MOTION IIT-JEE
(Where Faith Counts the Success)

TARGET IIT-JEE
HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-II CLASS XII & XIII (DATE 09-08-09)

MATHEMATICS

Q.	1	2	3	4	5	6	7	8	9
A.	C	D	C	A	C,D	A,C	A,B	A,B,C	A,B,D
Q.	10						11		
A.	(A)–R, (B)–S, (C)–Q, (D)–Q						(A)–R, (B)–S, (C)–P, (D)–Q		
Q.	12	13	14	15	16	17	18	19	
A.	7	3	2	3	2	3	5	3	

PHYSICS

20. A 21. C 22. A 23. D 24. B 25. A,C

26. B,C 27. A 28. A,C

29. $A \to P$; $B \to P,Q$; $C \to P, S$; $D \to R, S$

30. (A) $\to$ Q,R,S ; (B) $\to$ Q, S ; (C) $\to$ P, Q, S

31. $\frac{1260}{47}V$ 32. 0.113 rad per sec 33. $\mu_K = 1 - \frac{1}{n^2}$ 34. 4.84 m

35. h = 0.73 R 36. = 2C/m^2 37. 8/3 mm 38. $\vec{v}_r = 8\sqrt{3}\,\hat{i} - (8\sqrt{3} - 5)\hat{j}$

CHEMISTRY

39. D 40. D 41. A 42. C 43. C,D 44. A,B,D 45. A,B,C

46. A,B,C,D 47. B,C

48. $A \to Q$, ; $B \to S$; $C \to P$; $D \to R$,

49. $A \to P,Q,R,S$; $B \to P,Q,R,S$; $C \to R,S$; $D \to P,R,S$

50. 226.76 kJ/mole 51. 0.04 52. $\frac{1}{3} \times 10^{-3}$ moles 53. – 13.68 kcal

54. $y = 8 \times 10^{-8}$ 55. 2 isomer 56. 0.7625 moles 57. n = 1

1. $f(x) = \frac{1}{4}(\alpha-\beta)^2 - x^2(\alpha+\beta)x + \alpha\beta$

$= \frac{(\alpha-\beta)^2+4\alpha\beta}{4} - \left\{x + \frac{\alpha+\beta}{2}\right\}^2$

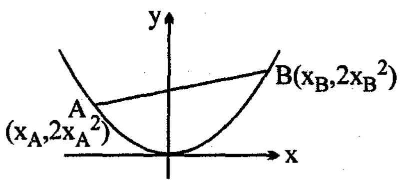
$+ \frac{(\alpha+\beta)^2}{4} = \frac{1}{2}(\alpha+\beta)^2 - \left\{x - \frac{a}{2}\right\}^2 \le \frac{a^2}{2}$

Thus, maximum value $f(x)$ is $\frac{a^2}{2}$ which is attained when $x = \frac{a}{2}$

2. We have $y = 2x^2$

$(AB)^2 = (x_B - x_A)^2 + (2x_B^2 - 2x_A^2)^2 = 5$

or $(x_B - x_A)^2 + 4(x_B^2 - x_A^2)^2 = 5$



differentiating w.r.t. x_A and denoting $\frac{dx_B}{dx_A} = D$

$2(x_B - x_A)(D-1) + 8(x_B^2 - x_A^2)(2x_BD - 2x_A) = 0$

put $x_A = 0$; $x_B = 1$

$2(1-0)(D-1) + 8(1-0)(2D-0) = 0$

$2D - 2 + 16D = 0 \quad \Rightarrow \quad D = 1/9$ Ans.

3. $f'(x) \cdot f'(x) - f(x) \cdot f''(x) = 0$

or $\frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2} = 0$

$\frac{d}{dx}\left[\frac{f(x)}{f'(x)}\right] = 0$

Integrating, $\frac{f(x)}{f'(x)} = C$ ....(1)

put $x = 0$, $\frac{f(0)}{f'(0)} = C$

$\Rightarrow$ $C = \frac{1}{2}$, hence $\frac{f(x)}{f'(x)} = \frac{1}{2}$

from (1) $2f(x) = f'(x)$

$\therefore$ $\frac{f'(x)}{f(x)} = 2$

again Integrating $\ln[f(x)] = 2x + k$

put x = 0 to get $k = 0$

$f(x) = e^{2x}$ **Ans.**

4. The given equation can be written as

$4\cos x(3\cos^2 x - 1) + 2\cos^2 x = 0$

$\Rightarrow 2\cos x(6\cos^2 x + \cos x - 2) = 0$

$\Rightarrow 2\cos x(3\cos x + 2)(2\cos x - 1) = 0$

$\Rightarrow$ either $\cos x = 0$ which gives $x = \pi/2$

or $\cos x = -2/3$, which gives no value of x for $0 \le x \le \pi/2$

or $\cos x = 1/2$, which gives $x = \pi/3$.

