TARGET IIT-JEE

HINT & SOLUTIONS

ANSWER KEY

PAPER-II CLASS XII (DATE 24-05-09)

MATHEMATICS

1. A **2.** D **3.** C **4.** A **5.** A,C,D **6.** A,B,C **7.** B,C
8. B,C,D **9.** A,D
10. (A) → (Q,T) ; (B) → (P,R,T); (C) → (P,R,S,T) ; (D) → (T)
11. (A) → (T) ; (B) → (P,S); (C) → (Q) ; (D) → (R)
12. 2 **13.** 8 **14.** 6 **15.** 4 **16.** 1 **17.** 8 **18.** 0
19. 4

PHYSICS

20. **21.** C **22.** B **23.** A **24.** D **25.** B,C **26.** A,D
27. A,B,C,D **28.** A,D
29. (A) → (P), (R), (B) → (R), (C) → (Q), (R), (S), (D) → (Q) (R)
30. (A) → (Q), (B) → (P), (R), C → (P),(R), D → Q
31. 0.66 h **32.** 9 kg **33.** (150cm, –30cm) **34.** 12 cm
35. ω = 2 rad/s
36. 6 cm
37. 50 km
38. 2

CHEMISTRY

39. C **40.** B **41.** A **42.** B **43.** A,B,C,D **44.** C,D **45.** A,D
46. A,B **47.** D
48. (A → p, q), (B → p,q), (C → p,s), (D → q,r)
49. (A → S), (B → Q), (C → R), (D → T)
50. 13 gm **51.** 10 **52.** 37 % **53.** 3 **54.** 44 **55.** 3 **56.** 79
57. 58

Sol.1 Since, $3^{\sec^2 x-1}\sqrt{9y^2-6y+2}\le 1$

$$\Rightarrow 3^{\sec^2}\frac{1}{3}\sqrt{9y^2-6y+2}\le 1$$

$$\Rightarrow 3^{\sec^2 x}\sqrt{y^2-\frac{2}{3}y+\frac{2}{9}}\le 1$$

$$\Rightarrow 3^{\sec^2 x}\sqrt{\left(y-\frac{2}{3}\right)^2+\frac{1}{9}}\le 1 \quad ....(1)$$

Now, $3^{\sec^2 x}\ge 3$ and $\sqrt{\left(y-\frac{1}{3}\right)^2+\frac{1}{9}}\ge\frac{1}{3}$

$$\therefore\ 3^{\sec^2 x}\sqrt{\left(y-\frac{1}{3}\right)^2+\frac{1}{9}}\ge 1 \quad ....(2)$$

Hence, the inequations (1) & (2) are satisfied if and only if

$$3^{\sec^2 x}\sqrt{\left(y-\frac{1}{3}\right)^2+\frac{1}{9}}=1$$

i.e. $\sec^2 x = 1$ and $y=\frac{1}{3}$

$\therefore\ x = 0, \pi, 2\pi, 3\pi$ and $y=\frac{1}{3}$

Sol.2 $\lim\limits_{\theta\to-1}\frac{\theta^2+\theta-2}{\theta+3}=-1=\lim\limits_{\theta\to-1}\frac{\theta^2+2\theta-1}{\theta+3}$

using Sequeeze Play theorem

$\lim\limits_{\theta\to-1}\frac{f(\theta)}{\theta^2}=-1;\qquad \lim\limits_{\theta\to-1}f(\theta)=-1]$

Sol.3 $\lim\limits_{x\to\infty}\ln\left(\frac{x^2+5x}{(cx+1)^2}\right)=-2 \quad\Rightarrow$

$$\lim_{x\to\infty}\ln\left(\frac{1+\frac{5}{x}}{\left(c+\frac{1}{x}\right)^2}\right)=-2$$

$$\ln\frac{1}{c^2}=-2 \quad\Rightarrow\quad -2\ln c=-2;$$

$\therefore\ c = e$

Sol.4 $\lim\limits_{x\to1}\frac{\sin^2(x^3+x^2+x-3)}{(x^3+x^2+x-3)^2}$

$$\frac{(x^3+x^2+x-3)^2}{1-\cos(x^2-4x+3)} = \quad (1)$$

$$\lim_{x\to1}\frac{(x^2-4x+3)^2}{1-\cos(x^2-4x+3)}\cdot\frac{(x^3+x^2+x-3)^2}{(x^2-4x+3)^2}$$

$= (1)\ (2)\ \lim\limits_{x\to1}\left(\frac{x^3+x^2+x-3}{x^2-4x+3}\right)^2 = 2l^2$ where

$= \lim\limits_{x\to1}\frac{3x^2+2x+1}{2x-4}$ (using L'Hospital's rule)

$$=\frac{6}{-2}=-3$$

$\therefore\quad L = 2(-3)^2 = 18$ Ans.

