



MOTION IIT-JEE

(Where Faith Counts the Success)

TARGET IIT-JEE HINT & SOLUTIONS

ANSWER KEY WITH SOLUTION

PAPER-I CLASS XII & XIII (DATE 09-08-09)

MATHEMATICS

| | | | | | | | | | | | | |
|----|--------------------------------|----|----|----|----|----|------------------------------------|---|-------|-----|-------|-----|
| Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| A. | A | C | B | C | C | A | A | B | B,C,D | A,C | A,B,D | C,D |
| Q. | 13 | 14 | 15 | 16 | 17 | 18 | | | | | | |
| A. | B | C | A | A | C | B | | | | | | |
| Q. | 19 | | | | | | 20 | | | | | |
| A. | (A)–QRS, (B)–PR, (C)–QR, (D)–R | | | | | | (A) – R, (B) – S, (C) – P, (D) – Q | | | | | |

PHYSICS

21. A 22. B 23. B 24. B 25. C 26. C 27. A
 28. D 29. A, C,D 30. A,D 31. B,C 32. A,B,C,D 33. D 34. A
 35. C 36. B 37. C 38. C
 39. (A) → P,R; (B) → Q, S; (C) → Q,R; (D) → (P,S)
 40. (A) → Q,S; (B) → P; (C) → Q,R,S; (D) → (R)

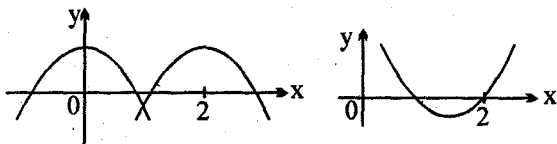
CHEMISTRY

41. C 42. B 43. A 44. D 45. C 46. B 47. B
 48. B 49. B,C,D 50. B 51. A,C 52. C,D 53. D 54. A
 55. D 56. B 57. A 58. B
 59. A → Q; B → R; C → P,S; D → Q
 60. A → P,Q; B → P,R,S; C → P,Q; D → P,Q

SOLUTIONS

MATHEMATICS

1. $f(0) \cdot f(2) < 0$
 $(k^2 + 5)(k^2 + 2k - 3)$
 $k^2 + 2k - 3 < 0$
 $(k + 3)(k - 1) < 0$
 $-3 < k < 1$



if one root is $x = 2$
then $f(2) = 0$
 $k^2 + 2k - 3 = 0$
 $\Rightarrow k = 1$ or $k = -3$

if $k = 1$
 $2x^2 - x - 6 = 0 \Rightarrow$
 $2x^2 - 4x + 3x - 6 = 0$
 $\Rightarrow 2x(x - 2) + 3(x - 2) = 0$
 $\Rightarrow x = 2$ or $x = -3/2$

other roots does not lie in $(0, 1)$
|||ly when $k = -3$
roots are $2, -7$

$\Rightarrow k = -3$ is also not possible

2. If $x = r \cos \theta$ and $y = r \sin \theta$ then $E = x^2 + y^2 = r^2$. Hence we have to minimise r^2 . Now in the given equation substituting $x = r \cos \theta$ and $y = r \sin \theta$, we get $r^2 = 4 \operatorname{cosec} 4\theta$

$\Rightarrow r^2 \Big|_{\min} = 4$ Ans.]

3. for $k \leq -1$, $D = 2$ and for $k \geq 0$,
 $D = 8k + 6 > 0 \forall k \geq 0$
 \Rightarrow roots are real and distinct.]