so the required difference $= \pi/2 - \pi/3 = \pi/6$

5. slope of the line $= -4$

$f'(x) = \frac{(2-x)+(x-1)}{(2-x)^2} = \frac{1}{(2-x)^2}$

$\therefore$ $\left.\frac{dy}{dx}\right]_{(x_1,y_1)} = \frac{1}{(2-x_1)^2}$

$\therefore$ $\frac{1}{(2-x_1)^2} \cdot (-4) = -1$

$2 - x_1 = 2$ or -2

$x_1 = 0$ or $x_1 = 4$

if $x_1 = 0$ then $y_1 = -\frac{1}{2}$

if $x_1 = 4$ then $y_1 = \frac{4-1}{2-4} = -\frac{3}{2}$

Hence the points are $\left(0, -\frac{1}{2}\right)$ and $\left(4, -\frac{3}{2}\right)$

6. (A) Take $f(x) = x^2 - 6x + 7$

(B) $\left.\begin{matrix} g(1) = 2 > 0 \\ g(3) = -6 < 0 \end{matrix}\right\}$ g is continuous

$\Rightarrow$ g must have a root

(C) Take $f(x) = \begin{cases} x+1 & \text{if } 1 \le x \le 2 \\ -5x+13 & \text{if } 2 < x \le 3 \end{cases}$

(D) $F(x) = f(x) - x$

7. given quadratic equation is an identity

$\therefore$ $\text{cosec}^2\theta = 4$ and $\cot\theta = -\sqrt{3}$

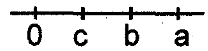
$\Rightarrow$ $\text{cosec}\,\theta = 2$ or -2 and $\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

8. $f(x) = Ax^2 + Bx + C$

$A = a + b - 2c = (a-c) + (b-c) > 0$

$\Rightarrow$ $A > 0$ $\Rightarrow$ mouth opens upwards now x = 1 is obvious solution therefore both roots are rational.

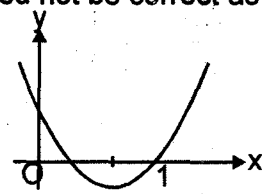


$$\underbrace{(b-a)}_{-ve}+\underbrace{(c-a)}_{-ve}<0 \quad \Rightarrow \quad B<0;$$

$$\text{vertex} = -\frac{B}{2A} > 0$$

hence abscissa 'a' of the vertex > 0

(D) need not be correct as with



a = 5, b = 4, c = 2, P < 0 and a = 6, b = 3, c = 2, P > 0

$\Rightarrow$ (A), (B) and (C) are correct.

9. Given f (x + y) = f (x) + f (y) + xy(x + y)....(1)

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(h)+xh(x+h)}{h} \qquad \text{[using (1)]}$$

$$= \lim_{h\to 0} x(x+h) + \lim_{h\to 0}\frac{f(h)}{h}$$

$$= x^2 + f'(0) \qquad (\text{ as } f(0) = 0)$$

$\therefore \quad f'(x) = x^2$

$$f(x) = \frac{x^3}{3} + C$$

but f (0) = 0 $\Rightarrow$ C = 0

Hence $f(x) = \frac{x^3}{3}$ $\Rightarrow$ A, B, D

10. (A) $$\lim_{x\to\infty}\frac{\ln x\cdot\int_{e^3}^{x}\frac{dt}{\ln t}}{x} \qquad \left(\frac{\infty}{\infty}\right) \text{ form}$$

using L'Hospital's rule

$$\ln x\cdot\frac{1}{\ln x} + \left(\int_{e^3}^{x}\frac{dt}{\ln t}\right)\cdot\frac{1}{x}$$

(1 + 0) = 1 Ans.

(B) $$l = \lim_{x\to\infty} e^{x^2+1}\left[e^{\sqrt{x^4+1}-(x^2+1)}-1\right]$$

now as $\lim_{x\to\infty}\sqrt{x^4+1}-(x^2+1)$

$$= \lim_{x\to\infty}\frac{x^4+1-(x^4+1+2x^2)}{\sqrt{x^4+1}+(x^2+1)}$$

$$= \lim_{x\to\infty}\frac{-2x^2}{x^2\left(\sqrt{1+(1/x^4)}+1+(1/x^2)\right)} = -1$$

now as $x\to\infty$

$\sqrt{x^2+1}-(x^2+1) \to -1$

$\infty \times \left(\frac{1}{e}-1\right) \to -\infty$ and hence limit does not exist **Ans.**