Sol.5 (A) $3\sin^2 x + 3\sin x \cdot \cos x + 7\cos^2 x$

$$=\frac{3(1-\cos 2x)}{2}+\frac{3\sin 2x}{2}+\frac{7(1+\cos 2x)}{2}$$

$$y=\frac{3\sin 2x+4\cos 2x}{2}+5$$

$y_{max} = 5/2 + 5 = 7.5$

$y_{min} = -5/2+5 = 2.5$

(B) $\frac{1}{\sin\pi/18}-\frac{\sqrt{3}}{\cos\pi/18}$

$$=\frac{2\left[\frac{1}{2}\cos\frac{\pi}{18}-\frac{\sqrt{3}}{2}\sin\frac{\pi}{18}\right]}{\frac{\sin(\pi/9)}{2}}$$

$$=\frac{4\left[\sin\frac{\pi}{6}\cos\frac{\pi}{18}-\cos\frac{\pi}{6}\sin\frac{\pi}{18}\right]}{\sin\frac{\pi}{9}}=4$$

(C) $\sqrt{4(\sin^2 x)^2 + \sin^2 2x} + 4\left\{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}^2$

$\sqrt{\frac{4(1-\cos 2x)^2}{4} + \sin^2 2x} + 2\left[1+\cos\left(\frac{\pi}{2}-x\right)\right]$

$\sqrt{1+1-2\cos 2x} + 2(1+\sin x)$

$\sqrt{2(2\sin^2 x)} + 2 + 2\sin x$

$2|\sin x| + 2 + 2\sin x$

$= -2\sin x + 2 + 2\sin x = 2$ **Ans**

(D) $4\cos^2\theta - 2\sqrt{2}\cos\theta - 1 = 0$

$\cos\theta = \frac{2\sqrt{2}\pm\sqrt{8+16}}{8} = \frac{\sqrt{2}\pm\sqrt{6}}{4}$

$\cos\theta = \frac{\sqrt{6}+\sqrt{2}}{4} \Rightarrow \theta = \frac{\pi}{12}; 2\pi - \frac{\pi}{12} = \frac{23\pi}{12}$

$\cos\theta = -\frac{\sqrt{6}-\sqrt{2}}{4} = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$

$\cos\theta = \cos(\pi - 5\pi/12)$; $\cos(\pi + 5\pi/12)$

$\theta = 7\pi/12$; $17\pi/12$

Sol.6 Put x = 3, $1 = f(3) . f(f(3)) = 2f(2)$

$\Rightarrow f(2) = \frac{1}{2}$

Put x = 2, $1 = f(2)\, f(f(2)) = \frac{1}{2} f\left(\frac{1}{2}\right)$

$\Rightarrow f\left(\frac{1}{2}\right) = 2$

Since f is continuous and $\frac{1}{2} < 1 < 2$, by I.V.T.

there exists $c \in \left(\frac{1}{2}, 2\right)$ such that f(c) = 1

$\Rightarrow f(c) . f(f(c)) = 1 \Rightarrow 1 . f(1) = 1 \Rightarrow f(1) = 1$

Sol.7 Using Sandwitch theorem :

$\lim_{x\to 0} p(x) = 0$ since $-|x| \le p(x) \le |x|$

Since p(x) is continuous, $p(0) = \lim_{x\to 0} p(x)$

$\therefore \quad a_0 = 0$

Now $\left|\frac{p(x)-p(0)}{x-0}\right| \le 1 \Rightarrow \lim_{x\to 0}\left|\frac{p(x)-p(0)}{x-0}\right| \le 1$

$\Rightarrow |p'(0)| \le 1 \Rightarrow |a_1| \le 1$

Sol.8 $\frac{f(x+1)-f(x)}{x+1-1} = 1 \Rightarrow$ the graph of f(x) has a constant slope of 1.

$\therefore \quad f(x) = x + b$ Since f(1) = 0, b = 0 and f(x) = x

Sol.9 $f(x) = \begin{cases} ax^2 + bx & -1 < x < 1 \\ \frac{a-b-1}{2} & x = -1 \\ \frac{a+b+1}{2} & x = 1 \\ \frac{1}{x} & x > 1 \text{ or } x < -1 \end{cases}$

for continuity at x = 1 a + b = 1 ....(1)

for continuity at x = – 1 a – b = – 1

$\Rightarrow$ a – b = – 1 ....(2)

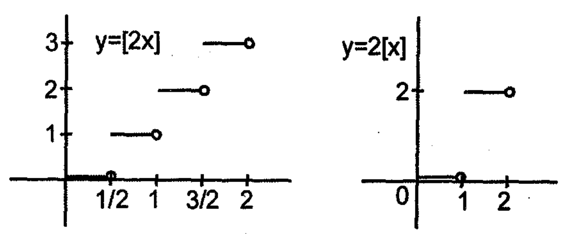
hence a = 0 and b = 1 Ans.

Sol.10 (A) Q, T (B) P, R, T

(C) P, R, S, T (D) T

(A) If x < 0, LHS $\in$ I but RHS is positive and less than $\frac{1}{4}+\frac{1}{5} < 1$. If x > 5 then LHS < $\frac{1}{4}$ while RHS is a positive integer. Hence we should check solution between 0 & 5. On checking we get x = 2 only.

(B)



On comparing the two graphs we find that $0 \le [2x] - 2[x] \le 1$

(C) Put y as x, $f(x + 2f(x)) = 2f(x) + x$

Let the degree of f(x) be $n > 1$.

then comparing the degree of the two sides we get $n^2 = n \Rightarrow 0$ or 1. Not possible.

