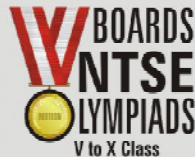हमारा विश्वास... हर एक विद्यार्थी है ख़ास

JEE MAIN JAN 2020

PAPER WITH SOLUTION
9th January 2020 _ SHIFT - 1

MATHEMATICS

1. The number of real roots of the equation, $e^{4x}+e^{3x}-4e^{2x} + e^x + 1 = 0$ is:

समीकरण $e^{4x}+e^{3x}-4e^{2x} + e^x + 1 = 0$ के वास्तविक मूलों की संख्या है :

(1) 1　　　　　(2) 3　　　　　(3) 4　　　　　(4) 2

Sol. **1**

let $e^x = t$

$= t^4 + t^3 - 4t^2 + t + 1 = 0$

$= \left(t^2 + \dfrac{1}{t^2}\right) + \left(t + \dfrac{1}{t}\right) - 4 = 0$

$= \left(t + \dfrac{1}{t}\right)^2 + \left(t + \dfrac{1}{t}\right) - 6 = 0$

$\Rightarrow u^2 + u - 6 = 0$

$\Rightarrow (u+3)(u-2) = 0$

$t + \dfrac{1}{t} - 2 = 0 \qquad ; \; t + \dfrac{1}{t} + 3 = 0 \,(\text{Not possible})$

$\Rightarrow t^2 - 2t + 1 = 0$

$\Rightarrow t = 1$

$\Rightarrow e^x = 1$

$\Rightarrow x = 0$

2. Negation of the statement :

$\sqrt{5}$ is an integer or 5 is irrational is :

(1) $\sqrt{5}$ is an integer and 5 is irrational.

(2) $\sqrt{5}$ is not an integer and 5 is not irrational.

(3) $\sqrt{5}$ is not an integer or 5 is not irrational.

(4) $\sqrt{5}$ is irrational or 5 is an integer.

कथन, $\sqrt{5}$ एक पूर्णांक है या अपरिमेय है' का निषेधन है :

(1) $\sqrt{5}$ एक पूर्णांक नहीं है या 5 अपरिमेय नहीं है।

(2) $\sqrt{5}$ एक पूर्णांक नहीं है या 5 अपरिमेय नहीं है।

(3) $\sqrt{5}$ एक पूर्णांक है और 5 अपरिमेय है।

(4) $\sqrt{5}$ अपरिमेय है या 5 एक पूर्णांक है।

Sol. **2**

$\sim(p \vee q) = \sim p \wedge \sim q$

3. The value of $\int\limits_{0}^{2\pi} \dfrac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$ is euqal

$\int\limits_{0}^{2\pi} \dfrac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$ का मान है :

(1) 4π (2) $2\pi^2$ (3) 2π (4) π^2

Sol. **4**

$I = \int\limits_{0}^{2\pi} \dfrac{x\sin^8 x}{\sin^8 x + \cos^8 x}dx$ ....(1)

$I = \int\limits_{0}^{2\pi} \dfrac{(2\pi - x)\sin^8 x}{\sin^8 x + \cos^8 x}dx$ ....(2)

On adding eq. 1 & 2

$2I = 2\pi\int\limits_{0}^{2\pi} \dfrac{\sin^8 x}{\sin^8 x + \cos^8 x}dx$

$I = 4\pi\int\limits_{0}^{\pi/2} \dfrac{\sin^8 x}{\sin^8 x + \cos^8 x}dx$ ....(3)

$I = 4\pi\int\limits_{0}^{\pi/2} \dfrac{\cos^8 x}{\cos^8 x + \sin^8 x}dx$ ....(4)

On adding eq. 3 and 4

$\Rightarrow 2I = 4\pi\int\limits_{0}^{\pi/2} 1.dx$

4. The value of

$\cos^3\left(\dfrac{\pi}{8}\right).\cos\left(\dfrac{3\pi}{8}\right) + \sin^3\left(\dfrac{\pi}{8}\right).\sin\left(\dfrac{3\pi}{8}\right)$ is :