(C) $$\lim_{n\to\infty}(-1)^n(-1)^{n-1}\sin\left(n\pi-\pi\sqrt{n^2+\frac{n}{2}+1}\right)$$

$$= \lim_{n\to\infty}(-1)^{2n-1}\sin\pi\left(\frac{\left(n-\sqrt{n^2+\frac{n}{2}+1}\right)\left(n+\sqrt{n^2+\frac{n}{2}+1}\right)}{n+\sqrt{n^2+\frac{n}{2}+1}}\right)$$

$$= \lim_{n\to\infty}(-1)^{2n-1}\sin\pi\left(\frac{n^2-n^2-\frac{n}{2}-1}{n\left(1+\sqrt{1+\frac{1}{2n}+\frac{1}{n^2}}\right)}\right)$$

$$= \lim_{n\to\infty}(-1)^{2n}\sin\pi\left(\frac{\frac{1}{2}+\frac{1}{n}}{1+\sqrt{1+\frac{1}{2n}+\frac{1}{n^2}}}\right)$$

as $n\to\infty$

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

also as $n\to\infty$, $\sin\frac{(n+1)\pi}{4n}\to\frac{1}{\sqrt{2}}$

$\therefore$ final answer is $\frac{1}{2}$ Ans.

(D) Use King P-5 add to get f (x) + f (1 – x) = 1

$\Rightarrow$ 2I = 1 $\Rightarrow$ I = 1/2 **Ans.**

11. (A) $g'(3^-) =$

$$\lim_{h\to 0}\frac{g(3-h)-g(h)}{-h} = \lim_{h\to 0}\frac{a\sqrt{4-h}-(3b+2)}{h} \quad \ldots(1)$$

for existence of limit $\lim_{h\to 0} N^r = 0$

$\therefore 2a - 3b = 2 \quad \ldots(2)$

now $g'(3^+) = \lim_{h\to 0}\frac{b(3+h)+2-(3b+2)}{h} = b \quad \ldots(3)$

substituting $3b + 2 = 2a$ in equation (1) $g'(3^-)$

$$= \lim_{h\to 0}\frac{a\sqrt{4-h}-2a}{-h} = \lim_{h\to 0}\left(\frac{(4-h)-4}{(-h)(\sqrt{4-h}+2)}\right) = \frac{a}{4}$$

hence $g'(3^-) = g'(3^+)$

$\frac{a}{4} = b \quad \Rightarrow \quad a = 4b \quad \ldots(4)$

from (2) and (4)

$8b - 3b = 2 \quad \Rightarrow \quad b = \frac{2}{5}$ and $a = \frac{8}{5}$

$\Rightarrow \quad a + b = 2 \quad \Rightarrow \quad$ **(R)**

(B) $\log(\log_3 10) = \log\left(\frac{1}{\log_{10} 3}\right) = -\log_{10}(\log_{10} 3)$

Given $f(-\log_{10}(\log_{10} 3)) = 5 \quad \ldots(1)$

now $f(x) = a\sin x + b\,x^{1/3} + 4$

$f(-x) = -a\sin x - b\,x^{1/3} + 4$

$f(x) + f(-x) = 8$

$f(\log_{10}(\log_{10} 3)) + f(-\log_{10}(\log_{10} 3)) = 8$

$f(\log_{10}(\log_{10} 3)) + 5 = 8$

$f(\log_{10}(\log_{10} 3)) = 3 \quad \Rightarrow \quad$ **(S)**

(C) Obviously zero $\quad \Rightarrow \quad$ **(P)**

(D) LHS = $f(x)$ and RHS = $f^{-1}(x)$

(property of inverse function)

hence $f(x) = f^{-1}(x) = x$

$\therefore (x-8)^{\frac{1}{\log_{10} 2}} = x \Rightarrow \frac{1}{\log_{10} 2}\log_{10}(x-8) = \log_{10}x$

$\Rightarrow \quad \frac{\log(x-8)}{\log_{10} x} = \log_{10}2$

$\log_x(x-8) = \log_{10}2$

$\Rightarrow \quad x = 10$ and $x - 8 = 2$

$\Rightarrow x = 10$

$\Rightarrow \quad$ only one solution $\quad \Rightarrow \quad$ **(Q)**

12. Let $y = \dfrac{\left(x+\frac{1}{x}\right)^4 - \left(x^4+\frac{1}{x^4}\right) - 1}{\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)}$.

Put $x + \frac{1}{x} = t$

$$\Rightarrow y = \frac{t^4 - [(t^2-2)^2 - 2] - 1}{t^2 + t^2 - 2} = \frac{t^4 - [t^4 - 4t^2 + 2] - 1}{2(t^2-1)}$$

$$\Rightarrow y = \frac{4t^2 - 3}{2(t^2-1)} \Rightarrow y = \frac{4t^2 - 4 + 1}{2(t^2-1)} = \left(2 + \frac{1}{2(t^2-1)}\right).$$

As $t = x + \frac{1}{x} \Rightarrow t^2 \ge 4 \Rightarrow t^2 - 1 \ge 3 \Rightarrow \frac{1}{t^2-1} \le \frac{1}{3}$.