4. Given $f'(x) + f(x) \leq 1$
multiplying by e^x
 $f'(x) e^x + f(x) e^x \leq e^x$

$\frac{d}{dx}(e^x \cdot f(x)) \leq e^x$

integrating between 0 and 1

$\int_0^1 \frac{d}{dx}(e^x f(x)) dx \leq \int_0^1 e^x dx$

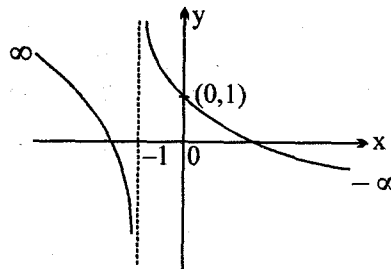
$e^x f(x) \Big|_0^1 \leq e^x \Big|_0^1$

$e \cdot f(1) - e^0 \cdot f(0) \leq e - 1$

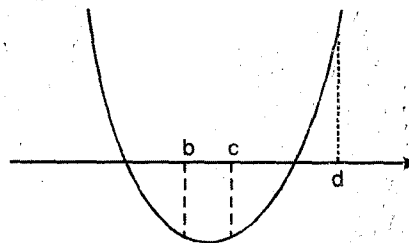
$f(1) \leq \frac{e-1}{e}$

5. $f'(x) = -\frac{3}{(x+1)^4} - 3 + \cos x < 0$

hence $f(x)$ is always decreasing, Also as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$
and as $x \rightarrow -\infty$, $f(x) \rightarrow +\infty$
hence one positive and one negative root
Graph is as shown



6. We can rewrite (1) as
 $ax^2 - a(b+c)x + abc + x - d = 0$
or $a(x-b)(x-c) + x - d = 0$
Let $f(x) = a(x-b)(x-c) + x - d$.
As $a > 0$, $y = f(x)$ represents a parabola which open upwards.



Also, $f(b) = b - d < 0$
 $f(c) = c - d < 0$, and $f(d) = a(d-b)(d-c) > 0$
Thus, $f(x) = 0$ has a root between $-\infty$ and b and a root between c and d .

7. $\alpha = \sin^{-1} \left[\cot \left(\theta + \frac{\pi}{6} + \frac{\pi}{4} \right) \right]$ where $\sin^2 \theta = \frac{2-\sqrt{3}}{4}$
 $= \sin^{-1} \left[\frac{\cot \theta \cot(\pi/4 + \pi/6) - 1}{\cot \theta + \cot(\pi/4 + \pi/6)} \right]$

Now, $\cot \theta = \sqrt{\frac{4}{2-\sqrt{3}} - 1} = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = 2 + \sqrt{3}$ and

$$\cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \frac{\cot\frac{\pi}{6} - 1}{1 + \cot\frac{\pi}{6}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} =$$

$$\frac{(\sqrt{3} - 1)^2}{2} = 2 - \sqrt{3}$$

So $\alpha = \sin^{-1} 0 = 0$.

8. The equation

$$\frac{1 - \sin x + \dots + (-1)^n \sin^n x + \dots}{1 + \sin x + \dots + \sin^n x + \dots} = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\Rightarrow \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1} = \frac{2 \sin^2 x}{2 \cos^2 x}$$

as $-1 < \sin x < 1$

$$\Rightarrow 1 - \sin x = \frac{\sin^2 x (1 + \sin x)}{1 - \sin^2 x}$$

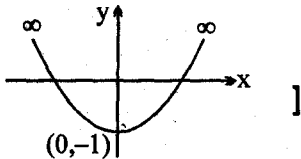
$$\Rightarrow (1 - \sin x)^2 = \sin^2 x \Rightarrow 1 - 2 \sin x = 0$$

$$\Rightarrow \sin x = 1/2 = \sin(\pi/6)$$

$$\Rightarrow x = n\pi + (-1)^n \pi/6$$

9. $f'(x) = \frac{2-x}{x^3}$ and $f''(x) = \frac{x-3}{x^4}$. Now interpret.

10. Graph of $y = f(x) \Rightarrow$ (A) and (C)



11. $(\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) > \frac{5}{8}$

$$= 1 - 3 \sin^2 x \cos^2 x > \frac{5}{8}$$

$$\Rightarrow 1 - \frac{5}{8} > 3 \sin^2 x \cos^2 x$$

$$\Rightarrow \frac{3}{8} > 3 \sin^2 x \cos^2 x$$

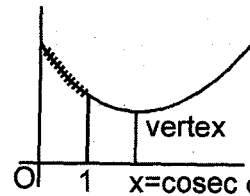
$$\Rightarrow 1 - 2 \sin^2 2x > 0 \Rightarrow \cos 4x > 0$$

$$4x \in \left(2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right) \text{ using graph}$$

$$x \in \left(\frac{n\pi}{2} - \frac{\pi}{8}, \frac{n\pi}{2} + \frac{\pi}{8}\right); n \in I$$