$\cos^3\left(\dfrac{\pi}{8}\right).\cos\left(\dfrac{3\pi}{8}\right) + \sin^3\left(\dfrac{\pi}{8}\right).\sin\left(\dfrac{3\pi}{8}\right)$ का मान है :

(1) $\dfrac{1}{\sqrt{2}}$ (2) $\dfrac{1}{2}$ (3) $\dfrac{1}{4}$ (4) $\dfrac{1}{2\sqrt{2}}$

Sol. **4**

$= \cos^3\dfrac{\pi}{8}\sin\dfrac{\pi}{8} + \sin^3\dfrac{\pi}{8}\cos\dfrac{\pi}{8}$

$= \left(\sin\dfrac{\pi}{8} \times \cos\dfrac{\pi}{8}\right) \times 1$

$= \dfrac{1}{2} \times \sin\dfrac{\pi}{4} = \dfrac{1}{2\sqrt{2}}$

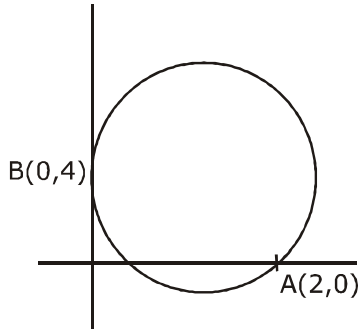**5.** A circle touches the y-axis at the point (0,4) and passes through the point (2,0). Which of the following lines is not a tangent to this circle ?

एक वृत्त y-अक्ष को बिन्दु (0, 4) पर स्पर्श करता है तथा बिन्दु (2, 0) से होकर जाता है। निम्न में से कौन सी रेखा इस वृत्त की स्पर्श रेखा नहीं है ?

(1) 3x-4y-24 = 0   (2) 4x - 3y + 17 = 0 (3) 3x + 4y - 6 = 0   (4) 4x +3y -8 = 0

Sol. **4**

$x^2 + (y-4)^2 + \lambda x = 0$

Passes through   (2,0)

$4 + 16 + 2\lambda = 0$

$\lambda = -10$

$x^2 + y^2 - 8x + 16 - 10x = 0$

$C(5,4), \quad r = \sqrt{25 + 16 + -16} = 5$

$\perp^r$ distance of (5,4) from line 4x + 3y − 8 = 0 ≠ radius

Option (4) is correct answer.

6. Let z be a complex number such that

$\left|\dfrac{z-i}{z+2i}\right| = 1$ and $|z| = \dfrac{5}{2}$. Then the value of $|z+3i|$ is :

माना z एक ऐसी सम्मिश्र संख्या है, कि $\left|\dfrac{z-i}{z+2i}\right| = 1$ है तथा $|z| = \dfrac{5}{2}$ है, तो $|z+3i|$ का मान है :

(1) $\dfrac{7}{2}$ (2) $\dfrac{15}{4}$ (3) $2\sqrt{3}$ (4) $\sqrt{10}$

Sol. **1**

$= \left|\dfrac{z-i}{z+2i}\right| \qquad = 1 \qquad , |z| = \dfrac{5}{2} \Rightarrow x^2 + (y-1)^2 = x^2 + (y+2)^2$

$\Rightarrow y = -\dfrac{1}{2} \qquad ...(1) \qquad \Rightarrow |z| = \dfrac{5}{2}$

$x^2 + y^2 = \dfrac{25}{4}$

from eq.(1)

$\Rightarrow x = \pm \sqrt{6}$

$z = \sqrt{6} - \dfrac{i}{2}$ or $-\sqrt{6} - \dfrac{i}{2}$

$|z + 3i| \left| \pm\sqrt{6} + \dfrac{5i}{2} \right| \Rightarrow \sqrt{6 + \dfrac{25}{4}} = \sqrt{\dfrac{49}{4}} = \dfrac{7}{2}$

7. Let C be the centroid of the triangle with vertices (3, -1), (1,3) and (2,4). Let P be the point of intersection of the lines x + 3y -1 = 0 and 3x-y+1 = 0.Then the line passing through the points C and P also passes through the point.