$\Rightarrow$ Maximum value of y is $2 + \frac{1}{3\times 2} = \frac{13}{6}$

13. Note that $x^2 - 3x + 7 > 0 \quad \forall \quad x \in R$

Also $x > -2/3$ and $x \ne -1/3$

Also $3x + 2 > 1 \Rightarrow x > -1/3$ and

$0 < 3x + 2 < 1$

$\Rightarrow -2/3 < x < -1/3$

Hence for $x > -1/3$

$\log_{3x+2}(x^2 - 3x + 7) < 1$

$\Rightarrow x^2 - 3x + 7 < 3x + 2$

$x^2 - 6x + 5 < 0$

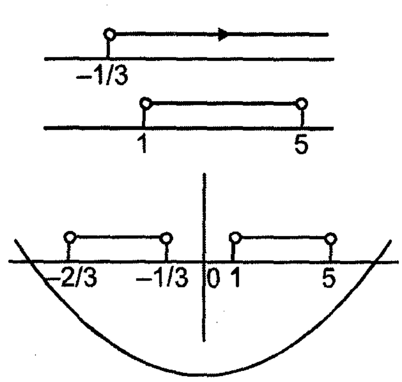
$(x-5)(x-1) < 0 \quad \Rightarrow \quad x \in (1, 5)$

for $-2/3 < x < -1/3$

$x \in \left(-\frac{2}{3}, -\frac{1}{3}\right)$

Hence the solution of the first inequality is

$$x \in \left(-\frac{2}{3}, -\frac{1}{3}\right) \cup (1, 5)$$



Now if any solution of the inequality is also the solution of,

$f(x) = x^2 + (5 - 2a)x - 10a \le 0$ then

$f(5) \le 0$ and

$f\left(-\frac{2}{3}\right) \le 0 \quad \Rightarrow \quad a \ge 5/2$

14. $f'(x) = (b^2 - 3b + 2)(-2\sin 2x) + b - 1 \neq 0$ for any $x \in R$

$\Rightarrow \quad (b-1)\{1-(b-2)(2\sin 2x)\} \neq 0$

$\Rightarrow \quad b \neq 1$ and $\left|\frac{1}{2(b-2)}\right| > 1$

$\Rightarrow \quad b \in \left(\frac{3}{2}, 2\right) \cup \left(2, \frac{5}{2}\right)$

When b = 2, f(x) = x + sin 3 $\Rightarrow \quad f'(x) = 1 \neq 0$

$\therefore \quad b \in \left(\frac{3}{2}, \frac{5}{2}\right) \Rightarrow$ integra value of b = 2

15. $$I = \int \frac{\sin^2\frac{\theta}{2}}{\cos\frac{\theta}{2}\sqrt{\cos^3\theta + \cos^2\theta + \cos\theta}}d\theta$$

$$= \int \frac{\left(2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right)\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta + \cos^2\theta + \cos\theta}}d\theta$$

$$= \int \frac{2\sin^2\frac{\theta}{2}\sin\theta\, d\theta}{2\cos^2\frac{\theta}{2}\sqrt{\cos^3\theta + \cos^2\theta + \cos\theta}}$$

Put $\cos\theta = t \quad \Rightarrow \quad -\sin\theta\, d\theta = dt$

Also $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2} = t$

$$\therefore I = \int \frac{\frac{1-t}{2}(-dt)}{(1+t)\sqrt{t^3+t^2+t}} = \frac{1}{2}\int \frac{(t^2-1)dt}{(t+1)^2\sqrt{t^3+t^2+t}}$$

$$= \frac{1}{2}\int \frac{\left(1-\frac{1}{t^2}\right)(dt)}{\left(t+\frac{1}{t}+2\sqrt{t+\frac{1}{t}+1}\right)}$$

Put $t + \frac{1}{t} + 1 = u^2 \Rightarrow \left(1 - \frac{1}{t^2}\right)dt = 2u\, du$

$$I = \frac{1}{2}\int \frac{2udu}{(1+u^2)u} = \tan^{-1} u = \tan^{-1}\sqrt{1+\frac{1}{t}+1} + c$$

$= \tan^{-1}(\cos\theta + \sec\theta + 1)^{1/2} + c$

So, $f(\theta) = \cos\theta + \sec\theta + 1 \geq 2 + 1 = 3$

16. $$\text{RHS} = \frac{(x-1)(x+1)(x^2+1)(x^4+1)....(2^{2^{2007}}+1)-1}{(x-1)}$$

