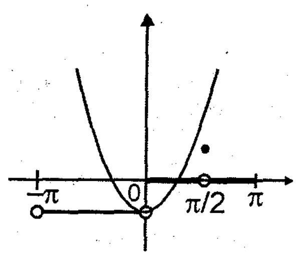
$\therefore k \cdot r^2 = \frac{\pi r^2}{2} \Rightarrow \quad k = \frac{\pi}{2}$ **Ans.** $\Rightarrow$ **(P)**

(D) $\quad (\hat{a}\cdot\hat{c}) - (\hat{a}\cdot\hat{c})\hat{c} = \frac{1}{\sqrt{2}}\hat{b} + \frac{1}{\sqrt{2}}\hat{c}$

$\therefore \hat{a}\cdot\hat{c} = \frac{1}{\sqrt{2}}$ and $\quad \hat{a}\cdot\hat{b} = -\frac{1}{\sqrt{2}}$

$\Rightarrow \hat{a} \wedge \hat{c} = \frac{\pi}{4};\ \hat{a} \wedge \hat{b} = \frac{3\pi}{4}$ **Ans.** $\Rightarrow$ **(S)**

**20.** (A) No. of into functions
= No. of into leaving two elements
+ No. of into leaving one element
$= {}^3C_1.1 + {}^3C_2(2^4 - 2) = 45$
No. of functions $= 3^4 = 81$
No. of onto functions $= 81 - 45 = 36$
Difference $= 45 - 36 = 9$




(B) $\quad \frac{x^2 - 3}{3} = [\sin x]$
one intersection points.

(C) $\quad y = \frac{x}{1+|x|}$ is differentiable for all x
$y = (x-2)(x+2)\,|(x-1)(x-2)(x-3)|$
is non differentiable at x = 1, 3.
So f(x) is non diff. at x = 1, 3

(D) $\quad f_n(x) = \tan x\left(\frac{1+\cos 2x}{\cos 2x}\right)(1 + \sec 4x)$ .....

$= \frac{\sin x \,.\, 2\cos^2 x}{\cos x \,.\, \cos 2x}\ (1 + \sec 4x)$ ......

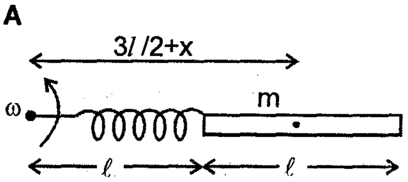
$= \tan 2x\ (1 + \sec 4x)\ .....(1+\sec 2^n x) = \tan 2^n x.$

$\lim_{x\to 0} \frac{f_3(x)}{2x} = \lim_{x\to 0} \frac{\tan 2^3 x}{2^3 x}\,.\,2^2 = 2^2$

Hence k = 2.

## PHYSICS

**21.** **A**

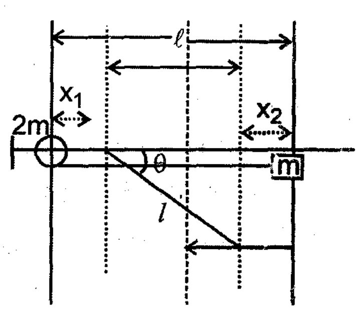



$kx = m\omega^2\left(\frac{3\ell}{2} + x\right) \Rightarrow x = \frac{\ell}{10}$

**22.** **B**
Work done = ΔGPE
$= (n+1)^2 mg\,\frac{a}{2} - (n+1)mg\frac{a}{2} = \frac{1}{2}\rho a^4 g(n^2 + n)$

**23.** **C**
$\ell = x_1 + \ell\cos\theta + x_2$
$x_2 = [\ell - x_1 - \cos\theta]$
$m_1x_1 = m_2x_2$
$2mx_1 = mx_2 =$

$x = \frac{\ell}{3}(1 - \cos\theta)$




**24.** **B**

$\frac{mv^2}{r} = \mu mg = \mu_0 mg\left(1 - \frac{r}{R}\right)$

$y = v^2 = \mu_0 g\left(r - \frac{r^2}{R}\right)$

For y to be maximum,

$\frac{dy}{dr} = 0 = 1 - \frac{2r}{R} \Rightarrow r = \frac{R}{2}$

$v_{max}^2 = \mu_0 g\left(\frac{R}{2} - \frac{R^2}{4R}\right) = \mu_0 g\frac{R}{4} \Rightarrow v_{max} = \frac{\sqrt{\mu_0 g R}}{2}$

**25.** A **26.** B

**27.** B
The liquid acts as a concave lens. Assembly will be a concave lens if liquid lens is more powerful then the glass lens.