माना शीर्षों (3, -1), (1,3) तथा (2,4) वाले त्रिभुज का केन्द्रक C है। माना रेखाओं x + 3y -1 = 0 तथा 3x-y+1 = 0 का प्रतिच्छेदन बिन्दु P है, तो बिन्दुओं C तथा P से गुजरने वाली रेखा, निम्न में से किस बिन्दु से भी गुजरती है?

(1) (-9, -7)　　　(2) (9,7)　　　(3) (7,6)　　　(4) (-9, -6)

Sol.　4

A(3,-1)

(2,2)

C

B
(1,3)

D
(2,4)
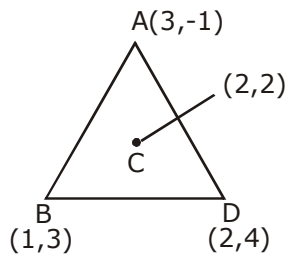
$P : x + 3y - 1 = 0 \ \ldots(1)$

$3x - y + 1 = 0 \ \ \ \ldots.(2)$

On solving eq.(1) and (2)

$x = -\dfrac{1}{5}, y = \dfrac{2}{5}$

eq. of line CP :

$= y - 2 = \left(\dfrac{\dfrac{2}{5} - 2}{-\dfrac{1}{5} - 2} \right)(x - 2)$

$\Rightarrow 8x - 11y + 6 = 0 \Rightarrow$ Clearly (−9,−6) satisfies it.

8. If the number of five digit numbers with distinct digits and 2 at the 10th place is 336k, then k is equal to :

यदि विभिन्न अंकों वाली पांच अंकों की संख्याओं, जिनका दहाई का अंक 2 है, की संख्या 336k है, तो k बराबर है :

(1) 4 (2) 6 (3) 8 (4) 7

Sol. **3**

| 8 | 8 | 7 | 1 | 6 |

↑ ↑ ↑ ↑ ↑

②

8 × 8× 7 × 6 = 336k

k = 8

9. The integral $\int \dfrac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to : (where C is a constant of integration)

समाकल $\int \dfrac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ बराबर है : (जहाँ C एक समाकलन अचर है)

(1) $\dfrac{1}{2}\left(\dfrac{x-3}{x+4}\right)^{3/7}+C$ (2) $\left(\dfrac{x-3}{x+4}\right)^{1/7}+C$ (3) $-\left(\dfrac{x-3}{x+4}\right)^{-1/7}+C$ (4) $-\dfrac{1}{13}\left(\dfrac{x-3}{x+4}\right)^{-13/7}+C$

Sol. **2**

$$\int \dfrac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$$

$$\int \dfrac{dx}{(x+4)^2\left(\dfrac{x-3}{x+4}\right)^{6/7}}$$

Put $\dfrac{x-3}{x+4}=t^7$

$$\left(\dfrac{(x+4)-(x-3)}{(x+4)^2}\right)dx = 7t^6dt$$

$$\dfrac{7}{(x+4)^2}dx = 7t^6dt$$

$$\int \dfrac{t^6}{t^6}dt = \left(\dfrac{x-3}{x+4}\right)^{\frac{1}{7}}+C$$

10. If $f(x) = \begin{cases} \dfrac{\sin(a+2)x + \sin x}{x} & ; x < 0 \\ b & ; x = 0 \\ \dfrac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} & ; x > 0 \end{cases}$

is continuous at x = 0 , then a + 2b is equal to :

यदि $f(x) = \begin{cases} \dfrac{\sin(a+2)x + \sin x}{x} & ; x < 0 \\ b & ; x = 0 \\ \dfrac{(x+3x^2)^{1/3} - x^{1/3}}{x^{4/3}} & ; x > 0 \end{cases}$

x = 0 पर संतत है, तो a + 2b का मान है :