$= (x^2-1)(x^2+1) ..... (x^{2^{2007}}+1) - 1$

$= (x^{2^2}-1)(x^{2^2}+1)......(x^{2^{2007}}+1) - 1$

hence $g(x) . (x^{(2^{2008}-1)} - 1) = \frac{(x^{2^{2008}}-1)-1}{(x-1)}$

$\therefore \quad g(2) . (2^{(2^{2008}-1)} - 1) = (2^{2^{2008}} - 1) - 1$

$g(2) . \frac{(2^{2^{2008}}-2)}{2} = (2^{2^{2008}} - 2) \Rightarrow g(2) =$ **Ans.**

17. $$y = \frac{2x^2+4x-2}{5x-3} = 2x + \frac{26x-10}{5x-3}$$

$$\Rightarrow \quad 25y = 10x + \frac{130x-50}{5x-3} = 10x + 26 + \frac{28}{5x-3}$$

Since $y \in I$, 28 is divisible by $5x - 3$

$\therefore \quad 5x - 3 = \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

Since $x \in I$, the only possibilities are x = 1, 2, –5.

The points are (1, 2), (2, 2) and (–5, –1).

18. Since α, β are the roots of $x^2 + px + q = 0$

$\therefore \quad \alpha + \beta = -p, \alpha\beta = q$ ...(1)

Also α, β are roots of $x^{2n} + p^n x^n + q^n = 0$

$\therefore \quad \alpha^n + \beta^n = = p^n$ and $\alpha^n\beta^n = q^n$ ...(2)

Now, α/β is are root of $x^n + 1 + (x+1)^n = 0$

$$\Rightarrow \quad \frac{\alpha^n}{\beta^n} + 1 + \left(\frac{\alpha}{\beta}+1\right)^2 = 0$$

$$\Rightarrow \quad \frac{\alpha^n+\beta^n}{\beta^n} + \frac{(\alpha+\beta)^n}{\beta^n} = 0$$

$\Rightarrow \quad (\alpha^n + \beta^n) + (\alpha + \beta)^n = 0$

$\Rightarrow \quad -p^n + (-p)^n = 0$ {from (1) and (2)}

This is true only if n is an even integer.

19. $$E_1 = \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$$

$$2\left(\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8}\right) = 2\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right)$$

$$= 2\left(1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)$$

$$= 2\left(1 - \frac{1}{2}\sin^2\frac{\pi}{4}\right) = 2\left(1 - \frac{1}{4}\right) = 2 \times \frac{3}{4} = \frac{3}{2}$$

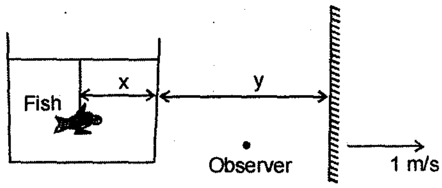
$$E_2 = \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

$$2\left(\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8}\right) = 2\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right)$$

$$= 2\left(1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) = 2\left(1 - \frac{1}{4}\right) = \frac{3}{2}$$

$E_1 + E_2 = 3$

20. A



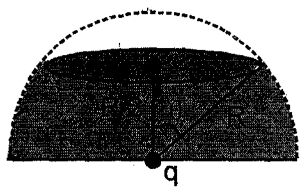
$$x_{fm} = -\left(\frac{x}{\mu} + y\right) \Rightarrow \frac{dx_{fm}}{dt} = -\left[\frac{1}{\mu}\frac{dx}{dt} + \frac{dv}{dt}\right]$$

$$\Rightarrow V_{OM} = -V_{IM} = -\left[\frac{1}{2} \times (-4) + 1\right]$$

$$-[V_I - V_m] = +1 \Rightarrow V_I = 0$$

21. C

$$\phi = \left[\frac{q}{2\varepsilon_0} - \frac{q}{\varepsilon_0} \times \frac{2\pi(1-\cos\theta)}{4\pi}\right]$$



22. A

The tension at joint is due to force exerted by the rod of linear density μ_2.

So $F = \int_{l}^{2l} \mu_2 dx \omega^2 x = \frac{3\mu_2\omega^2 l^2}{2}$

23. D

24. B

Relative speed $v^2 = (a\,\omega\cos\omega t)^2 + (a\omega\sin\omega t - a\omega)^2$

$\therefore \quad v = a\omega\sqrt{2(1-\sin\omega t)}$

when $t = \pi/\omega$

$v \rightarrow = a\,\omega\sin\pi - a\omega = -a\,\omega$

$v\uparrow a\omega\cos\pi = -a\,\omega$

Hence at $t = \pi/\omega$ relative velocity is parallel to the line $y = x$

25. A,C **26.** B,C

27. A

$W_{net} = \Delta K$

$2mg\,L\sin\alpha - \mu_1 mg\cos\alpha\,L - \mu_2 mg\cos\alpha\,L = 0$

$2\sin\alpha = (\mu_1 + \mu_2)\cos\alpha$

$\mu_1 + \mu_2 = 2\tan\alpha$

28. A,C

Let equilibrium extension be x_0

$\Rightarrow 2kx_0\cos\theta = mg \Rightarrow x_0 = \frac{mg}{2k\cos\theta}$

Let the spring be displaced below by an amount x from equilibrium position.