Let $n = 1$ then $1^2 = 1$ $\therefore$ f(x) is linear polynomial, $f(x) = ax + b$

$$f((2a + 1)x + b) = 2(ax + b) + x$$

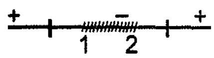
$$\Rightarrow \quad a(2a + 1)x + 2ab + b = (2a + 1)x + 2b$$

$$a(2a + 1) = 2a + 1 \Rightarrow a = -\frac{1}{2}, 1$$

Comparing coefficients of x and constant term,

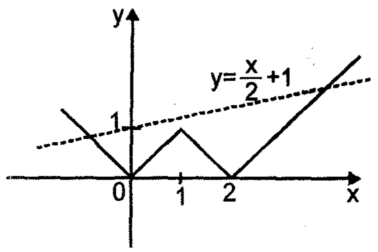
In both cases $b = 0$ $\quad \therefore f(0) = 0$

(D) $f(1) < 0$ and $f(2) < 0$

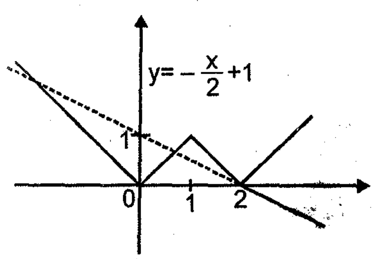


Sol.11. (A) T (B) P, S

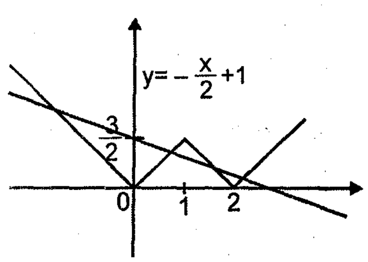
(C) Q (D) R



$p = \frac{1}{2}, c = 1.$ Two solutions

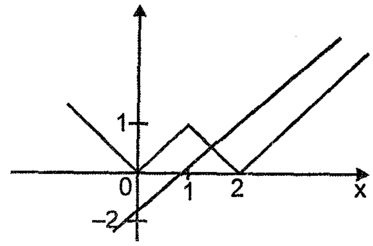


$p = \frac{1}{2}, c = 1.$ Three solutions

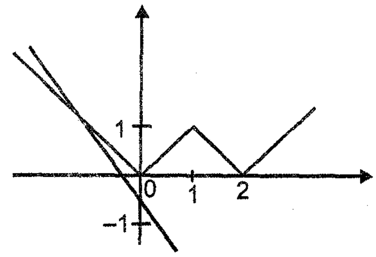


$\frac{-3}{4} < p < \frac{-1}{2}, c = \frac{3}{2}.$

More than three solutions



$p = 1, -2 < c < a$ One solution



One solution

Sol.12

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1 - \frac{x^6}{2!} \ldots\ldots -1 + x^6 + \ldots\ldots}{x^2\left(1 + x^2 + \frac{x^4}{2!} \ldots\ldots -1 - x^2\right)} = 1$$

$$\lim_{x\to 1} g(x) = \lim_{x\to 1} \frac{e(e^{x^2-1})(x^2 - 1)}{e(x^2 - 1)(x - 1)} = 2$$

Using theorem on composition of limits,

$$\lim_{x\to 0} g(f(x)) = 2$$

Sol.13

The inequality is $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$ ....(1)

The L.H.S. is valid if

(i) $x^2 - 10x + 22 > 0 \Rightarrow x < 5 - \sqrt{3}$ or $x > 5 + \sqrt{3}$

(ii) $\frac{x}{2} > 0 \Rightarrow x > 0$

Now the inequatity (1) will be solved for two cases of $\log_2\left(\frac{x}{2}\right)$.

Case 1 :

$$0 < \log_2\left(\frac{x}{2}\right) < 1 \Rightarrow 1 < \frac{x}{2} < 2 \Rightarrow 2 < x < 4$$

$$\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$$

$\Rightarrow x^2 - 10x + 22 < 1 \Rightarrow x^2 - 10x +$
$21 < 0 \Rightarrow 3 < x < 7$. The common solution is $3 < x < 4$.

Case 2 : $\log_2\left(\frac{x}{2}\right) > 1 \Rightarrow \frac{x}{2} > 2 \Rightarrow x > 4.$

The inequatity is then $\log_{\log_2\left(\frac{x}{2}\right)}(x^2 - 10x + 22) > 0$

$\Rightarrow x^2 - 10x + 22 > 1 \Rightarrow x^2 - 10x + 21 > 0$
$\Rightarrow x < 3$ or $x > 7$.
The common solution is $x > 7$.
$\therefore$ The value of x from two cases are $x \in (3, 4) \cup (7, \infty)$
Now taking intersection with intital values of x, we get $x \in (7, \infty)$

Sol.14 $\lim_{x\to 0} \frac{x(1 + a\cos x) - b\sin x}{\{f(x)\}^3} = 1$

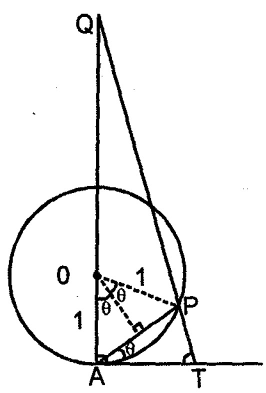
$$\Rightarrow \lim_{x\to 0} \frac{x + ax\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots - b\left\{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\right\}\right]}{\{f(x)\}^3} = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\frac{1 + a - b}{x^2} + \left(-\frac{a}{2!} + \frac{b}{3!}\right) + x^2\left(\frac{a}{4!} - \frac{b}{5!}\right) + \ldots\ldots}{\left\{\frac{f(x)}{x}\right\}^3} = 1$$