(1) 1   (2) 0   (3) -1   (4) -2

Sol. **2**

$f(0^-) = \lim\limits_{h \to 0} \dfrac{\sin(a+2)h + \sin h}{h}$

$= \lim\limits_{h \to 0} \dfrac{2\sin\left(\dfrac{(a+3)}{2}\right)h \cos\left(\dfrac{a+1}{2}\right)h}{\dfrac{(a+3)}{2} \times h} \times \dfrac{(a+3)}{2} = a + 3$

$= f(0^+) = \lim\limits_{h \to 0} \dfrac{(h+3h^2)^{1/3} - h^{1/3}}{h^{4/3}}$

$= f(0^+) = \lim\limits_{h \to 0} \dfrac{1 + h - 1}{h} = 1$

$= a + 3 = 1 = b$

a = -2 , b = 1

a + 2b = 0

11. The product

$2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \dots \text{to } \infty$ is equal to :

गुणनफल $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \dots \infty$ तक बराबर है :

(1) $2^{\frac{1}{4}}$ (2) $2^{\frac{1}{2}}$ (3) 2   (4) 1

Sol. **2**

$= 2^{1/4} \cdot 2^{1/8} \cdot 2^{1/16} \cdot 2^{1/32} \ldots\ldots$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\ldots\ldots} \qquad = 2^{\frac{\frac{1}{4}}{1-\frac{1}{2}}} = 2^{1/2}$

12. If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $f(0) = 0$, then $f(1)$ is equal to :

यदि $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ है तथा $f(0) = 0$ है, तो $f(1)$ का मान है :

(1) $\dfrac{\pi+2}{4}$ (2) $\dfrac{1}{4}$ (3) $\dfrac{\pi+1}{4}$ (4) $\dfrac{\pi-1}{4}$

Sol. **3**

$= f'(x) = \tan^{-1}\left[\dfrac{1+\sin x}{\cos x}\right]$

$= f'(x) = \tan^{-1}\left[\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right]$

Now, $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$

$\qquad 0 < \dfrac{\pi}{4}+\dfrac{x}{2} < \dfrac{\pi}{2}$

$\Rightarrow f'(x) = \left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

$= f(x) = \dfrac{\pi x}{4}+\dfrac{x^2}{4}+C$

$f(0) = 0 \qquad \Rightarrow C = 0$

$= f(x) = \dfrac{\pi x}{4}+\dfrac{x^2}{4} \quad = f(1) = \dfrac{\pi+1}{4}$

13. Let f be any function continuous on [a, b] and twice differentiable on (a,b). If for all $x \in (a,b), f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a,b), \dfrac{f(c)-f(a)}{f(b)-f(c)}$ is greater than :

माना f कोई फलन है जोकि [a, b] में संतत तथा (a, b) में दो बार अवकलनीय है। यदि सभी $x \in (a,b)$ के लिए $f'(x) > 0$ तथा $f''(x) < 0$ हैं, तो किसी भी $c \in (a,b)$ के लिए $\dfrac{f(c)-f(a)}{f(b)-f(c)}$ निम्न में से किससे बड़ा है ?

(1) $\dfrac{c-a}{b-c}$ (2) $\dfrac{b-c}{c-a}$ (3) $\dfrac{b+a}{b-a}$ (4) 1

Sol. 1

$f'(x) > 0$ & $f''(x) < 0$

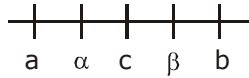Apply LMVT on $[a,c]$

$$f'(\alpha) = \frac{f(c) - f(a)}{c - a}$$

Apply LMVT on $[c, b]$

$$f'(\beta) = \frac{f(b) - f(c)}{b - c}$$

$f''(x) < 0 \Rightarrow f'(x)$ is decreasing

$f'(\alpha) > f'(\beta)$

$$\frac{f(c) - f(a)}{c - a} > \frac{f(b) - f(c)}{b - c}$$

$$\frac{f(c) - f(a)}{f(b) - f(c)} > \frac{c - a}{b - c}$$

14. Let the observations $x_i \, (1 \leq i \leq 10)$ satisfy the equations, $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If μ and λ are the mean and the variance of the observations, x_1-3, x_2 - 3, ...x_{10}- 3, then the ordered pair (μ, λ) is equal to :