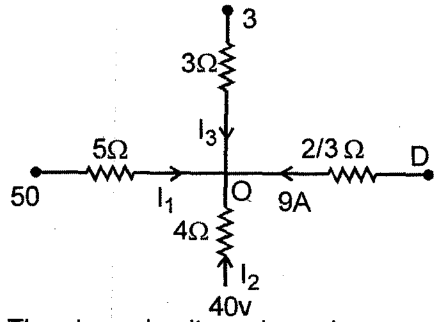
The restoring force $= 2Kx\cos^2\theta = -m\frac{d^2x}{dt^2}$.

$$\Rightarrow \omega = \sqrt{\frac{2k\cos^2\theta}{m}}$$

29. $A \rightarrow P$; $B \rightarrow P,Q$; $C \rightarrow P, S$; $D \rightarrow R, S$

30. $(A) \rightarrow Q,R,S$; $(B) \rightarrow Q, S$; $(C) \rightarrow P, Q, S$

31.



The given circuit can be radrawn.

Apply junction rule at point O

$I_1 + I_2 + I_3 = 9$

Potential of point O be x then

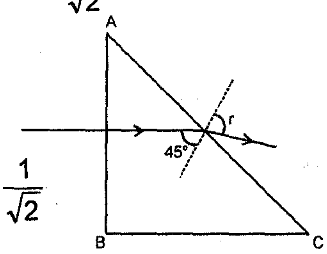
$$I_1 = \frac{50-x}{5},\quad I_3 = \frac{30-x}{3},\quad I_2 = \frac{40-x}{4}$$

$$9 = \frac{600 - 12x + 600 - 20x + 600 - 15x}{60}$$

$$x = \frac{1800-540}{47} = \frac{1260}{47}\text{V}$$

32. $\frac{\sin r}{\sin 45°} = n \Rightarrow \sin r = \frac{n}{\sqrt{2}}$

$$\Rightarrow r = \sin^{-1}\left(\frac{n}{\sqrt{2}}\right)$$



$$\omega = \frac{\sqrt{2}}{\sqrt{2-n^2}}\left(\frac{dn}{dt}\right)\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2-n^2}}\left(\frac{dn}{dt}\right) = \frac{1}{\sqrt{2-1.1^2}}(0.1) = 0.113 \text{ rad per sec}$$

33. On smooth inclined plane

$$L = 0 + \frac{1}{2}g\sin\theta t^2 \Rightarrow t = \sqrt{\frac{2L}{g\sin\theta}}$$

on rough inclined surface :

$$L = 0 + \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)t^2$$

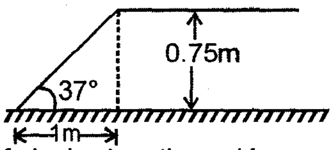
$$t' = \sqrt{\frac{2L}{g\sin\theta - \mu_k g\cos\theta}}$$

given $t' = nt$

$$\frac{2L}{g(\sin\theta - m_k\cos\theta)} = n^2\frac{2L}{g\sin\theta}$$

$$\frac{1}{1-\mu_k} = n^2 \Rightarrow 1 - \mu_k = \frac{1}{n^2} \Rightarrow \mu_k = 1 - \frac{1}{n^2}$$

34.



Speed of plank when the rod leaves the plank

$v^2 = 2 \times 8 \times 1 = 16$

$\Rightarrow v = 4$ m/s

By constraint relation speed of the rod,

$u/4 = \tan 37° \Rightarrow u = 3$ m/s (vertically upwards)

Time period of its flight $= \frac{2 \times 3}{10} = 0.6\,s$

$l = 1 + 4 \times (0.6) + \frac{1}{2} \times 8 \times (0.6)^2 = 4.84$ m

35. $\omega = \frac{1}{\sqrt{\tan\theta}} \Rightarrow T = 2\pi\sqrt{\tan\theta} = 2\pi\sqrt{\frac{l}{g}}$

$\therefore\ g = \frac{l}{\tan\theta}$

$\therefore\ \frac{g_{ground}}{g_h} = \frac{\tan\theta_h}{\tan\theta_{ground}} = \frac{(R+h)^2}{R^2} = 0.73$ hr

$\frac{\tan 60°}{\tan 30°} = \left(\frac{R+h}{R}\right)^2 \Rightarrow h = 0.73\,R$

36. Consider a Gaussian surface of radius $r(R_1 < r < R_2)$

The net charge enclose is

$q_{net} = q + \int_{R_1}^{r}(4\pi r^2)\frac{b}{r}dr$ or $q_{net} = q + 2\pi b(r^2 - R_1^2)$

