JEE MAIN JAN 2020

PAPER WITH SOLUTION

9th January 2020 _ SHIFT - II

MATHEMATICS

1. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if I_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and I_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $I_2 : I_1$ is equal to :

$\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$ के प्रसार में, यदि x से स्वतंत्र पद का निम्नतम मान I_1 है जब $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ तथा x से स्वतंत्र पद का निम्नतम मान I_2 है जब $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, तो अनुपात $I_2 : I_1$ बराबर है :

(1) 1 : 16 (2) 8 : 1 (3) 1 : 8 (4) 16 : 1

Sol. **4**

$$T_9={}^{16}C_8\left(\frac{x}{\cos\theta}\right)^8\left(\frac{1}{x\sin\theta}\right)^8={}^{16}C_8\left(\frac{1}{\sin\theta\cos\theta}\right)^8$$

$$\Rightarrow \frac{{}^{16}C_8.2^8}{(\sin2\theta)^8}$$

if $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$ $\therefore$ $2\theta\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

$$\ell_1=16_{C_8}.2^8$$

if $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$ $\therefore$ $2\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$

$$\ell_2=\frac{{}^{16}C_8.2^8}{(1/\sqrt{2})^8}={}^{16}C_8.2^8.2^4$$

$$\frac{\ell_2}{\ell_1}=2^4=(16:1)$$

2. Let a function $f:[0,5]\to R$ be continuous f(1) = 3 and F be defined as :

$F(x)=\int_1^x t^2g(t)\,dt$, where $g(t)=\int_1^t f(u)\,du$

Then for the function F, the point x = 1 is :

(1) a point of inflection (2) a point of local minima

(3) not a critical point. (4) a point of local maxima

माना एक फलन $f : [0, 5] \to R$ संतत है, $f(1) = 3$ है तथा F, $F(x) = \int_1^x t^2 g(t)\,dt$, द्वारा परभाषित है, जहाँ

$g(t) = \int_1^t f(u)\,du$

है, तो फलन F के लिए, बिन्दु $x = 1$ एक :
(1) क्रांतिक बिन्दु नहीं है।
(2) स्थानीय निम्निष्ठ बिन्दु है।
(3) नति परिवर्तन (inflection) बिन्दु है।
(4) स्थानीय उच्चिष्ठ बिन्दु है।

Sol. **2**

$F(x) = \int_1^x t^2 g(t)\,dt$ $\qquad$ $g(t) = \int_1^x f(u)\,du$

$F'(x) = x^2.\,g(x)$ $\qquad$ $g'(t) = f(t)$

$F'(1) = 1.g(1) = 0$

$F''(x) = 2xg(x) + x^2.f(x)$

$F''(1) = 2g(1) + f(1) = 0 + 3 = 3$

Local Minima

3. Let [t] denote the greatest integer $\leq t$ and $\lim\limits_{x\to 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2] \sin(\pi x)$ is discontinuous, when x is equal to :

माना [t] महत्तम पूर्णांक $\leq t$ को दर्शाता है तथा $\lim\limits_{x\to 0} x\left[\frac{4}{x}\right] = A$ है। तो फलन $f(x) = [x^2] \sin(\pi x)$ असंतत है, जब x बराबर है :

(1) $\sqrt{A+21}$

(2) $\sqrt{A+1}$

(3) $\sqrt{A+5}$

(4) $\sqrt{A}$

Sol. **2**

$$\lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right)$$

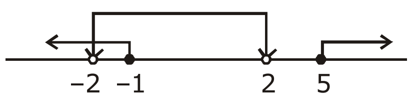
$$\lim_{x\to 0} \left(4 - x\left\{\frac{4}{x}\right\}\right)$$

4 – 0 × finite
A = 4
$f(n) = [x^2] \sin(\pi x)$
In option 1, 3, 4 values are integer and Integral Multiple of π in sine is always zero.
$\therefore$ f(x) is disc. at $\sqrt{A+1}$

4. If $A = [x \in R : |x| < 2]$ and
$B = \{x \in R : |x - 2| \geq 3\}$; then :
यदि $A = [x \in R : |x| < 2]$ तथा
$B = \{x \in R : |x - 2| \geq 3\}$, तो :
(1) $A - B = [-1, 2)$
(2) $B - A = R - (-2, 5)$
(3) $A \cap B = (-2, -1)$
(4) $A \cup B = R - (2, 5)$