12. Abscissa corresponding to the vertex is given by

$$x = \frac{1}{\sin \alpha} > 1 \text{ is the vertex}$$



the graph of $f(x) = (\sin \alpha)x^2 - 2x + b$ as shown $\forall x \leq 1$

\therefore minimum of $f(x) = (\sin \alpha)x^2 - 2x + b - 2$ must be greater than zero where minimum is at $x = 1$ i.e. $\sin \alpha - 2 + b - 2 \geq 0$; $b \geq 4 - \sin \alpha, \alpha \in (0, \pi)$

$$b \geq 4 \text{ as } \sin \alpha > 0 \text{ in } (0, \pi)$$

Hence (C) and (D) are correct

13. Given cubic

$$f(x) = (x - 1)(x - \cos \theta)(x - \sin \theta)$$

\therefore roots are 1, $\sin \theta$ and $\cos \theta$

$$\therefore x_1^2 + x_2^2 + x_3^2 = 1 + \sin^2 \theta + \cos^2 \theta = 2 \text{ Ans.}$$

$$\text{now if } 1 = \sin \theta \Rightarrow \theta = \frac{\pi}{2}$$

$$\text{if } 1 = \cos \theta \Rightarrow \theta = 0, 2\pi$$

$$\text{and if } \sin \theta = \cos \theta \Rightarrow \tan \theta = 1$$

$$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

14. \therefore number of values of θ in $[0, 2\pi]$ is Ans. 5 Ans.

15. again maximum possible difference between the two roots is 2

$$\frac{1 - \sin \theta}{\text{when } \theta = 3\pi/2} \text{ or } \frac{1 - \cos \theta}{\text{when } \theta = \pi}$$

16. $y = e^{a+bx^2}$, passes through (1, 1)

$$1 = e^{a+b} \Rightarrow a + b = 0$$

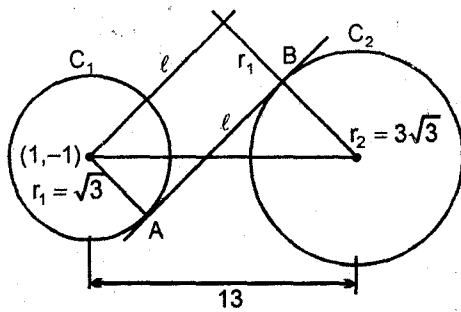
$$\text{also } \frac{dy}{dx} \Big|_{(1,1)} = -2$$

$$e^{a+bx^2} \cdot 2bx = -2 \Rightarrow e^{a+b} \cdot 2b(1) = -2$$

$$\Rightarrow b = -1 \text{ and } a = 1$$

$$\Rightarrow (a, b) = (1, -1)$$

$$\Rightarrow a^2 + b^2 = 2 \text{ Ans.}$$



17.

hence $C_1: (x-6)^2 + (y+1)^2 = (\sqrt{3})^2$ and $C_2: (x-6)^2 + (y-11)^2 = (3\sqrt{3})^2$ are separated

$$AB^2 = l^2 = d^2 - (r_1 + r_2)^2 = 169 - (4\sqrt{3})^2 = 121$$

AB = 11 Ans.

18. again $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$; $f(1) = 1$

$$= \frac{f(x) \left[\frac{f(x+h)}{f(x)} - 1 \right]}{h}$$

$$= \frac{f(x)}{x} \lim_{h \rightarrow 0} \left[\frac{f\left(1 + \frac{h}{x}\right) - 1}{\frac{h}{x}} \right] = \frac{f'(1) \cdot f(x)}{x} = \frac{2f(x)}{x}$$

(as $f(1) = f^2(1)$ but $f(1) \neq 0 \Rightarrow f(1) = 1$)

$$\frac{f'(x)}{f(x)} = \frac{2}{x}$$

$$\ln(f(x)) = 2 \ln x + C$$

$$x = 1, f(1) = 1 \Rightarrow C = 0$$

$$f(x) = x^2$$

$$\therefore I = \int_b^a f(x) d(\ln x) = \int_{1/e}^e x^2 d(\ln x)$$

$$= \int_{1/e}^e x dx = \left[\frac{x^2}{2} \right]_{1/e}^e = \frac{e^2 - e^{-2}}{2} \text{ Ans.}$$