$\Rightarrow 1 + a - b = 0$ and $-\frac{a}{2!} + \frac{b}{3!} = 1$

$\therefore b - 3a = 6$

Sol.15



$AP = AT = 2\sin\theta$

$$\angle PTA = \frac{180° - \theta}{2}$$

$\therefore$ In ΔAQT, $\frac{AQ}{AT}$

$= \tan\left(\frac{180° - \theta}{2}\right)$

$\Rightarrow AQ = AT\cot\frac{\theta}{2} = \frac{2\sin\theta}{\cot\frac{\theta}{2}}$

As $P \to A$, $\theta \to 0^+$

$$\therefore \lim_{\theta\to 0^+} AQ = \lim_{\theta\to 0^+} \frac{\frac{2\sin\theta}{\theta} \times 2}{\frac{\cot\frac{\theta}{2}}{\frac{\theta}{2}}} = 4$$

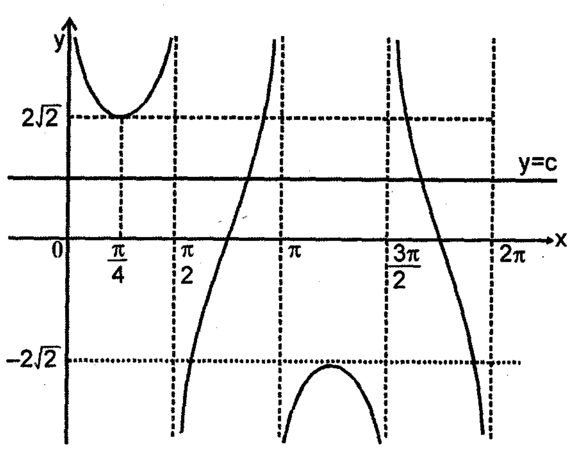
Sol.16 $S_n = \sum_{r=1}^{n} \sqrt{\frac{r^4 + 2r^3 + 3r^2 + 2r + 1}{r^2(r+1)^2}}$

$$= \sum_{r=1}^{n} \frac{r^2 + r + 1}{r^2 + r} = \sum_{r=1}^{n}\left(1 + \frac{1}{r^2 + r}\right)$$

$$= n + \sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) = n + 1 - \frac{1}{n+1}$$

$$\lim_{n\to\infty} \frac{S_n}{n} = \lim_{n\to\infty} \frac{n + 1 - \frac{1}{n+1}}{n} = 1$$

Sol.17 Using graphical addition, the graph of y = secθ + cosecθ is shown



For two real roots $-2\sqrt{2} < c < 2\sqrt{2}$

$\Rightarrow \quad c^2 < 8$

Sol.18 $\text{LHL} = \lim_{x \to a^-} \sqrt{a^2 - x^2} \cot\left(\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}\right)$

$= \lim_{h \to 0} \sqrt{2ah - h^2} \cot\left(\frac{\pi}{2}\sqrt{\frac{h}{2a-h}}\right)$

Put $x = a - h$

$$= \lim_{h \to 0} \frac{\sqrt{2ah - h^2}}{\frac{\tan\left(\frac{\pi}{2}\sqrt{\frac{h}{2a-h}}\right)}{\left(\frac{\pi}{2}\sqrt{\frac{h}{2a-h}}\right)}\left(\frac{\pi}{2}\sqrt{\frac{h}{2a-h}}\right)} = \frac{4a}{\pi}$$

$$\text{RHL} = \lim_{x \to a^+}\left(\frac{\sqrt{x}-\sqrt{a}}{\sqrt{x^2-a^2}} + \frac{\sqrt{x-a}}{\sqrt{x^2-a^2}}\right)$$

$$= \lim_{x \to a^+}\left(\frac{x-a}{\sqrt{x-a}\sqrt{x+a}\left(\sqrt{x}+\sqrt{a}\right)} + \frac{1}{\sqrt{x+a}}\right) = \frac{1}{\sqrt{2a}}$$

Now $\frac{4a}{\pi} = \frac{1}{\sqrt{2a}} \Rightarrow a^3 = \frac{\pi^2}{16} \qquad \therefore [a^3] = 0$

Sol.19 Since, $p(x)$ is a factor of $f_1 = x^4 + 6x^2 + 25$ and of $f_2 = 3x^4 + 4x^2 + 28x + 5$

Hence, it will also be a factor of $E = 3(f_1) - (f_2)$ (say)

$\therefore \quad E = 3(x^4 + 6x^2 + 25) - (3x^4 + 4x^2 + 28x + 5)$

$\Rightarrow \quad E = 14x^2 - 28x + 70$

$\Rightarrow \quad E = 14(x^2 - 2x + 5)$

$\therefore \quad p(x) = x^2 - 2x + 5$

i.e. $\quad p(1) = 4$

20. $\vec{R}$ is the resultant of $\vec{A}$ & $\vec{B}$, then

$R^2 = A^2 + B^2 + 2AB\cos\theta$ ...(1)

$(A + B) = 2(A - B)$

$B = \frac{A}{3}$ ...(2)

$R = \frac{A+B}{2} \Rightarrow R = \frac{4A}{6}$ ...(3)

From (1), (2) & (3)

$$\frac{16A^2}{36} = A^2 + \frac{A^2}{9} + 2A \times \frac{A}{3}\cos\theta$$

$\cos\theta = -1$

21. [C]

22. [B]