Sol. **2**

$A = \{x \in (-2, 2)\}$
$B = \{|x - 2| \geq 3\}$
$\Rightarrow x - 2 \geq 3 \cup x - 2 \leq -3$
$x \geq 5 \cup x \leq -1$



5. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to :

माना धनात्मक पदों की एक गुणोत्तर श्रेढ़ी का n वां पद a_n है। यदि $\sum_{n=1}^{100} a_{2n+1} = 200$ तथा $\sum_{n=1}^{100} a_{2n} = 100$, तो $\sum_{n=1}^{200} a_n$ बराबर है :

(1) 175 (2) 225 (3) 300 (4) 150

Sol. **4**

$\sum_{n=1}^{100} a_{2n+1} = 200$

$a_3 + a_5 .... + a_{201} = 200$

$a_2 + a_4 + ... + a_{200} = 100$

So

$ar^2 + ar^4 .... + ar^{200} = 200$

$ar^2(1 + r^2 .... + r^{198}) = 200$ .....(i)

and

$ar + ar^3 .... + ar^{199} = 100$

$ar(1+r^2..........+r^{198}) = 100$ ....(ii)

$\sum_{n=1}^{200} a_n = a_1 + a_2..........+ a_{200}$

$= a + ar .... ar^{199}$

$\Rightarrow a\dfrac{\{r^{200}-1\}}{r-1}$

using eq. (i) $a.2\dfrac{\{2^{200}-1\}}{3} = 100$

$a\,(2^{100} - 1) = 150$

$a = \dfrac{150}{2^{200}-1}$

$\sum_{n=1}^{200} a_n = \dfrac{150}{2^{200}-1} \times (2^{200}-1)$

$= 150$

6. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

यदि 10 भिन्न गेंदें, 4 भिन्न बक्सों में यादच्छया रखी जानी है, तो इनमें से दो बक्सों में मात्र 2 तथा 3 गेंदों के होने की प्रायिकता है :

(1) $\dfrac{965}{2^{11}}$ (2) $\dfrac{965}{2^{10}}$ (3) $\dfrac{945}{2^{10}}$ (4) $\dfrac{945}{2^{11}}$

Sol. **Bonus**

10 different balls in 4 different boxes.

10

2,3,0,5 2,3,1,4 2,3,2,3

$$\frac{1}{4^{10}}\left(4!\times\frac{10!}{2!\times 3!\times 0!\times 5!} + 4!\times\frac{10!}{2!\times 3!\times 1!\times 4!} + 4!\times\frac{10!}{(2i)^2\,!2!\times(3!)^2\times 2!}\right)$$

$$= \frac{17\times 945}{2^{15}}$$

7. If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$; y (1) = 1 ; then a value of x satisfying y (x) = e is :

यदि $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$; y (1) = 1 है, तो y (x) = e को सन्तुष्ट करने वाला x का एक मान है :

(1) $\sqrt{2}\,e$ (2) $\frac{1}{2}\sqrt{3}\,e$ (3) $\sqrt{3}\,e$ (4) $\frac{e}{\sqrt{2}}$

Sol. **3**

$$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$$

y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2+v^2x^2} = \frac{1-v-v^3}{1+v^2}$$

$$\frac{(1+v^2)dv}{v^3} = -\frac{dx}{x}$$

$$-\frac{1}{2v^2} + \ell nv = -\ell nx + C$$

$$-\frac{x^2}{2y^2} = -\ell nx + C$$

x = 1, y = 1

$\therefore$ C = $-\frac{1}{2}$

a + y = e

$\therefore$ $x = \sqrt{3}\,e$

8. The following system of linear equations
7x + 6y – 2z = 0
3x + 4y + 2z = 0
x – 2y – 6z = 0, has
(1) Infinitely many solutions, (x, y, z) satisfying y = 2z.
(2) Infinitely many solutions, (x, y, z) satisfying x = 2z.
(3) No solutions
(4) Only the trivial solution.
निम्नलिखित रेखिय समीकरणों
7x + 6y – 2z = 0
3x + 4y + 2z = 0
x – 2y – 6z = 0, की निकाय रखती है
(1) अनन्त रूप से कई हल, (x, y, z) है जो y = 2z को सन्तुष्ट करते है।
(2) अननत रूप से कई हल, (x, y, z) है जो x = 2z को सन्तुष्ट करते है।
(3) कोई हल नही
(4) केवल तुच्छ (trivial) हल

