DESIGN OF THE NEW QUESTION PAPER

Time : 3 Hours *Max. Marks : 100*

The weightage of marks over different dimensions of the question paper shall be as follows :

A. Weightage to Different Topics/Content Units

S.No.	*Topics*	*Marks*
1.	Relations and Functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors & Three-Dimensional Geometry	17
5.	Linear Programming	06
6.	Probability	10
	Total	**100**

B. Weightage to Different Forms of Questions

S.No.	*Forms of Questions*	*Marks for Each Question*	*No. of Questions*	*Total Marks*
1.	Very Short Answer Questions (VSA)	01	10	10
2.	Short Answer Questions (SA)	04	12	48
3.	Long Answer Questions (LA)	06	07	42
	Total		**29**	**100**

C. Scheme of Options

There will be no overall choice. However, an internal choice in any four questions of four marks each and any two questions of six marks each has been provided.

D. Difficulty Level of Questions

S.No.	*Estimated Difficulty Level*	*Percentage of Marks*
1.	Easy	15
2.	Average	70
3.	Difficult	15

PAPER-1

Time limit :- 3 hrs

MAX MARKS - 100

General Instruction.

Q.1 to Q.10 carry ~~two~~ one marks each.
Q.11 to Q.22 carry four marks each.
Q.23 to Q.29 carry six marks each.

SECTION-A

Q.1. Find the value of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.

Q.2. If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$ find $(AB)'$.

Q.3. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given $a_{ij} = \dfrac{(i+j)^2}{2}$.

Q.4. Find the value of the derivative of $\sec^{-1}(e^x)$ w.r.t. x at the point $x = 1$.

Q.5. Write down whether the function f given by $f(x) = \log \cos x$ is strictly increasing or strictly decreasing on $\left(0, \dfrac{\pi}{2}\right)$.

Q.6. Evaluate : $\displaystyle\int_0^{1/2} \frac{1}{2+8x^2}\,dx$.

Q.7. Write the order and degree of the differential equation : $\left[5 + \left(\dfrac{dy}{dx}\right)^2\right]^{5/3} = x^5 \dfrac{d^2y}{dx^2}$

Q.8. Find the value of x, for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Q.9. A coin is tossed 5 times. What is the probability that a head appears an even number of times ?

Q.10. A purse contains 4 copper coins, 3 silver coins. The second purse contains 6 copper coins and 2 silver coins. A coin is taken out of any purse, find the probability that it is a copper coin.

SECTION-B

Q.11. Prove that : $\sin^{-1}\dfrac{4}{5} + \sin^{-1}\dfrac{5}{13} + \sin^{-1}\dfrac{16}{65} = \dfrac{\pi}{2}$.

Q.12. Prove that : $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$.

Or

If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

Q.13. If $y = (\cos x)^{\log x} + (\log x)^x$, find $\frac{dy}{dx}$.

Q.14. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(*i*) parallel to the line $2x - y + 9 = 0$

(*ii*) perpendicular to the line $5y - 15x = 13$.

Q.15. Find the value of k so that the function f is continuous at $x = 5$.

$$f(x) = \begin{cases} kx + 1, & \text{if } x \le 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

Q.16. Evaluate : $\int \frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} dx.$

Q.17. Evaluate : $\int_0^{\pi} \log(1 + \cos x)\, dx$.

Or

Evaluate : $\int_{-1}^{2} |x^3 - x|\, dx$.

Q.18. For any two vectors $\vec{a}$ and $\vec{b}$, show that $(1 + |\vec{a}|^2)(1 + |\vec{b}|^2) = |(1 - \vec{a}.\vec{b})|^2 + |\vec{a} + \vec{b} + (\vec{a} \times \vec{b})|^2$

Or

Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is

Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

Q.19. Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Q.20. Solve the differential equation : $\cos^2 x \frac{dy}{dx} = \tan x - y$.

Or

Solve the differential equation : $\left[x\sqrt{x^2 + y^2} - y^2\right] dx + xy\, dy = 0$.

Q.21. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that th student knows the answer given that he answered it correctly ?

Q.22. Three coins are tossed simultaneously. Consider the event E 'three heads or three tails' F 'at least two heads' and G 'at most two heads'. Of the pairs (E, F), (E, G) and (F, G), which are independent and which are dependent ?

SECTION-C

Q.23. State whether the function is one-one, onto or bijective. $f: R \to R$ defined by $f(x) = 1 + x^2$. Justify your answer.

Or

Show that the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$. Given by : $R = \{(a, b) : |a - b|$ is a multiple of $4\}$ is an equivalence relation. Find the set of all elements related to 1.

Q.24. Show that the semi-vertical angle of a right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

Q.25. Make a rough sketch of the region given below and find its area using integration.