Using Gauss's Law

$E(4\pi r^2) = \frac{q_{net}}{\varepsilon_0} = \frac{q + 2\pi b(r^2 - R_1^2)}{\varepsilon_0}$

or $E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{b}{2\varepsilon_0}\left(1 - \frac{R_1^2}{r^2}\right)$

For E = constant, $b = \frac{q}{2\pi R_1^2} = 2C/m^2$

37. 8/3 mm

$\mu_w d\sin\theta = t(\mu_g - 1)$

38. Velocity of rain $\vec{v}_r = v_x\hat{i} + v_y\hat{j}$, $\vec{v}_m = 5\hat{i}$

$\vec{v}_{rm} = (v_x - 5)\hat{i} + v_y\hat{j}$

$\frac{v_y}{v_x - 5} = -\tan 45°$ (Given)

$v_y = -(v_x - 5)$ ...(i)

In second case :

$\vec{v}'_{rm} = \frac{16\sqrt{3}}{5}\hat{i} - \frac{16}{2}\hat{j}$

$\vec{v}'_m = (v_x - 8\sqrt{3})\hat{i} + (v_y + 8)\hat{j}$

Given resultant $\vec{v}'_m$ is in vertical direction

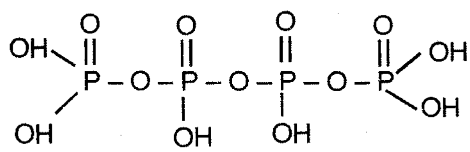
$\Rightarrow v_x = 8\sqrt{3}$ Hence from (i)

$v_y = -(8\sqrt{3} - 5) \Rightarrow \vec{v}_r = 8\sqrt{3}\,\hat{i} - (8\sqrt{3} - 5)\hat{j}$

CHEMISTRY

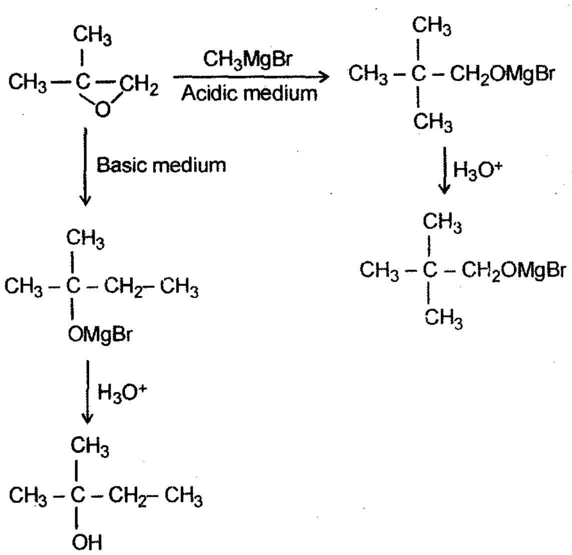
39. D

$H_4P_4O_{13} \Rightarrow$



Bridge O = 3

40. D



Sol.

41. A $Ba\overset{14}{C}O_3 + H_2SO_4 \longrightarrow BaSO_4 + H_2O + \overset{14}{C}O_2$

$CH_2 = CHBr \xrightarrow[x\ H_2O^+]{Mg/ether} CH_2 = CH - \overset{14}{C}OOH$

$\downarrow LiAlH_4$

$CH_2 = CH - \overset{14}{C}H_2OH$

(Z)

42. C

$[Co(CN)_6]^{3-} \Rightarrow d^2sp^3$ ($\mu = 0$)

$[Fe(CN)_6]^{3-} \Rightarrow d^2sp^3$ ($\mu = \sqrt{3}$)

$[Mn(CN)_6]^{3-} \Rightarrow d^2sp^3$ ($\mu = \sqrt{8}$)

$[Cr(CN)_6]^{3-} \Rightarrow d^2sp^3$ ($\mu = \sqrt{15}$)