Sol. 2

$$\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

$7\{-24+4\}-6\{-18-2\}-2\{-6,-4\}$

$\Delta = -140 + 120 + 20 = 0$

Also Δ_1, Δ_2, Δ_3 are zero

Infinite Solutions

from equation (i) + 3 equation (iii)

$10x - 20z = 0$

$x = 2z$.

9. Let a, b $\in$ R $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to :

माना a, b $\in$ R, $a \neq 0$ इस प्रकार हैं कि समीकरण $ax^2 - 2bx + 5 = 0$ का α पुनरावत्त मूल है, जो समीकरण $x^2 - 2bx - 10 = 0$ का भी एक मूल है। यदि β इस समीकरण का दूसरा मूल है, तो $\alpha^2 + \beta^2$ बराबर है :

(1) 24 (2) 25

(3) 26 (4) 28

Sol. 2

$ax^2 - 2bx + 5 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \alpha \end{matrix}$ $4b^2 - 4a.5 = 0$

$x^2 - 2bx - 10 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$ $b^2 = 5a$

$a\alpha^2 - 2b\alpha + 5 = 0$

$\alpha^2 - 2b\alpha - 10 = 0$

$-\quad +\qquad\qquad +$

$(a-1)\alpha^2 + 15 = 0$

$(a-1)5a + 15a^2 = 0$

$20a^2 - 5a = 0$

$5a(4a-1) = 0$

$a = \dfrac{1}{4}$ $\quad\therefore\ b^2 = \dfrac{5}{4}$

$\alpha + \beta = 2b$

$\alpha^2 + \beta^2 + 2\alpha\beta = 4b^2 = 5$

$\alpha^2 + \beta^2 = 5 - 2(-10) = 25$

10. The length of the minor axis (along y - axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the lines, x + 6 y = 8, then its eccentricity is :

मानक रूप में एक दीर्घवत्त के लघु अक्ष (y - अक्ष के अनुदिश) की लम्बाई $\frac{4}{\sqrt{3}}$ है। यदि दीर्घवत्त, रेखा x + 6 y = 8 को स्पर्श करता है, तो इसकी उत्केन्द्रता है :

(1) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (2) $\frac{1}{2}\sqrt{\frac{5}{3}}$ (3) $\sqrt{\frac{5}{6}}$ (4) $\frac{1}{2}\sqrt{\frac{11}{3}}$

Sol. **4**

$2b = \frac{4}{\sqrt{3}} \Rightarrow \quad b = \frac{2}{\sqrt{3}}$

$y = -\frac{x}{6} + \frac{4}{3} \Rightarrow mx \pm \sqrt{a^2m^2 + b^2}$

$m = -\frac{1}{6}$

$a^2m^2 + \frac{4}{3} = \frac{16}{9} \Rightarrow a^2 = 16$

$e^2 = 1 - \frac{4/3}{16} = 1 - \frac{1}{12} \Rightarrow e = \sqrt{\frac{11}{12}}$

11. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of the tangent to it at B is :

यदि परवलय $y^2 = 8x$ की एक नाभि जीवा AB का एक छोर $A\left(\frac{1}{2}, -2\right)$ पर है, तो B पर इसकी स्पर्श–रेखा का समीकरण है :

(1) x – 2y + 8 = 0
(2) 2x + y – 24 = 0
(3) x + 2y + 8 = 0
(4) 2x – y –24 = 0

Sol. **1**

$y^2 = 8x$ A(1/2, – 2)

a = 2

$4t_1 = -2$

$t_1 = -1/2$

$t_1 t_2 = -1$

$t_2 = 2$

$\therefore$ B (8, 8)

$\therefore$ 8y = 4(x + 8)