$\{(x, y) : 0 \leq y \leq x^2 + 3; 0 \leq y \leq 2x + 3; 0 \leq x \leq 3\}$

Q.26. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machine for at the most 12 hours a day ?

Q.27. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A . adj A = | A | I. Also find A^{-1}.

Q.28. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

Q.29. Find the vector equation of a plane through the intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 5$ and $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 3$ and passing through the point (2, 1, –2).

Or

Find the distance of the point (1, –2, 3) from a plane $x - y + z = 5$ measured parallel to the line $\frac{x+1}{2} = \frac{y+3}{3} = \frac{z+1}{-6}$.

ANSWERS

1. $\pm\sqrt{3}$

2. $\begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$

4. $\frac{1}{\sqrt{e^2-1}}$

5. Strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

6. $\frac{\pi}{6}$

7. Order 2 and degree 3

8. $x = \pm\frac{1}{\sqrt{3}}$

9. $\frac{1}{2}$

10. $\frac{37}{56}$

12. ***Or*** $x = 8, y = 8; A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

13. $(\cos x)^{\log x}\left\{\frac{\log \cos x}{x} - \tan x \log x\right\} + (\log x)^x\left\{\log(\log x) + \frac{1}{\log x}\right\}$

14. (*i*) $y - 2x - 3 = 0$ (*ii*) $36x + 12y - 227 = 0$

15. $\frac{9}{5}$

16. $-\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + c$

17. $-\pi \log 2$ ***Or*** $\frac{11}{4}$

19. $\frac{\pi^2}{16}$

20. $y = (\tan x - 1) + (ce^{-\tan x})$ ***Or*** $\sqrt{x^2 + y^2} = x \log\left|\frac{c}{x}\right|$

21. $\frac{12}{13}$

22. The events (E and F) are independent. The events (E and G) and (F and G) are dependent.

23. Neither one-one nor onto. ***Or*** $\{1, 5, 9\}$

25. $\frac{50}{3}$ sq. units.

26. Maximum profit = Rs. 73.50, when 3 packages of nuts and 3 packages of bolts are produced.

27. $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

28. $(-3\hat{i} + 5\hat{j} + 2\hat{k})$

29. $\vec{r}.(3\hat{i} + 2\hat{j} + 0\hat{k}) = 8$ ***Or*** 1 unit.

PAPER-2

Time limit :- 3 Hrs MAX MARKS - 100

General Instructions

Q.1 to Q.10 carry one mark each.

Q.11 to Q.22 carry four marks each.

Q.23 to Q.29 carry six marks each.

SECTION-A

Q.1. If A and B are symmetric matrices of the same order, write whether the matrix (AB + BA) is symmetric or skew-symmetric.

Q.2. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, write whether AB is O or I.

Q.3. If A is a square matrix of order 3 such that $|\text{adj } A| = 81$, find $|A|$.

Q.4. Find the value of the derivative of $e^{\tan^{-1}\sqrt{x}}$ w.r.t. x at the point $x = 1$.

Q.5. Write whether the function $f(x) = -3x + 8$ is strictly increasing or decreasing on R.

Q.6. Find the order and degree of the differential equation $xy\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - y\left(\dfrac{dy}{dx}\right) = 0$.

Q.7. Write the value of $\int \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\,dx$.

Q.8. Find the unit vector in the direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Q.9. A four digit number is formed using the digits 1, 2, 3, 4 with no repetitions. Find the probability that the number is divisible by 4.

Q.10. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs none will fuse after 150 days of use.

SECTION-B

Q.11. If $\tan^{-1}\dfrac{x-1}{x-2} + \tan^{-1}\dfrac{x+1}{x+2} = \dfrac{\pi}{4}$, then find the value of x.

Q.12. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$.

Or

If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$. Show that either $a + b + c = 0$ or $a = b = c$.

Q.13. If $x = a\left(\cos t + \log \tan \frac{t}{2}\right), y = a \sin t$, find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$..

Q.14. Find the equation of the tangent and normal to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where it cuts the axis of x.

Q.15. Show that the function $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{when } x \neq 0 \\ 2, & \text{when } x = 0 \end{cases}$ is not continuous at $x = 0$.

Q.16. Evaluate : $\int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

Q.17. Using properties of definite integrals, evaluate $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$.

Or

Evaluate : $\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$

Q.18. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 6, |\vec{b}| = 5$ and $|\vec{c}| = 7$, find the angle between $\vec{b}$ and $\vec{c}$.

Or

If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitude, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

Q.19. Evaluate : $\int_0^{\pi/2} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$.

Q.20. Sole the differential equation $(1 + y^2)(1 + \log x)\, dx + x\, dy = 0$, satisfying the given condition : $y = 1$ when $x = 1$.