माना प्रेक्षण $x_i \, (1 \leq i \leq 10)$ समीकरणों $\sum_{i=1}^{10} (x_i - 5) = 10$ तथा $\sum_{i=1}^{10} (x_i - 5)^2 = 40$ को संतुष्ट करते हैं। यदि μ तथा λ, प्रेक्षणों x_1-3, x_2 - 3, ...x_{10}- 3 के क्रमशः माध्य तथा प्रसरण हैं, तो क्रमित युग्म (μ, λ) बराबर है :

(1) (3,3) (2) (3, 6) (3) (6,6) (4) (6,3)

Sol. 1

$$\sum_{i=1}^{10} (x_i - 5) = 10$$

$$x_1 + x_2 + ... + x_{10} = 10 + 50 = 60$$

$$\sum_{i=1}^{10} (x_i - 5)^2 = 40$$

$$(x_1^2 + x_2^2 + ... + x_{10}^2) + 250 - 10(60) = 40$$

$$x_1^2 + ... + x_{10}^2 = 640 - 250 = 390$$

$$\text{Mean} = \frac{x_1 + ... + x_{10} - 30}{10}$$

$$= \frac{60 - 30}{10} = 3 = \mu$$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\overline{x}\right)^2$$

$$= \frac{\left(x_1 - 3\right)^2 + \dots + \left(x_{10} - 3\right)^2}{10} - 3^2$$

$$= \frac{390 + 90 - 6 \times 60}{10} - 3^2 = 3$$

15. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm³ /min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which the thickness of ice decreases, is :

एक 10 cm त्रिज्या वाली गोलाकार लोहे की गेंद को बर्फ की एक समान मोटाई वाली परत से लेप किया गया है, जो कि 50 cm³ /min की दर से पिघलती है। जब बर्फ की परत की मोटाई 5 cm है, उस समय बर्फ की मोटाई के घटने की दर (cm/min में), है :
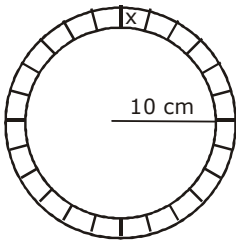
(1) $\dfrac{1}{18\pi}$ (2) $\dfrac{5}{6\pi}$ (3) $\dfrac{1}{36\pi}$ (4) $\dfrac{1}{54\pi}$

Sol. **1**

$$V = \frac{4}{3}\pi\left\{(10 + x)^3 - 10^3\right\}$$

$$\frac{dv}{dt} = 4\pi(10 + x)^2 \frac{dx}{dt}$$

$$50 = 4\pi(10 + x)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{50}{4\pi(15)^2} = \frac{50}{4\pi \times 15 \times 15} = \frac{1}{18\pi} \, cm/min$$

16. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adj A and C = 3A , then $\dfrac{|adjB|}{|C|}$ is equal to :

यदि आव्यूह $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, B = adj A तथा C = 3A हैं, तो $\dfrac{|adjB|}{|C|}$ का मान है :

(1) 8 (2) 2 (3) 16 (4) 72

Sol. **1**

$$\frac{|adj(adjA)|}{|c|} = \frac{|A|^{(n-1)^2}}{|c|}$$

$$d = \frac{(6)^{(3-1)^2}}{3^3 |A|} = \frac{6^4}{6 \times 27} = 8$$

17. If for all real triplets (a, b, c), $f(x) = a + bx + cx^2$; then $\int_0^1 f(x)dx$ is equal to :

यदि सभी वास्तविक त्रिकों (a, b, c) के लिए, $f(x) = a + bx + cx^2$ है, तो $\int_0^1 f(x)dx$ बराबर है :

(1) $2\left\{3f(1) + 2f\left(\frac{1}{2}\right)\right\}$ (2) $\frac{1}{2}\left\{f(1) + 3f\left(\frac{1}{2}\right)\right\}$ (3) $\frac{1}{3}\left\{f(0) + f\left(\frac{1}{2}\right)\right\}$ (4) $\frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$