2y = x + 8

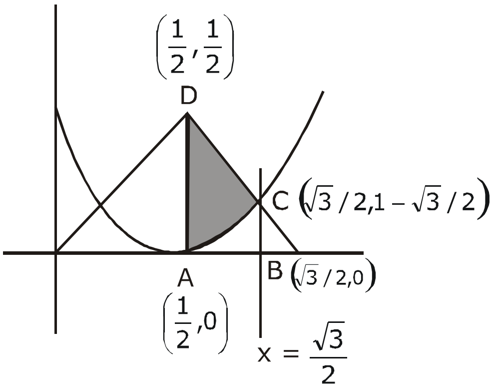
x – 2y + 8 = 0

12. Given : $f(x) = \begin{cases} x & , \ 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \ x = \frac{1}{2} \\ 1-x & , \ \frac{1}{2} < x \le 1 \end{cases}$ and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curve, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :

दिया है : $f(x) = \begin{cases} x & , \ 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \ x = \frac{1}{2} \\ 1-x & , \ \frac{1}{2} < x \le 1 \end{cases}$ तथा $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$ तो रेखाओं $2x = 1$ तथा $2x = \sqrt{3}$ के बीच, वक्रों $y = f(x)$ तथा $y = g(x)$ द्वारा प्रतिबद्ध क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{1}{2} + \frac{\sqrt{3}}{4}$ (2) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (3) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (4) $\frac{\sqrt{3}}{4} - \frac{1}{3}$

Sol. 4



Required area = Area of trapezium ABCD - $\int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2}\right)^2 dx$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left[\left(x - \frac{1}{2}\right)^3\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{\sqrt{3}}{4} - \frac{1}{3}$$

13. If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively :

यदि $p \to (p \wedge \sim q)$ असत्य है, तो p तथा q के क्रमशः सत्यमान हैं :

(1) T, F
(2) T, T
(3) F, F
(4) F, T

Sol. **2**

p	q	$\sim q$	$p \wedge \sim q$	$p \to (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

14. If z be a complex number satisfying $|Re(z)| + |Im(z)| = 4$, then $|z|$ cannot be :

यदि z एक ऐसी सम्मिश्र संख्या है जो $|Re(z)| + |Im(z)| = 4$ को सन्तुष्ट करती है, तो $|z|$ नहीं हो सकता :

(1) $\sqrt{7}$ (2) $\sqrt{10}$ (3) $\sqrt{8}$ (4) $\sqrt{\dfrac{17}{2}}$

Sol. **1**

$z = x + iy$

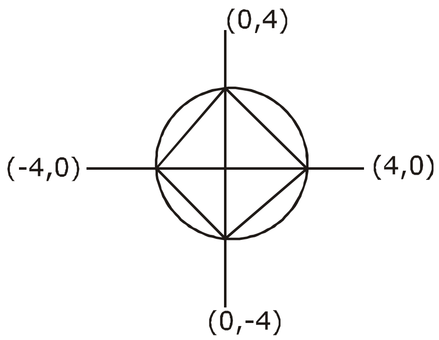
$|x| + |y| = 4$

Minimum value of $|z| = 2\sqrt{2}$

Maximum value of $|z| = 4$

$|z| \in \left[\sqrt{8}, \sqrt{16}\right]$

So $|z|$ can't be $\sqrt{7}$



15. Let f and g be differentiable function on R such that fog is the identity function. If for some a, b $\in$ R $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

माना R पर अवकलनीय फलन f तथा g इस प्रकार है कि fog तत्समक फलन है। यदि किसी a, b, $\in$ R के लिए $g'(a) = 5$ तथा $g(a) = b$ हैं, तो $f'(b)$ बराबर है :

(1) 5 (2) $\dfrac{1}{5}$ (3) $\dfrac{2}{5}$ (4) 1

Sol. **2**

$f(g(x)) = x$

$f'(g(x)).\ g'(x) = 1$

$x = a$

$f'(g(a)) - g'(a) = 1$

$f'(b) = 1/5$

16. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to :

यदि $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ है, जहाँ C एक समाकलन अचर है, तो क्रमित युग्म $(\lambda, f(\theta))$ बराबर है :