IV > III > II > I i.e.,

43. C,D

$N_2O_3 + H_2O \rightarrow HNO_2$

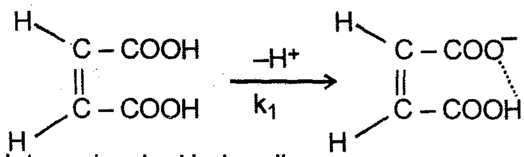
$Cl_2O + H_2O \rightarrow HOCl$

$ClO_3 + H_2O \rightarrow HClO_4 + HClO_3$

$ClO_2 + H_2O \rightarrow HClO_2 + HClO_3$

44. **A,B,D**

45. **A,B,C**



Intramolecular H - bonding

46. **A,B,C,D**

47. **B,C**

48. **A → Q, ; B → S ; C → P ; D → R,**

$Ba(SCN)_2 \longrightarrow SO_4^{-2} + HCN$

$Ba^{+2} + S^{-} - C^{+3} \equiv N^{-3}$ $H - C^{+3} \equiv N^{-3}$

+1 –1   –1

Only oxidation no. of sulphur is changing E_{wt} = M/12

(B) $B_2S_3 \rightarrow Bi^{+5} + S$

B^{+3} s^{-2}

change = 2 × 2 + 2 × 3 = 10

Ewt. = M/10

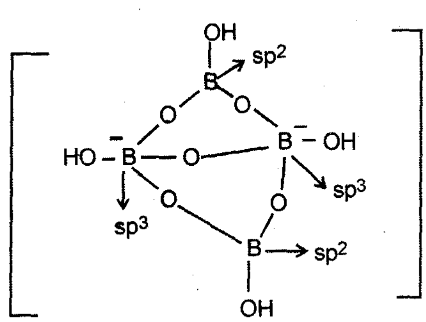
(C) $FeC_2O_4 \rightarrow Fe^{+3} + CO_2$

⇒ Ewt. = M/3

$Cr\,I_3 \rightarrow Cr_2O_7^{-2}$

Ewt. = M/27

49. **A → P,Q,R,S ; B → P,Q,R,S ; C → R,S ; D → P,R,S**

2Na+  2H₂O

50. $2C_{(s)} + H_{2(g)} \rightarrow C_2H_{2(g)}$

$\Delta H° = 2\Delta H_c°C_{(s)} + \Delta H_c° H_2 - \Delta H_c° C_2H_2$

= 2 (–393.51) – 285.85 + 1299.63

= –787.02 – 285.85 + 1299.63 = 226.76 kJ/mole

51. 72 ml of FeC_2O_4 + 75 ml of $FeSO_4$

meq. of $KMnO_4$ = 20 × 0.04 × 6 = 4.8

4.8 = meq. of FeC_2O_4 + meq of $FeSO_4$

4.8 = 75 x × 3 + 75 y ...(1)

= 225 x + 75 y

Zn reduces the Fe^{+3} to Fe^{2+}

Meq. of Fe^{+2} = meq. of $KMnO_4$

75x + 75y = 36 × 0.02 × 5

75x + 75y = 3.6 ...(ii)

225x + 75y = 4.8 ...(i)

150x = 1.2 ⇒ $x = \frac{1.2}{150} = 0.008M$

Normality of FeC_2O_4 = 3 × 0.008 = 0.024

75 y = 3 ⇒ y = 3/75 = 1/25 = 0.04

Normality of $FeSO_4$ = 0.04

52. $HNO_3 \xrightarrow[\Delta]{Cu} NO \xrightarrow{\Delta} NO_3^-$

Meq of $FeSO_4$ = Meq. of Excess of $kMnO_4$

40 × 0.1 = 4

Now Meq. of $kMnO_4$ = meq. of $FeSO_4$

100 × M × 5 = 50 × 0.1

10M = 0.1 ⇒ M = 0.01

Meq. of $kMnO_4$ used for NO

= 100 × 0.01 × 5 – 4

= 1 = meq. of NO

m. moles of NO = 1/3

moles of No = $\frac{1}{3} \times 10^{-3}$ moles

53. Heat of Neutralisation = $-\frac{C\Delta t}{75 \times 10^{-3}}$

$= \frac{-300 \times 3.42}{75 \times 10^{-3}} = -13.68$ kcal

54. $H_2 + Br_2 \rightleftharpoons 2HBr$

4   2   O

2   Y   4

$K = \frac{(4)^2}{2.y} \Rightarrow 10^8 = \frac{16}{2 \times y} \Rightarrow y = 8 \times 10^{-8}$

55. $Ph{-}CHO \xrightarrow[H_2O]{CH_3MgBr} Ph - CH(OH) - CH_3 \xrightarrow{H^+} Ph - CH = CH_2 \xrightarrow{HBr} Ph - \overset{*}{C}H(Br) - CH_3$

= 2 isomer

56. $LiCl.3NH_3(s) \rightleftharpoons LiCl.NH_3(s) + 2NH_3$

kp = 9 atm²

$k_p = p_{NH_3}{}^2 \Rightarrow p_{NH_3} = 3$ atm

Pv = nRT ⇒ 3 × 5 = n × 0.0821 × 320

$3 \times 5 = n \times \frac{1}{12} \times 320 = 0.5625$

for complete reaction

moles of NH_3 = 0.5625 + 0.2 = 0.7625 moles

57. $13.84 = \frac{\text{moles wt. of} \times \text{Vap}}{\text{mole wt. of air}}$

$13.84 = \frac{400n}{29} = \frac{6.92 \times 29}{200} = n$ ⇒ n = 1®