23. [A]

The resultant intensity at a point on the screen due to superposition of two waves of intensity I_1 and I_2 is given by :

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where δ is the phase difference between the waves.

$$\delta = \frac{2\pi}{\lambda} \times (\text{path difference}) = \frac{2\pi}{\lambda}(S_2P - S_1P)$$

$$= \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) = kx$$

where $(S_2P - S_1P) = \frac{xd}{D}$ and $k = \frac{2\pi d}{\lambda D}$ a constant

$\therefore$ $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos kx$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\left(2\cos^2\frac{kx}{2} - 1\right)$$

$$= (\sqrt{I_1} - \sqrt{I_2})^2 + 4\sqrt{I_1 I_2}\cos^2\frac{kx}{2}$$

$$= A + B\cos^2\frac{kx}{2}$$

where $A = (\sqrt{I_1} - \sqrt{I_2})^2$ and $B = 4\sqrt{I_1 I_2}$ are another constants.

24. [D]

Sol. With knowing the actual value of μ we can not find f_1 and f_2.

25. [B,C]

26. [A,D]

Since wave numbers of the two waves are different, their wave velocities will also be different. Their direction will also be different.

27. [A,B,C,D]

28. [A,D]

Velocity of light in denser medium is less than the velocity of light in air.

29. (A) → (P), (R), (B) → (R), (C) → (Q), (R), (S), (D) → (Q) (R)

30. A → Q, B → P,R, C → P,R, D → Q

31. time t_1 to cross river $= \frac{1}{3\cos\theta}$

$x = v_x \cdot t_1$

$$x = (5 - 3\sin\theta) \times \frac{1}{3\cos\theta}$$

time t_2 to reach P by walking $= \frac{x}{5} = \frac{5 - 3\sin\theta}{15\cos\theta}$

For time to be minimum $\frac{d(t_1 + t_2)}{d\theta} = 0$

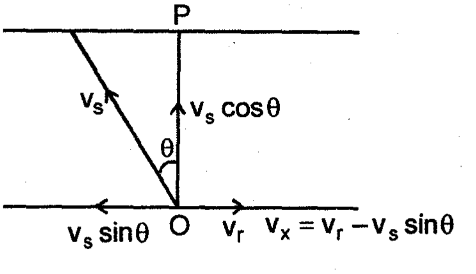
$$= \frac{d}{d\theta}\left(\frac{1}{3\cos\theta} + \frac{5 - 3\sin\theta}{15\cos\theta}\right)$$

or $\frac{2}{3}\sec\theta\tan\theta - \frac{1}{5}\sec^2\theta = 0$

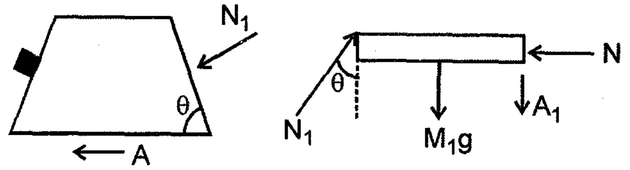
or $\sin\theta = \frac{3}{5}$

$$t = \frac{1}{3\cos\theta} + \frac{5 - 3\sin\theta}{15\cos\theta}$$

$$= \frac{1}{3\left(\frac{4}{5}\right)} + \frac{5 - 3 \times \frac{3}{5}}{15 \times \frac{4}{5}} = \frac{5}{12} + \frac{16}{12 \times 5} = \frac{41}{60} = 0.66\,h$$



32.



Let M_1 be the mass of the rod.

For rod → $M_1g - N_1 \cos\theta = M_1A_1$ ...(i)

For wedge → $N_1 \sin\theta = (M + M) A$ ....(ii)

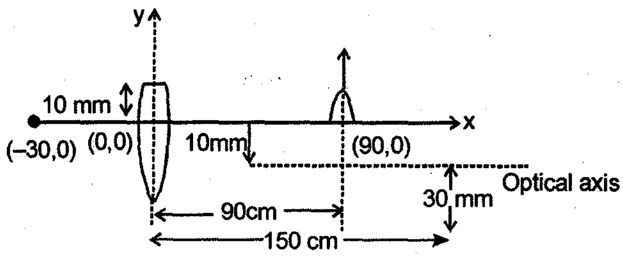
$A = g \tan\theta$ ....(iii)

relation between A_1 and A

$A_1 = A \tan\theta$

So by solving these equation $M_1 = 3M = 9$ kg

33.



For first lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{30}=\frac{1}{60} \Rightarrow v_1 = 60 \text{ cm}$$

For second lens

u = 30 cm, f = +20 cm

$$\frac{1}{f}=\frac{1}{v_2}-\frac{1}{u} \Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{30}$$

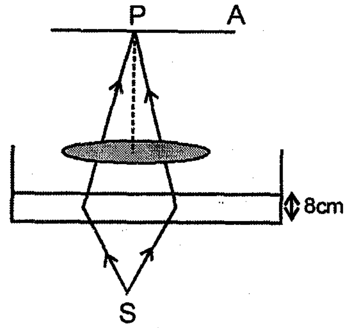
$$\Rightarrow \frac{1}{v}=\frac{3-2}{60}=\frac{1}{60}$$

∴ $v_2 = 60$ cm

⇒ m = – 2

So co-ordinates are (150 cm, –30 mm)

34. Suppose that the object distance (between the object and the lens) be u and the image distance v. Then,



$|u| + |v| = 50$

$$\frac{1}{|v|}-\frac{1}{|u|}=\frac{1}{f} \quad ...(i)$$

When water is poured into the vessel,

$$\frac{1}{(|v|+6)}-\frac{1}{-(|u|-2)}=\frac{1}{f} \quad ...(ii)$$

Solving the equations, we get

$|u| = 20$ cm

$|v| = 30$ cm and f = 12 cm

35. ω = 2 rad/s

36. 6 cm

37. 50 km

38. **2**

When each block is displaced by a very small distance x, the spring will be compressed by 2x (?)