19. $f_n(x) + f_n(y) = \frac{x^n + y^n}{x^n y^n} \dots \dots \dots (1)$

Put $x = y = 1$ in eq. (1) $\Rightarrow f_n(1) = 1$

put $y = 1$ in eq. (1) $\Rightarrow f_n(x) = \frac{1}{x^n}$

$$\Rightarrow f_{2k}(\operatorname{cosec} \theta) = \frac{1}{(\operatorname{cosec} \theta)^{2k}} = (\sin \theta)^{2k}$$

$$(A) \sum_{k=1}^{\infty} f_{2k}(\operatorname{cosec} \theta) + \sum_{k=1}^{\infty} f_{2k}(\sec \theta) = (\sin^2 \theta + \sin^4 \theta + \dots \infty) + (\cos^2 \theta + \cos^4 \theta + \dots \infty)$$

$$= \frac{\sin^2 \theta}{1 - \sin^2 \theta} + \frac{\cos^2 \theta}{1 - \cos^2 \theta} = \tan^2 \theta + \cot^2 \theta \geq 2$$

\Rightarrow Possible values are 2, 3, 4

$h(x) = \frac{\operatorname{sgn} x}{x^n}$. For $h(x)$ to be even $h(x) = h(-x)$

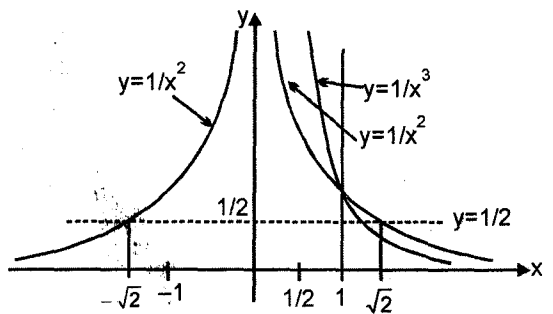
$$(B) h(-x) = h(x) \Rightarrow \operatorname{sgn}(-x) \frac{1}{(-x)^n} = -\operatorname{sgn} x \frac{1}{(-x)^n}$$

$$\frac{1}{(-x)^n} = \operatorname{sgn} x \frac{1}{x^n}$$

\Rightarrow n is odd. Ans. n = 1, 3

now, $f_2(x) = \frac{1}{x^2}$ and $f_3(x) = \frac{1}{x^3}$

$$(C) g(x) = \begin{cases} \frac{1}{x^3} & \frac{1}{2} \leq x \leq 1 \\ \frac{1}{x^2} & 1 < x \leq \sqrt{2} \\ \frac{1}{2} & \sqrt{2} < x \leq 2 \end{cases}$$



$$\int_{1/2}^2 g(x) dx = \int_{1/2}^1 \frac{1}{x^3} dx + \int_1^{\sqrt{2}} \frac{1}{x^2} dx + \int_{\sqrt{2}}^2 \frac{1}{2} dx$$

$$= \frac{7}{2} - \sqrt{2} \quad \text{Ans. } 2, 3 \quad (3.5 - 1.141 \approx 2)$$

(D) $g(x)$ is not differentiable at $x = 1, \sqrt{2}, -\sqrt{2}$

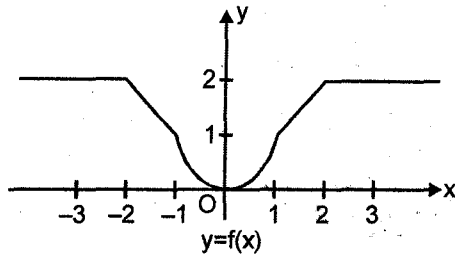
20. (A) $2 \log x - 3 \log y = 1$
 $2 \log x + 3 \log y = 7$

$4 \log x = 8 \Rightarrow \log x = 2;$

$\therefore \log y = 1$

$\therefore \log x + \log y = 3 \Rightarrow \log(xy) = 3$

Ans. \Rightarrow (R)



(B)

$$f(x) = \begin{cases} 2 & -5 \leq x \leq -2 \\ |x| & 1 < |x| < 2 \\ x^2 & -1 \leq x \leq 1 \end{cases}$$

function not derivable at $x = -2, -1, 1, 2 \Rightarrow 4$

Ans. \Rightarrow (S)

(C) $I = \int_0^2 \frac{2x^3 - 6x^2 + 9x - 5}{x^2 - 2x + 5} dx$ (put $x - 1 = t$)

ax $x \rightarrow 0, t = -1$ and $x \rightarrow 2, t \rightarrow 1$.