**28.** A

**29.** A,C,D **30.** A,C,D **31.** A,B

**32.** B,C
As the particle moves along horizontal plane under the influence of constant force F,

$W_F = \int F.ds = \int F\,ds\cos\theta = \int F.dx = F\int_{x_i}^{x_f} dx = F(1)$

$W_F = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2F}{m}}$

**33.** A **34.** D **35.** C

**Sol.** **[33, 34, 35]**

$\therefore F_{net} = 0 \quad$ so $\quad \Delta\vec{V}_{cm} = 0$

so $\quad V_{cm_f} = V_{cm_i} = 0$

$V_A = V_B = \vec{V}_{AP} + \vec{V}_P = 5 - V_P$

So $0 = m_A v_A + m_B v_B - m_p v_p$

$= 80\,[5 - v_p] + 50\,[5 - v_p] - 20 v_p$

$0 = 650 - 150\, v_p$

$v_p = \dfrac{65}{15} = 4.3$ m/s

M.C. I

$0 = m_A v_A - (m_B + m_p)\, v_p$

$0 = 80\,[5 - v_p] - 20 v_p$

$v_p = \dfrac{8}{3}$

M.C. II

$(m_B + m_p)\, v_p = -m_B\,(5 - v'p) + m_p v'_p$

$\Rightarrow \quad 70\, v_p = 70\, v'_p - 50 \times 5$

$\Rightarrow \quad v'_p = 6.2$ m/sec

M.C. I[st] $\quad 0 = m_B v_B - (m_A + m_B)\, v_p$

$0 = 50\,[5 - v_p] - (80 + 20)\, v_p$

$\Rightarrow v_p = 5/3$ m/sec

M.C II

$(m_A + m_B)\, v_P = -m_A\,(5 - v'_p) + m_p v'_p$

$100\, v_p = -80\,(5 - v'_p) + 20\, v'_p$

$\Rightarrow \quad v'_p = 5.7$ m/s

**36.** A

Flux through one disc is $\dfrac{Q}{2\epsilon_0}(1 - \cos\alpha)$

Here $\alpha_1 = \alpha_2 = 60°$

So flux through both disc

$= 2 \times \dfrac{Q}{2\epsilon_0}(1 - \cos 60°) = \dfrac{Q}{2\epsilon_0}$

Hence flux through curved surface

$= \dfrac{Q}{\epsilon_0} - \dfrac{Q}{2\epsilon_0} = \dfrac{Q}{2\epsilon_0}$

**37.** D

Here again $\alpha_1 = \alpha_2 = 60°$

Flux through right side surface including disc will be $\dfrac{Q}{2\epsilon_0}$ and

flux through right side disc $= \dfrac{Q}{2\epsilon_0}(1 - \cos 60°) = \dfrac{Q}{4\epsilon_0}$

Flux through right side curved surface $= \dfrac{Q}{2\epsilon_0} - \dfrac{Q}{4\epsilon_0} = \dfrac{Q}{4\epsilon_0}$

Similarly flux through left side curved surface $= \dfrac{Q}{4\epsilon_0}$

**38.** C

$\alpha_1 = 60°\,;\ \alpha_2 = 60°$

So flux through curves surface is equal to

$$\frac{Q}{\epsilon_0} - \frac{Q}{\epsilon_0}\left[\left(\frac{1 - \cos 60°}{2}\right) + \left(\frac{1 - \cos 30°}{2}\right)\right] = \frac{Q}{\epsilon_0}\left(\frac{\sqrt{3} + 1}{4}\right)$$

**39.** **A → P,Q,R,S ; B → P,Q ; C → Q,R ; D → Q,R,S**

**40.** **A → P,R,S ; B → P,Q,R,S ; C → P ; D → P,Q**

**(A)** $V_{cm} = \dfrac{3 \times 2 + 6 \times 0}{3 + 6} = \dfrac{6}{9} = \dfrac{2}{3}$ m/s

P so maximum deformation and blocks have velocites same as C.M.

So, (P) (R) (S)

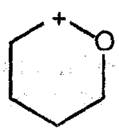
**(B)** (P) (R)

$\dfrac{2}{3} = \dfrac{3 \times (-2/3) + 6 \times v'}{9} \quad \Rightarrow \quad v' = \dfrac{4}{3}$ m/sec

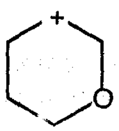
$\dfrac{1}{2} \times 3 \times (2)^2 = \dfrac{1}{2} \times 3 \times (2/3)^2 + \dfrac{1}{2} \times 6 \times (4/3)^2 + \dfrac{1}{2} k(0)^2 \Rightarrow Q$

## CHEMISTRY

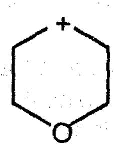
**41.** B



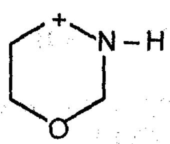
(i)