39. [C]

Moles of $NaHC_2O_4$ = moles of $H_2C_2O_4$ = moles of $Na_2C_2O_4$

$$\frac{x}{112}=\frac{y}{90}=\frac{20-(x+y)}{134}$$

Meq. of NaOH = Meq. of $H_2C_2O_4$ + Meq of $NaHC_2O_4$

$$40 \times 0.2 = \frac{y}{90} \times 2 \times 1000 + \frac{x}{112} \times 1 \times 1000$$

$$8 = \frac{x}{112} \times 2 \times 1000 + \frac{x}{112} \times 1000 = \frac{3x \times 1000}{112}$$

$$\Rightarrow \frac{x \times 1000}{112}=\frac{8}{3}$$

Meq. of $KMnO_4$ = Meq. of $NaHC_2O_4$ + meq of $H_2C_2O_4$ + Meq. of $Na_2C_2O_4$

$$v \times 0.4 \times 5 = \frac{x}{112} \times \frac{40}{20} \times 2 \times 1000$$

$$+ \frac{y}{90} \times \frac{40}{20} \times 2 \times 1000 + \frac{20-(x+y)}{134} \times \frac{40}{20} \times 1000 \times 2$$

$$= \frac{8}{3} \times 4 + 4 \times \frac{8}{3} + 4 \times \frac{8}{3} = \frac{32}{3}+\frac{32}{3}+\frac{32}{3}$$

$v \times 2 = 32 \times 3/3$

v = 16 ml

40. [B]

$CH_4 : C_2H_6 : H_2 = 1 : 2 : 3$

$CH_4 + C_2H_6 + H_2 + O_2 \rightarrow CO_2 + H_2O$

x 2x 3x

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

$C_2H_6 + 7/2\ O_2 \rightarrow 2CO_2 + 3H_2O$

$H_2 + 1/2\ O_2 \rightarrow H_2O$

Volume of CO_2 produced = 5x

10x + 2.5 = 2x + 7/2 × 2x + 3/2 x

= 10.5 x ⇒ x = 5

If H_2 is removed then volume of O_2 used

$$= 2x + \frac{7}{2} \times 2x = 9x = 45 \text{ ml}$$

41. [A]

$$A \xrightarrow{k_1 = 10^{-5}\ sec^{-1}} B \xrightarrow{k_2} C$$

when the concentration of B become constant then

$k_1A = k_2B$

If the conc. of A & B are equal then

$k_1 = k_2 = 10^{-5}\ sec^{-1}$

$$t_{1/2} \text{ for B to C} = \frac{0.693}{k_2} = 6.93 \times 10^4$$

42. [B]

$$k_{300} = \frac{2.303}{20}\log\left(\frac{100}{87.5}\right)$$

$$k_{315} = \frac{2.303}{2.5}\log\left(\frac{100}{87.5}\right)$$

$$\frac{K_{315}}{K_{300}} = \frac{20}{2.5} = 8$$

$$\log\frac{K_{315}}{K_{300}} = \frac{E_a}{2.303R}\left[\frac{1}{300}-\frac{1}{315}\right]$$

$$\log 8 = \frac{Ea}{2.303R}\left[\frac{1}{300}-\frac{1}{315}\right]$$

$$3 \log 2 = \frac{Ea}{2.303 \times 2 \times 10^{-3}}\left[\frac{15}{300 \times 315}\right]$$

$$3 \log 2 = \frac{Ea}{2.303 \times 2 \times 10^{-3}}\left[\frac{15}{300 \times 315}\right]$$

$$3 \log 2 = \frac{Ea}{2.303 \times 2 \times 6300 \times 10^{-3}}$$

Ea = 3 × 0.3010 × 2.303 × 2 × 6.3

= 26.4 kcal.

43. [A,B,C,D]

$$FeS_2 + MnO_4^- \xrightarrow{H^+} Fe^{3+} + 2SO_2 + Mn^{2+}$$

Hence, n factor of FeS_2 = 11 and that of $KMnO_4$ = 5

So, equivalent weight of $KMnO_4 = \frac{M}{5}$ and that of $FeS_2 = \frac{M}{11}$

Also, $KMnO_4 + SO_2 \xrightarrow{H^+} Mn^{2+} + SO_4^{2-}$

gm eq. of $KMnO_4$ = gm eq. of SO_2

0.1×5 = moles of $SO_2 \times 2$

moles of $SO_2 = 0.5$

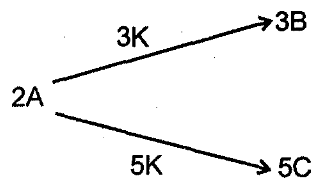
volume of SO_2 at STP = $0.25 \times 22.4 = 5.6$ lit

Also, moles of FeS_2 used = $\frac{1}{2}$ mole of SO_2 produced

$= \frac{1}{2} \times 0.25 = 0.125$ moles

Hence (A, B, C, D)