$$= \int_{-1}^1 \frac{2(1+t)^3 - 6(1+t)^2 + 9(1+t) - 5}{t^2 + 4} dx = \int_{-1}^1 \frac{2t^3 + 3t}{t^2 + 4} dx$$

$\Rightarrow I = 0$ Ans. \Rightarrow (P)

(D) Consider $4^{x^2} + 4^{(x-1)^2}$

AM \geq GM for two positive numbers

4^{x^2} and $4^{(x-1)^2}$

$$\frac{4^{x^2} + 4^{(x-1)^2}}{2} \geq [4^{x^2} \cdot 4^{(x-1)^2}]^{1/2}$$

$$= 2^{x^2} \cdot 2^{(x-1)^2}$$

$$= 2^{x^2 + (x-1)^2}$$

$$4^{x^2} + 4^{(x-1)^2} \geq 2^{x^2 + (x-1)^2 + 1}$$

now $z = x^2 + (x-1)^2 + 1$

$$\frac{dz}{dx} = 2x + 2(x-1) = 0 \text{ for maximum}$$

or minimum $\Rightarrow x = \frac{1}{2}$

hence $z_{\min} = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$

$4^{x^2} + 4^{(x-1)^2}$ has the minimum value $= 2^{3/2}$

hence $f(x) \geq \log_2(2)^{3/2} = \frac{3}{2}$

$y \geq \frac{3}{2} \Rightarrow$ range is $\left[\frac{3}{2}, \infty\right) \Rightarrow$

$a = \frac{3}{2}$ Ans. \Rightarrow (Q)

PHYSICS

21. A

$$2\mu_1 t = (2n-1) \frac{\lambda}{2}, \quad t = \frac{(2n-1) \frac{\lambda}{2}}{2\mu_1}$$

$t = 120 \text{ nm}, 360 \text{ nm}, \dots$

22. B

Maximum tension in string $= T = mg + m\omega^2 A$

$$\mu(3mg) = mg + m \left(\frac{2\pi}{\pi/2} \right)^2 \Rightarrow \mu = \frac{13}{15}$$

23. B

As initially string is vertical, angular amplitude will be θ and

$$g_{\text{eff}} = g \cos \theta$$

$$\therefore v_{\text{max}} = \sqrt{2g_{\text{eff}} l (1 - \cos \theta)} = \sqrt{2gl \cos \theta (1 - \cos \theta)}$$

24. B

Resistent of cylinder $R = \int_0^l \frac{1}{\sigma a} dx = \frac{2\sqrt{l}}{3a\sigma_0}$

$$I = \frac{E}{R}$$

$$\text{Electric field} = \frac{J}{\sigma} = \frac{I}{a\sigma} = \frac{E\sqrt{x}}{Ra\sigma_0 l} = \frac{E(\sqrt{l})}{\frac{2\sqrt{l}}{3a\sigma_0} a\sigma_0 l} = \frac{3E}{2l}$$

25. C

Net flux is zero, so $q + \lambda R \sqrt{2} = 0$

26.

27.

A

Velocity of vertical rod with respect to horizontal rod is

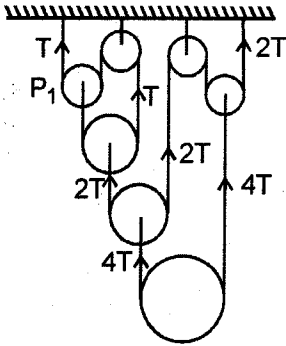
$$\vec{v}_{rel} = v\hat{i} - (-2v\hat{j})$$

$$\therefore \vec{v}_{rel} = \frac{v\hat{i} + 2v\hat{j}}{\sqrt{v^2 + (2v)^2}} = \frac{\hat{i} + 2\hat{j}}{\sqrt{5}}$$