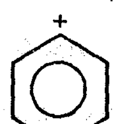
(ii)



(iii)



(iv)



(v)

Stability

IV > I > III > II > V

Hence Ease of $S_N1$ reaction IV > I > III > II

**42.** B

cis alkene + cis addition → meso product

It is valid only for symmetrical alkene.

cis + trans addition → Racemic

**43.** B

(A) 0.5 mole of $NH_2(CH_2)_4\,CH(OH)-CH(OH)-(CH_2)_4NH_2$

active H = 4

moles of $CH_4 = 4 \times 0.5 = 2$

volume = 2 × 22.4 = 44.8 lit

(B) 1.5 mole of Me $NH(CH_2)_3$ NH Me

active H = 2

moles of $CH_4 = 1.5 \times 2 = 3$

volume of $CH_4 = 3 \times 22.4 = 67.2$ lit

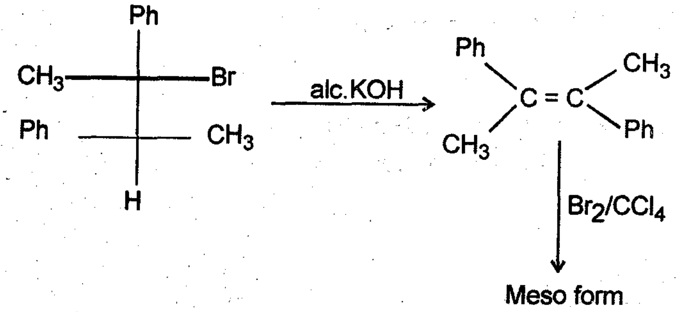
(C) 1 mole of HO $(CH_2)_4\,NH_2$

moles of active H = 2

moles of $CH_4 = 2$

volume of $CH_4 = 44.8$ lit.

**44.** B




**45.** D

$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(s)$

$4p°$ $2p°$

$kp = (4p^0)^2 2p^0 = 32p^{0^3}$

when temp decreases the reaction proceeds in backward direction.

$kp = (4p° - 2x)(2p° - x)$

$kp < 32p^{0^3}$

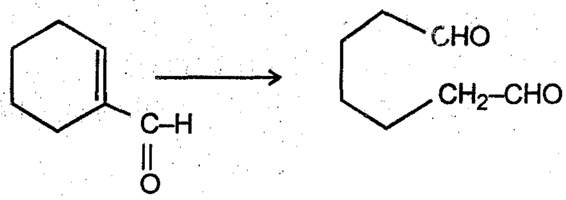
Total pressure = $(6p° - 3x) < 6p°$

**46.** C

C – C ≡ C – C ≡ C – C – C

Total Carbon atoms which are linearly arranged = 6

**47.** A




**48.** A

**49.** B,C,D

**50.** A,B,D

$H_2SO_4 + 2NH_4OH \rightarrow (NH_4)_2SO_4 + 2H_2O$

50 meq. 40 meq.