(1) $(1, 1+\tan\theta)$ (2) $(-1, 1+\tan\theta)$ (3) $(-1, 1-\tan\theta)$ (4) $(1, 1-\tan\theta)$

Sol. **2**

$$\int \frac{\sec^2\theta\, d\theta}{\left(\frac{2\tan\theta}{-\tan^2\theta} + \frac{1+\tan^2\theta}{1-\tan^2\theta}\right)}$$

$$\Rightarrow \int \frac{\sec^2\theta(1-\tan^2\theta)d\theta}{(1+\tan\theta)^2}$$

$$\Rightarrow \int \frac{\sec^2\theta(1-\tan^2\theta)d\theta}{(1+\tan\theta)}$$

$\tan\theta = t$

$$\int \frac{1-t}{1+t}dt = \int -1 + \frac{2}{1+t}\, dt$$

$= -t + 2\ln(1+t) + C$

$= -\tan\theta + + 2\log(1+\tan\theta) + C$

$\Rightarrow \lambda = -1$ and $f(x) = 1 + \tan\theta$

17. A random variable X has the following probability distribution :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

Then $P(X > 2)$ is equal to :

एक यादच्छिक चर X का प्रायिकता बंटन निम्न है :

X	:	1	2	3	4	5
P(X)	:	K^2	2K	K	2K	$5K^2$

तो $P(X > 2)$ बराबर है :

(1) $\frac{1}{36}$ (2) $\frac{7}{12}$ (3) $\frac{23}{36}$ (4) $\frac{1}{6}$

Sol. **3**

$\sum p_i = 1 \Rightarrow 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

$6k^2 + 6k - k - 1 = 0$

$(6k - 1)(k + 1) = 0$

$\Rightarrow k = -1$(rejected) ; $k = \frac{1}{6}$

$p(x > 2) = k + 2k + 5k^2$

$= \frac{1}{6} + \frac{2}{6} + \frac{5}{36} = \frac{6+12+5}{36} = \frac{23}{36}$

18. Let $a - 2b + c = 1$. If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then :

माना $a - 2b + c = 1$ है। यदि $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$ है, तो :

(1) $f(-50) = 501$ (2) $f(50) = 1$ (3) $f(-50) = -1$ (4) $f(50) = -501$

Sol. **2**

Apply $R_1 = R_1 + R_3 - 2R_2$

$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} \Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

19. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then :

यदि $0 < \theta < \frac{\pi}{4}$ के लिए $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ तथा $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$ हैं, तो :

(1) $x(1 - y) = 1$ (2) $y(1 - x) = 1$ (3) $y(1 + x) = 1$ (4) $x(1 + y) = 1$

Sol. **2**

$x = \sum_{n=0}^{\infty} (-1)^n . \tan^{2n}\theta$

$= 1 - \tan^2\theta \tan^4\theta$

$x = \frac{1}{1+\tan^2\theta} \Rightarrow x = \cos 2\theta$

$y = 1 + \cos^2\theta + \ldots\ldots\ldots.$

$= \frac{1}{1-\cos^2\theta} = \operatorname{cosec}^2\theta$

$\therefore\ y(1 - x) = \operatorname{cosec}^2\theta\ (1 - \cos 2\theta) = 1$

20. If x = 2 sin θ – sin 2θ and y = 2 cos θ – cos 2θ, $\theta \in [0, 2\pi]$, then $\frac{d^2y}{dx^2}$ at θ = π is :

यदि x = 2 sin θ – sin 2θ तथा y = 2 cos θ – cos 2θ, $\theta \in [0, 2\pi]$ हैं, तो θ = π पर $\frac{d^2y}{dx^2}$ का मान है :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $-\frac{3}{8}$ (4) $-\frac{3}{4}$

Sol. **Bonus**

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta, \quad \frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\frac{dy}{dx} = \frac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta} = \frac{2\cos\frac{3\theta}{2}.\sin\theta/2}{2\sin 3\theta/2.\sin\theta/2}$$

$$= \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\operatorname{cosec}^2\frac{3\theta}{2}.\frac{3}{2}.\frac{d\theta}{dx} \Rightarrow \frac{-3/2}{-2-2} = \frac{3}{8}$$ **[No Ans. Matching]**

21. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b}.\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $\left|\vec{a} \times (\vec{b} \times \vec{c})\right|$ is equal to ________.