44. **[C,D]**



$$-\frac{1}{2}\frac{dA}{dt} = \frac{1}{3}\frac{dB}{dt} + \frac{1}{5}\frac{dC}{dt} = 3k[A] + 5k[A] = 8kA$$

$$-\frac{dA}{dt} = 8kA \times 2 = 16\ kA$$

$$A = A_0\, e^{-16\,kt}$$

$$\frac{1}{3}\frac{dB}{dt} = 3kA, \quad \frac{dB}{dt} = 9kA = 9kA_0\, e^{-16kt}$$

$$B = \frac{9}{16}[1 - e^{-16kt}]$$

Similarly

$$C = \frac{25}{16}[1 - e^{-16kt}]$$

Hence $\frac{B}{C} = \frac{9}{25}$

% of B = $\frac{9}{25+9} \times 100 = 26.47\%$

45. **[A,D]**

$\frac{k_1}{k_2} = \frac{1}{2} \Rightarrow \frac{B}{C} = \frac{1}{2} \Rightarrow$ % of B = $\frac{1}{3} \times 100 = 33.33$ %

% of = $\frac{2}{3} \times 100 = 66.67$ %

after a long time

$[B] = 2 \times \frac{1}{3} = \frac{2}{3}$, $[C] = 2 \times \frac{2}{3} = \frac{4}{3}$

Angle of rotation = $\frac{2}{3} \times (-72) + \frac{4}{3} \times 42$

$= \frac{2}{3} \times (-72) + 56$

$= -48 + 56 = 8$

Hence dextro rotatory

(d) after 75% completion

$[A] = 1/2$, $[B] = 0.5$, $[C] = 1.0$

Angle of rotation = $\frac{1}{2} \times 60 + 0.5 \times (-72) + 1 \times 42$

$= 30 - 36 + 42 = 36°$

46. **[A,B]**

(a) $k_{eff} = \frac{2k_2}{k_3}\left(\frac{k_1}{k_5}\right)^{1/5}$

$E_{eff} = E_2 - E_3 + \frac{E_1}{5} - \frac{E_5}{5}$

$= 100 - 25 + 10 - 4$

$= 110 - 29 = 81$ kJ

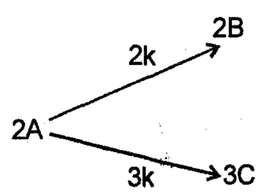
(b) $k_{eff} = \frac{2k_2}{k_3}\cdot\left(\frac{k_1}{k_5}\right)^{1/5}$

$$= \frac{2.A_2 e^{-E_2/RT}}{A_3 e^{-E_3/RT}}\left(\frac{A_1 e^{-E_1/RT}}{A_5 e^{-E_5/RT}}\right)^{1/5}$$

$$= \frac{2A_2}{A_3}\cdot\left(\frac{A_1}{A_5}\right)^{1/5} \cdot e^{\frac{-[E_2 + E_1/5 - E_3 - E_5/5]}{RT}}$$

$$A_{eff} = \frac{2A_2}{A_3}\cdot\left(\frac{A_1}{A_5}\right)^{1/5}$$

C.



$$-\frac{1}{2}\frac{dA}{dt} = \frac{1}{2}\frac{dB}{dt} + \frac{1}{3}\frac{dC}{dt}$$

$= 2kA + 3kA$

$$-\frac{dA}{dt} = 10kA$$

$k_{eff} = 10k$

$A = A_0 e^{-10kt}$

degree of dissociation = $1 - e^{-10kt}$

47. **[D]**

A multitude of He^+ will produce 10 possible emission spectral lines. A single He^+ can produce maximum 4 lines (not more)

$$5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$$

$$E_n = -13.6\frac{Z^2}{n^2} = -13.6 \times \frac{2^2}{5^2}$$

$$= -13.6 \times \frac{4}{25} = -2.176 \text{ eV.}$$

$\therefore$ I E = + 2.176 eV > 2 eV

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R.4\left(\frac{1}{4^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{\lambda} = 4R\left(\frac{9}{16 \times 25}\right)$$

$$\lambda = \frac{16 \times 25}{4 \times 9 \times R} = \frac{100}{9R} > \frac{10}{R}$$

$$r_n = 0.53\frac{n^2}{Z} = 0.53 \times \frac{5^2}{Z} = 6.625 \text{ Å} > 6 \text{ Å}$$

48. **(A → p, q), (B → p,q), (C → p,s), (D → q,r)**

$0.53 \times 10^6 = 2.16 \times 10^6 \text{ ms}^{-1} \times \frac{1}{n}$;

$n = \frac{2.16 \times 10^6}{0.53 \times 10^6}$; $n \approx 4$

$$\lambda = \frac{h}{mv} = \frac{6.634 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.53 \times 10^6} = 1.374 \times 10^{-9} \text{ m}$$

$\lambda = \frac{h}{\sqrt{2mKE}}$ $\therefore$

$$kE = \frac{1}{2}mv^2 = \frac{1}{2}(9.1 \times 10^{-31}) \times (0.53 \times 10^6)^2$$

$$= 1.278 \times 10^{-19} \text{ J}$$

49. **(A → S), (B → Q), (C → R), (D → T)**

50. meq. of Fe^{+2} = meq of $KMnO_4$

$$= 20 \times \frac{1}{5} \times 5 = 20$$

Meq. of Fe^{+2} = meq of Fe^{+3}

= meq. of N_2H_4

= 20

Meq. of N_2H_4 for 200 ml sol.