Or

Solve the differential equation : $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$

Q.21. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results reports that 30% of all students who resided in hostel attained A grade and 20% of day scholars attained A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hostler ?

Q.22. Two cards are drawn simultaneously (or successively without replacement) from a well-shuffled pack of 52 cards. Find mean, variance and standard deviation of the number of kings.

SECTION-C

Q.23. Show that the signum function $f: R \to R$, given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.

Or

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

Q.24. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}\,r$.

Q.25. Make a rough sketch and find the area of the region using integration.

$\{(x, y) : x^2 + y^2 \leq 16x; y^2 \geq 8x; x \geq 0, y \geq 0\}$

Q.26. A soft drink firm has two bottling plants located at the places P and Q. Each plant produces three different kinds of soft drinks A, B and C. The capacity of the plants in number of bottles per day are as follows :

Soft drink	Plant	
	P	Q
A	3,000	1,000
B	1,000	1,000
C	2,000	6,000

A market survey indicates that during the month of April, there will be a demand for 24,000 bottles of A, 16,000 bottles of B and 48,000 bottles of C. The operating cost per day of running plants at P and Q are respectively Rs. 6,000 and Rs. 4,000. Find graphically, how many days should the firm run each plant is April, so that the production cost is minimised while still meeting the market demand.

Q.27. Solve the system of linear equations by matrix method

$x - 2y = 10; 2x + y + 3z = 8; -2y + z = 7$

Q.28. Find the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line

$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

Or

Show that the lines :

$\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ and $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ are coplanar. Also, find the equation of the plane containing them.

Q.29. Prove that the image of the point $(3, -2, 1)$ in the plane $3x - y + 4z = 2$ lies on the plane $x + y + z + 4 = 0$.

ANSWERS

1. Symmetric
2. O
3. 9
4. $\frac{1}{4} e^{\pi/4}$
5. Strictly decreasing
6. Order 2 and degree 1
7. $\frac{1}{2} \log | e^{2x} + e^{-2x} | + c$
8. $\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
9. $\frac{1}{4}$
10. $\left(\frac{19}{20}\right)^5$
11. $\pm\frac{1}{\sqrt{2}}$
12. $\pm 4\sqrt{3}$
13. $\frac{2\sqrt{2}}{a}$
14. $(7, 0)$; $20y - x + 7 = 0, y + 20x - 140 = 0$
16. $-e^x \cot \frac{x}{2} + c$
17. $\frac{\pi^2}{2ab}$ ***Or*** $\sin^{-1}\left(\frac{e^x + 2}{3}\right) + c$
18. $\cos^{-1}\left(\frac{-19}{35}\right)$
19. $\frac{\pi}{4}$
20. $\frac{1}{2}(1 + \log x)^2 + \tan^{-1} y = \frac{\pi}{4} + \frac{1}{2}$ ***Or*** $x \sin\left(\frac{y}{x}\right) = c$
21. $\frac{9}{13}$
22. $\mu = \frac{34}{221}$, $\sigma^2 = \frac{6800}{(221)^2}$, S.D. = 0.37
25. $\frac{16}{3}(3\pi - 8)$ sq. units
26. Firm should run plant P and 4 days and Q for 12 days, Minimum cost = Rs. 72,000
27. $x = 4, y = -3, z = 1$
28. $(1, 2, 3)$; $\sqrt{14}$ ***Or*** $x + y + z = 0$

PAPER- 3

Time Limit :- 3 Hrs.

MAX MARKS 100

General Instructions.

(i) Q.1 to Q.10 carry one mark each.

(ii) Q.11 to Q.22 carry four marks each.

(iii) Q.23 to Q.29 carry six marks each.

SECTION-A

Q.1. If $f : R - \{0\} \rightarrow R - \{0\}$ be defined as $f(x) = \dfrac{3}{x}$, then find $f^{-1}(x)$.

Q.2. Find the value of : $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

Q.3. For what value of x, the following matrix is singular $\begin{bmatrix} 8-x & x+4 \\ 1 & 5 \end{bmatrix}$?

Q.4. Give an example of two matrices A and B such that $A \neq 0$, $B \neq 0$ and $AB = BA = 0$.

Q.5. A matrix A of order 3×3 has determinant 15. What is the value of $|5A|$?

Q.6. Write the value of $\int e^{3\log x}(x^4)\,dx$.

Q.7. Write the value of $\int_0^{\pi/2} \log \tan x \, dx$.

Q.8. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors, then write the value of $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2$.

Q.9. What is the angle between vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively? Given $\vec{a}.\vec{b} = 3$.

Q.10. Write the value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{k} \times \hat{i}) + \hat{k}.(\hat{j} \times \hat{i})$.

SECTION-B

Q.11. Let L be the set of all lines in xy plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Or

Let $A = N \times N$ and * be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Find the identity element for * on A, if any.