$HCl + NaOH \longrightarrow NaCl + H_2O + 57$ kJ

$CH_3COOH + NH_4OH \longrightarrow CH_3COONH_4 + H_2O + 48.1$ kJ

Heat evolved = $C\,\Delta t = 1.5 \times 1.4 = 2.10$ kJ

Heat of neutralisation of $H_2SO_4$ & $NH_4OH$ is

$$-\frac{2.1}{40} \times 1000 = -52.5$$

Heat of neutralisation of HCl & $NH_4OH$ is –52.5 KJ

(B) Enthalpy of dissociation of $NH_4OH$ is

= (57 – 52.5) = 4.5 kJ/mole

(C) Enthalpy of dissociation of $CH_3COOH$ is

$(57 - x - y) = 48.1$

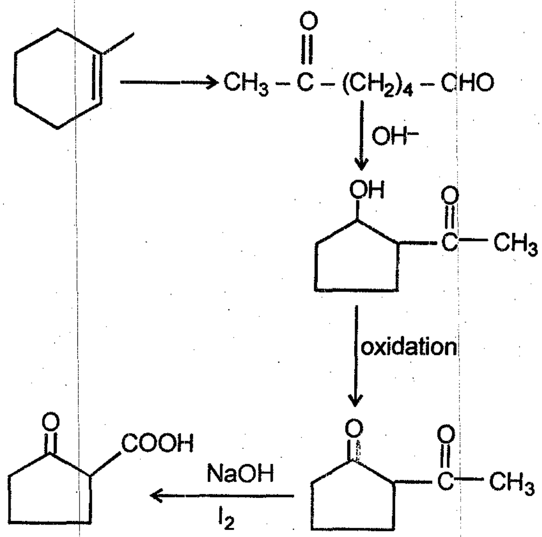
$57 - 4.5 - y = 48.1$

$y = 57 - 52.6 = 4.4$ kJ

(D) $2H^+ + 2OH^- \rightarrow 2H_2O(l)$ $\Delta H = -57 \times 2 = -114$

**51.** C,D

**Sol.**




**52.** D

**53.** B

104.5% $H_2SO_4$

$SO_3 + H_2O \rightarrow H_2SO_4$

$$\frac{4.5}{180} = 0.25 \text{ mole}$$

moles $SO_3$ = 0.25

wt = 0.25 × 80 = 20 gm

% of $SO_3$ = 20%

**54.** C

112% $H_2SO_4$ means total wt of $H_2SO_4$ = 112 gm

$H_2SO_4 + Na_2CO_3 \rightarrow H_2O + CO_2 + Na_2SO_4$

$\frac{112}{98}$ $\frac{5.3}{106} = 0.05$ mole

total moles of $CO_2$ = 0.05

$$1 \times v = 0.05 \times \frac{1}{12} \times 300$$

$$v = \frac{15}{12} \simeq 1.25 \text{ lit.}$$

**55.** **B**

**Sol.** $SO_3 + H_2O \longrightarrow H_2SO_4$

$n_1 \times 2 + n_2 \times 2 = 54 \times 0.4 \times 10^{-3}$

$n_1 + n_2 = 10.8 \times 10^{-3} = 1.08 \times 10^{-2}$ ...(1)

$n_1 \times 80 + n_2 \times 98 = 1$

$n_1 + 1.225\, n_2 = 1.25 \times 10^{-2}$ ...(2)

$0.225\, n_2 = 0.17 \times 10^{-2} = 0.755 \times 10^{-2}$

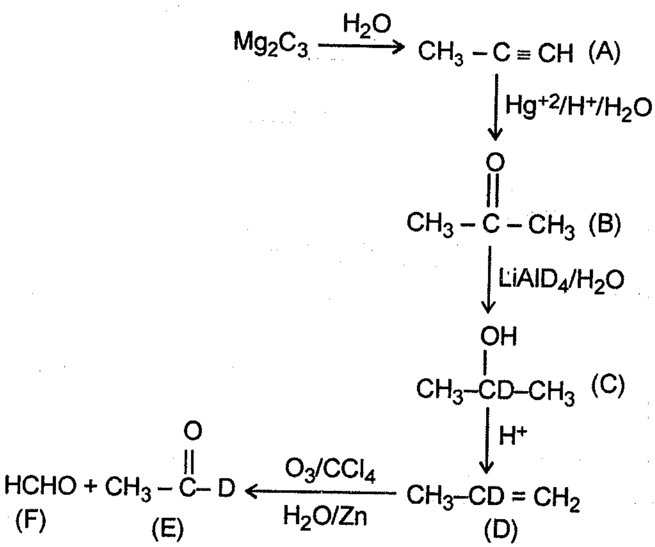
wt. of $H_2SO_4 = 0.74$ gm

% of $SO_3 = 26\%$

**56.** **C**

**57.** **C**

**58.** **B**

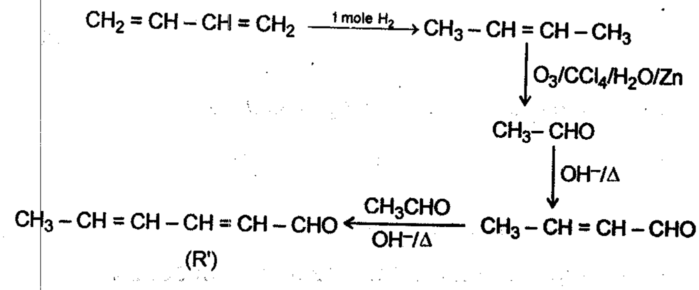


**56.** $B + E \xrightarrow{OH^-/\Delta} P$

P is $CH_3-\overset{O}{\overset{||}{C}}-CH=CD-CH_3$

Total stereo isomers = 2

**57.**



$CH_3-\underset{*}{CH}(Br)-CH=CH-CH=CH-\underset{*}{CH}(Br)-CH_3$

(X)

∴ Total stereo isomers of X is 10

**58.**

| | | | |
|---|---|---|---|
| R | Cis | Trans | R |
| S | Cis | Trans | S |
| S | Cis | Trans | R |
| R | Cis | Trans | S |

Total enantiomers = 4

**59.** **A → P,Q ; B → R,S ; C → P,Q ; D → R**

**60.** **A → Q, R, S ; B → P, R,S ; C → R,S ; D → P,R,S**