$$= \frac{20 \times 200}{10} = 400 \text{ meq.}$$

moles of N_2H_4 = $\frac{400 \times 10^{-3}}{4} = 0.1$

moles of $N_2H_6SO_4$ = 0.1

wt. of $N_2H_6SO_4$ = 130 × 0.1 = 13 gm

51. $K = Ae^{-Ea/RT}$

$K_c = Ae^{-Ec/RT}$

$$\frac{k_c}{k} = e^{(Ea-Ec)/RT}$$

$$\log\frac{k_c}{k} = \frac{Ea - Ec}{2.303RT} = \frac{83.68 - 25.1}{2.303 \times 8.31 \times 10^{-3} \times 310}$$

$$\log\frac{x}{y} = 10$$

52. $CS_2 + H_2S + \frac{9}{2}O_2 \rightarrow CO_2 + 3SO_2 + H_2O$

x mole y mole

PV = nRT for mix of CO_2, SO_2 & H_2O

$$\frac{748.8}{760} \times 60 = n \times 0.0821 \times 600$$

$= 0.821$ n

n = 1.21

$\overset{+4}{SO_2} \rightarrow \overset{+6}{SO_3}$

meq. of I_2 = meq. of SO_2

700 × 2 = meq. of SO_2

1400 = meq. of SO_2, moles of SO_2 = $\frac{1.4}{2} = 0.7$

moles of S = 0.7

2x + y = 0.7

moles of H_2O = y

moles mix = moles of CO_2 + moles of SO_2 + moles of H_2O

$1.21 = x + 2x + y + y$

$= x + y + 0.7$

$x + y = 1.21 - 0.7 = 0.51$

$2x + y = 0.7$

$x = 0.19$

$y = 0.32$

$$\% \text{ of } CS_2 = \frac{x}{x+y} \times 100 = \frac{0.19}{0.51} \times 100 = 37\%$$

53. $\frac{r_A}{r_B} = \frac{2}{1}$

$$\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}} = \frac{2}{1} \Rightarrow \frac{M_B}{M_A} = \frac{4}{1}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{V_{rms}(A)}{V_{rms}(B)} = \sqrt{\frac{T_A \cdot M_B}{M_A \, T_B}} = \sqrt{\frac{2}{1} \times \frac{4}{1}} = 2\sqrt{2} = 2.83 \simeq \mathbf{3}$$

54. $$k_{27^\circ} = \frac{2.303}{20} \log\left(\frac{100}{75}\right)$$

$$k_{127^\circ} = \frac{2.303}{20} \log\left(\frac{100}{A}\right)$$

$$\frac{k_{127}}{k_{27}} = \frac{\log 100/A}{\log 100/75}$$

$$\log \frac{k_{127}}{k_{27}} = \frac{7}{2.303R}\left[\frac{1}{300} - \frac{1}{400}\right]$$

$$= \frac{7}{2.303 \times 8.31 \times 10^{-3} \times 1200} = \frac{7 \times 10}{2.303 \times 8.31 \times 12}$$

$= 0.3 = \log 2$

$$\frac{k_{127}}{k_{27}} = 2 = \frac{\log 100/A}{\log\left(\frac{100}{75}\right)}$$

$$2. \log 4/3 = \log 100/A \Rightarrow \log \frac{16}{9} = \log \frac{100}{A}$$

$$\frac{16}{9} = \frac{100}{A} \Rightarrow A = \frac{100 \times 9}{16} = 56.25\ \%$$

% decompostion = 43.75 $\simeq$ 44

55. number of waves = orbit no. = 3

56. $H \rightarrow H^+ + e^-$ $\quad IE_1 = 13.6$ eV

$He \rightarrow He^+ + e^-$ $\quad IE_1 = 24.6$ eV

We have to determine the value of

$He^{+2} + e^- \rightarrow He^+$ $\quad \Delta H = a$

$He^+ + e^- \rightarrow He$ $\quad \Delta H = b$

$He^{+2} + 2e^- \rightarrow He$ $\quad \Delta H = (a + b)$

IE_1 of $He^+ = IE_{1\,H} \times 2^2 = 13.6 \times 4 = 54.4$

a = –54.4 ev

$He^+ + e^- \rightarrow He$

IE = 24.6 eV

b = –24.6 eV

total energy given

= –54.4 – 24.6

= –79 eV = 79 ev

57. H_2 = 1.12 at = 0.05 mole = 0.1 gm

D_2 = 1.12 at = 0.05 mole = 0.2 gm

After deffusion H_2 left = 0.05 gm

H_2 diffusel in given time

= 0.1 – 0.05

= 0.05 gm

$$\frac{w_{H_2}}{t_{H_2}} \cdot \frac{t_{D_2}}{w_{D_2}} = \sqrt{\frac{M_{H_2}}{M_{D_2}}} = \sqrt{\frac{2}{4}}$$

$$\Rightarrow \frac{0.05}{w_{D_2}} = \sqrt{\frac{1}{2}} \Rightarrow w_{D_2} = 1.4 \times 0.05 = 0.07 \text{ gm}$$

% of D_2 in IInd bulb

$$= \frac{0.07}{0.07 + 0.05} \times 100 = 58\%$$