\therefore unit vector along friction force = $-\hat{v}_{rel}$

28. D

29. A, C, D



On $P_1 \Rightarrow 2T = T \Rightarrow T = 0$

30. A, D
31. B, C

$$\vec{F} = q\vec{E} = qE_0\hat{i}$$

$$\vec{v}(t) = v_0\hat{j} + \frac{qE_0}{m}t\hat{i}$$

$$y(t) = v_0t$$

$$x(t) = \frac{1}{2} \frac{qE_0}{m} t^2$$

$$\text{When } x = x_0 = \frac{1}{2} \frac{qE_0}{m} t^2$$

$$t = \frac{2x_0}{v_0}$$

$$\text{so } |\vec{v}| = \sqrt{2} v_0$$

$$|\vec{v}| = v_0\hat{i} + v_0\hat{j}$$

$$a = \sqrt{(a_{||})^2 + (a_{\perp})^2}$$

$$\Rightarrow \left(\frac{qE_0}{m}\right)^2 = \left(\frac{\vec{a} \cdot \vec{v}}{|\vec{v}|}\right)^2 + (a_{\perp})^2$$

$$\Rightarrow \frac{1}{4} \frac{v_0^4}{x_0^2} = \frac{1}{8} \frac{v_0^4}{x_0^2} + (a_{\perp})^2$$

$$\Rightarrow a_{\perp} = \frac{v_0^2}{2\sqrt{2}x_0} = \frac{|\vec{v}|^2}{R}$$

$$R = 4\sqrt{2}x_0$$

32. A, B, C, D

33. D

34. A

35. C

$$T^2 \propto R^3$$

$$T_e^2 = kR_e^3; T_m^2 = kR_m^3; T^2 = kR^3$$

$$R = \frac{R_e + R_m}{2}$$

$$\Rightarrow T^2 = k \left[\frac{T_e^{2/3}}{k^{1/3}} + \frac{T_m^{2/3}}{k^{1/3}} \times \frac{1}{2} \right]^3$$

$$\Rightarrow T = \left[\frac{T_e^{2/3} + T_m^{2/3}}{2} \right]^{3/2}$$

$$E_e = -\frac{GM_s M_e}{2R_e} = -\frac{GM_s M}{2R} = \frac{2R_e E_e}{M_e} \times \frac{M}{2 \left(\frac{R_e + R_m}{2} \right)}$$

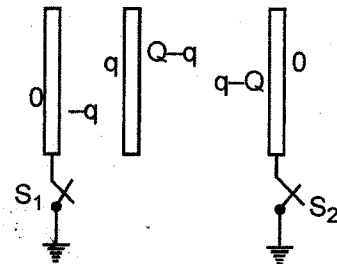
$$= \frac{2M}{M_e} \left(\frac{R_e}{R_e + R_m} \right) E_e$$

36. B

37. C

38. C

$$\frac{(Q-q)3d}{\epsilon_0 A} = \frac{qd}{\epsilon_0 A} \Rightarrow q = \frac{3Q}{4}$$



Sol. (A) \rightarrow P, R; (B) \rightarrow Q, S; (C) \rightarrow Q, R; (D) \rightarrow (P, S)
In case of sine wave, node forms at $x = 0$

40.

Sol. (A) \rightarrow Q, S; (B) \rightarrow P; (C) \rightarrow Q, R, S; (D) \rightarrow (R)

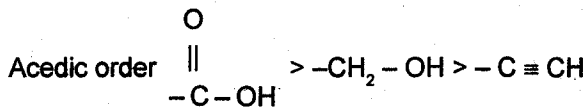
41. C
42. B

Internal energy of (A + B) = Internal energy of the mixture.

$$n_A C_V T_A + n_B C_V T_B = (n_A + n_B) C_V T_{mix}$$

$$T_{mix} = \frac{n_A T_A + n_B T_B}{n_A + n_B}$$

43. A



44. D
45. C

m moles of NaOH = $V_1 x$
m moles of $Ba(OH)_2 = V_2 y$
m. moles of HCl = $100 \times 0.1 = 10$
HCl/ neutralize NaOH completely
m moles of HCl/ remaining after the Reaction with NaOH

$$= 10 - V_1 x = \text{M.eq. of } Ba(OH)_2$$

$$10 - V_1 x = 2V_2 y$$

$$10 = V_1 x + 2V_2 y$$

$$\frac{V_1}{V_2} = \frac{1}{4}, \quad \frac{x}{y} = 4$$

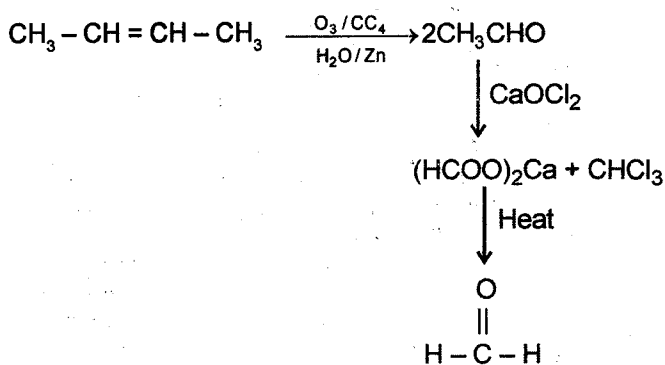
$$\frac{V_1 x}{V_2 y} = 1 \Rightarrow V_1 x = V_2 y$$