Q.12. Prove that : $\tan^{-1}\left[\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right] = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x, 0 < x < 1$.

Q.13. If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1+xyz=0$.

Q.14. Find the value of k so that the function f is continuous at the indicated point

$$f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases} \quad \text{at } x = \pi.$$

Q.15. If $(\cos x)^y = (\sin y)^x$, find $\dfrac{dy}{dx}$.

Q.16. Verify mean value theorem, if $f(x) = x^3 - 5x^2 - 3x$ is the interval $[a, b]$, where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

Q.17. Evaluate : $\displaystyle\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\, d\theta.$

Or

Integrate the function : $\displaystyle\int \frac{x-3}{(x-1)^3}\, e^x\, dx.$

Q.18. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{6}$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$.

Or

If with reference to the right-handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express β in the form $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where β_1 is parallel to α and β_2 is perpendicular to α.

Q.19. Find the shortest distance between the lines, whose equations are :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Q.20. A man is known to speak truth 3 out of 4 times. He throws die and reports that it is a six. Find the probability that it is actually a six.

Q.21. Show that the given differential equation : $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ is homogenous and solve it.

Or

Find a particular solution of the differential equation : $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x \ (x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$.

Q.22. Form the differential equation of the family of circles having centre on y-axis and radius 3 units.

SECTION-C

Q.23. Two bags A and B contain 4 white; 3 black balls and 2 white; 2 black balls respectively. From bag A two balls are transferred to bag B. Find the probability of drawing :

(*i*) 2 white balls from bag B ?

(*ii*) 2 black balls from bag B ?

(*iii*) 1 white and 1 black ball from bag B ?

Or

A coin is biased so that the head is 3 times as likely as to occur as a tail. If the coin is tossed twice, find the probability distribution for the number of tails. Also find E (X) and variance.

Q.24. Find the equation of the line through the point (–1, 2, 3) which is perpendicular to the lines :

$$\frac{x}{2} = \frac{y-1}{-3} = \frac{z+2}{-2} \text{ and } \frac{x+3}{-1} = \frac{y+2}{2} = \frac{z-1}{3}$$

Q.25. Find the area of the region bounded by the curves $y = x - 1$ and $(y - 1)^2 = 4(x + 1)$.

Or

Using integration, find the area of the region given by $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$.

Q.26. Evaluate : $\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$.

Q.27. The sum of the lengths of the hypotenuse and a side of a right triangle is given. Show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

Q.28. Using elementary row transformation, find the inverse of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

Q.29. A manufacture company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8,000 on each piece of model A and Rs. 12,000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realize a maximum profit ? What is the maximum profit per week ?

ANSWERS

1. $f^{-1}(x) = \frac{3}{x}$ for all $x \in R - \{0\}$ **2.** $-\frac{\pi}{3}$ **3.** $x = 6$

4. $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **5.** 1875 **6.** $\frac{x^8}{8} + c$

7. 0 **8.** $|\vec{a}|^2 |\vec{b}|^2$ **9.** $\frac{\pi}{6}$ **10.** 1

11. $y = 2x + c, c \in R$ ***Or*** Identity element does not exist. **14.** $\frac{3}{4}$

15. $\frac{\log \sin y + y \tan x}{\log \cos x - x \cot y}$ **16.** $c = \frac{7}{3} \in (1, 3)$. There is no value of $c \in (1, 3)$ for which $f'(c) = 0$

17. $3 \log (2 - \sin \theta) + \frac{4}{2 - \sin \theta} + c$ ***Or*** $\frac{e^x}{(x-1)^2} + c$

18. ***Or*** $\vec{\beta} = 2\hat{i} + \hat{j} - \hat{k} = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right) = \vec{\beta}_1 + \vec{\beta}_2$ **19.** $\frac{1}{\sqrt{6}}$

20. $\frac{3}{8}$ **21.** $ye^{\frac{x}{y}} + x = c$ ***Or*** $y \sin x = 2x^2 - \frac{\pi^2}{2} (\sin x \neq 0)$

22. $(x^2 - 9)(y')^2 + x^2 = 0$

23. (*i*) $\frac{5}{21}$ (*ii*) $\frac{4}{21}$ (*iii*) $\frac{4}{7}$ ***Or***

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$E(X) = \frac{1}{2}; \sigma^2 = \frac{3}{8}$.

24. $\frac{x+1}{5} = \frac{y-2}{4} = \frac{z-3}{-1}$ **25.** $\frac{64}{3}$ sq. units ***Or*** $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq. units

26. $x \cdot \log(\log x) - \frac{x}{\log x} + c$ **28.** $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

29. The company should produce 12 pieces of model A and 6 pieces of model B to get maximum profit of Rs. 1,68,000.