माना तीन सदिश $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ इस प्रकार है कि $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b}.\vec{c} = 10$ तथा $\vec{b}$ और $\vec{c}$ के बीच का कोण $\frac{\pi}{3}$ है। यदि $\vec{a}$, सदिश $\vec{b} \times \vec{c}$ पर लम्बवत है, तो $\left|\vec{a} \times (\vec{b} \times \vec{c})\right|$ बराबर है ________।

Sol. **30**

$$\vec{b}.\vec{c} = 10$$

$$\Rightarrow |\vec{b}||\vec{c}|\cos\left(\frac{\pi}{3}\right) = 10 \Rightarrow 5.|\vec{c}|.\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$$

Also, $\vec{a}.(\vec{b} \times \vec{c}) = 0$

$$\left|\vec{a} \times (\vec{b} \times \vec{c})\right| = |\vec{a}||\vec{b} \times \vec{c}|\sin\left(\frac{\pi}{2}\right)$$

$$\sqrt{3} \times |\vec{b}||\vec{c}|\sin\frac{\pi}{3} \times 1 = \sqrt{3} \times 5 \times 4 \times \frac{\sqrt{3}}{2} = 30$$

22. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, ($k > 0$) touch each other at a point, then the largest value of k is ______________.
यदि वक्र $x^2 - 6x + y^2 + 8 = 0$ तथा $x^2 - 8y + y^2 + 16 - k = 0$, ($k > 0$) एक दूसरे को एक बिन्दु पर स्पर्श करते हैं, तो k अधिकतम मान है.................. ।

Sol. **36**

Two circle touches each other if $C_1C_2 = |r_1 \pm r_2|$

Distance between $C_2(3,0)$ and $C_1(0,4)$ is either $\sqrt{k}+1$ or $|\sqrt{k}-1|$ $(C_1C_2 = 5)$

$\Rightarrow \sqrt{k}+1=5$ or $|\sqrt{k}-1| = 5 \Rightarrow k = 16$ or $k = 36$

23. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containg the lines

$$\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3} \text{ and } \frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda \in R)$$

is equal to $\frac{k}{\sqrt{633}}$, then k is equal to

यदि समतल $23x - 10y - 2z + 48 = 0$ तथा रेखाओं $\frac{x+1}{2}=\frac{y-3}{4}=\frac{z+1}{3}$ और $\frac{x+3}{2}=\frac{y+2}{6}=\frac{z-1}{\lambda}(\lambda \in R)$

को अंतर्विष्ट करने वाले समतल के बीच की दूरी $\frac{k}{\sqrt{633}}$ है, तो k बराबर है............ ।

Sol. **3**
distance between (-1,3,1) and Plane

is $\left|\frac{-23-30+2+48}{\sqrt{23^2+10^2+2^2}}\right| = \frac{3}{\sqrt{633}}$

$k = 3$

24. The number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16......, 709 is
दो संमातर श्रेढ़ियों 3, 7, 11, ... 407 तथा 2, 9, 16......, 709 में उभयनिष्ठ (common) पदों की संख्या है.......... ।

Sol. **14**
3, 7, 11, ... 407 $\quad d = 4$
2, 9, 16 .... 709 $\quad d = 7$
1st term commom of both series = 23

c.d = 28

$407 = 23 + (n - 1)\, 28$

$$\frac{384}{28}+1=n$$

$n = 14.$

25. If $C_r = {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to ________.

यदि $C_r = {}^{25}C_r$ तथा $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, तो k बराबर है ________ ।

Sol. **51**

$$\sum_{r=0}^{25}(4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r.{}^{25}C_r + \sum_{r=0}^{25}{}^{25}C_r$$

$$= 4\sum_{r=1}^{25} r \times \frac{25}{r}\;{}^{24}C_{r-1} + 2^{25} = 100\sum_{r=1}^{25}{}^{24}C_{r-1} + 2^{25}$$

$$= 100.\,2^{24} + 2^{25} = 2^{25}(50+1) = 51.2^{25}$$