$$10 = 3V_2 y \Rightarrow V_2 y = \frac{10}{3}$$

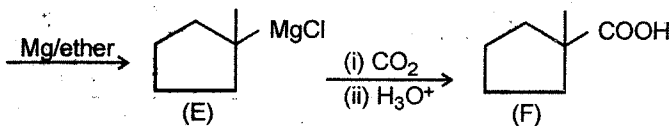
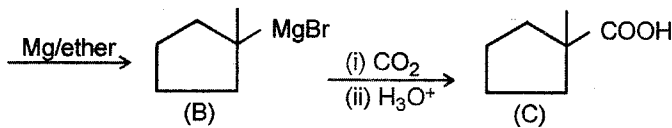
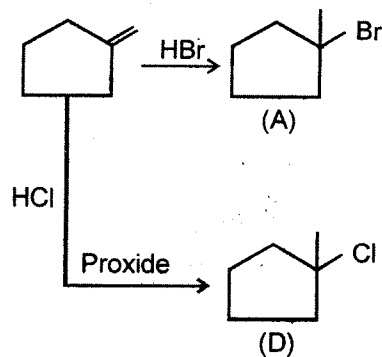
meq. acid neutralize by $Ba(OH)_2 = 2V_2 y$

$$\text{fraction} = \frac{20}{3 \times 10} = \frac{2}{3} = 0.67$$

46. B



47. B



48. B

$$\text{Meq. of } KMnO_4 = \frac{20 \times 0.05 \times 5}{25} \times 1000$$

$$= \text{meq. of } FeSO_4 \cdot 7H_2O$$

$$\text{wt. of } FeSO_4 \cdot 7H_2O = 80 \times 278 \times 10^{-3}$$

$$\% \text{ of } FeSO_4 = \frac{80 \times 278 \times 10^{-3}}{32} \times 100 = 69.5\%$$

49. B,C,D

50. B

Catalyst decreases the activation energy of both by backward and forward reaction

\therefore concentration of reactant and product remain unchanged by using catalyst

51. A,C

52. C,D

53. D

54. A

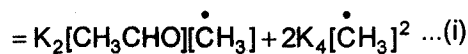
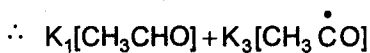
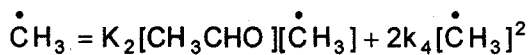
55.

D

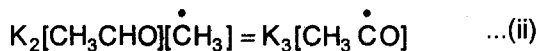
Rate of formation of



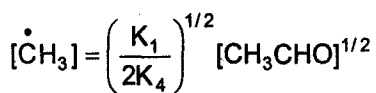
Rate of consumption of



Similarly,



From (i) and (ii)



$$\therefore \frac{d[\text{CH}_4]}{dt} = K_2[\text{CH}_3\text{CHO}][\dot{\text{C}}\text{H}_3]$$

$$= K_2[\text{CH}_3\text{CHO}] \left(\frac{K_1}{2K_4} \right)^{1/2} [\text{CH}_3\text{CHO}]^{1/2} = K[\text{CH}_3\text{CHO}]^{3/2}$$

56. B

57. A

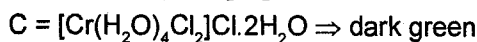
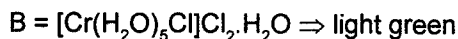
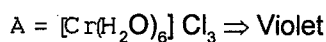
58. B

V I B B G G Y O R

Strong

field ligand \longrightarrow weak field ligand

As the no. of the Cl increases in coordination sphere ligand field decreases

59. (A) \rightarrow Q, (B) \rightarrow R, (C) \rightarrow (P,S), (D) \rightarrow (Q)60. A \rightarrow P,Q ; B \rightarrow P,R,S ; C \rightarrow P,Q ; D \rightarrow P,Q