Sol. **4**

$$\Rightarrow \int_0^1 (a + bx + cx^2)dx$$

$$\left(ax + \frac{bx^2}{2} + \frac{x^3}{3}\right)_0^1$$

$$a + \frac{b}{2} + \frac{c}{3} \Rightarrow \frac{6a + 3b + 2c}{6}$$

$$\Rightarrow \frac{1}{6}\left\{f(0) + f(1) + 4f\left(\frac{1}{2}\right)\right\}$$

18. If e_1 and e_2 are the eccentricities of the ellipse, $\dfrac{x^2}{18} + \dfrac{y^2}{4} = 1$ and the hyperbola, $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to :

यदि e_1 तथा e_2 क्रमशः दीर्घवृत्त $\dfrac{x^2}{18} + \dfrac{y^2}{4} = 1$ तथा अतिपरवलय $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ की उत्केंद्रताएँ है तथा (e_1, e_2) दीर्घवृत्त $15x^2 + 3y^2 = k$ पर स्थित एक बिन्दु है, तो k का मान है :

(1) 14   (2) 17   (3) 15   (4) 16

Sol. **4**

$$e_1^2 = 1 - \frac{4}{18} = \frac{7}{9}$$

$$= e_1 = \frac{\sqrt{7}}{3} \quad ....(1)$$

$$= e_2 = \sqrt{1 + \frac{4}{9}} = \frac{\sqrt{13}}{3} \quad ....(2)$$

$$\Rightarrow 15(e_1)^2 + 3(e_2)^2 = k$$

$$\Rightarrow 15\left(\frac{7}{9}\right) + 3\left(\frac{13}{9}\right) = k$$

$$\Rightarrow k = 16$$

19. In a box, there are 20 cards, out of which 10 are labelled as A and the remaining 10 are labelled as B. Cards are drawn at random one after the other and with replacement, till a second A-card is obtained. The probability that the second A-card appears before the third B-card is :

एक बक्से में 20 कार्ड हैं जिनमें से 10 पर A अंकित किया गया है तथा शेष 10 पर B अंकित किया गया है। बक्से में से यादच्छया एक के बाद एक (प्रतिस्थापना सहित) कार्ड तब तक निकाले गए जब तक कि दूसरा A से अंकित कार्ड न आ जाए। दूसरे A से अंकित कार्ड के तीसरे B से अंकित कार्ड से पहले आने की प्रायिकता है :

(1) $\dfrac{15}{16}$ (2) $\dfrac{11}{16}$ (3) $\dfrac{9}{16}$ (4) $\dfrac{13}{16}$

Sol. **2**
Required Probability

$$= P(A \cap A) + P(A \cap B \cap A) + P(B \cap A \cap A) + P(B \cap B \cap A \cap A) + P(A \cap B \cap B \cap A) + P(B \cap A \cap B \cap A)$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{3}{16} = \frac{11}{16}$$

20. If for some α and β in R, the intersection of the following three planes

x + 4y - 2z = 1
x + 7y - 5z = β
x + 5y + αz = 5

is a line in R^3, then $\alpha + \beta$ is equal to :

यदि R में किन्हीं α तथा β के लिए, निम्न तीन समतलों

x + 4y - 2z = 1

x + 7y - 5z = β

x + 5y + α z = 5

का प्रतिच्छेदन, R^3 में एक रेखा है, तो $\alpha + \beta$ का मान है :

(1) 0   (2) -10   (3) 10   (4) 2

Sol. **3**

This system has infinite soluition, so

$\Delta = 0$

$$\begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -5 \\ 1 & 5 & \alpha \end{vmatrix} = 0$$

$1(7\alpha + 25) - 4(\alpha + 5) - 2(5 - 7) = 0$

$7\alpha + 25 - 4\alpha - 20 + 4 = 0$

$3\alpha + 9 = 0$

$\alpha = -3$

also,

$$\Delta_3 = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 4 & \beta \\ 1 & 5 & 5 \end{vmatrix} = 0$$

$= 1(35 - 5\beta) - 4(5 - \beta) + (-2) = 0$

$= 13 - \beta = 0$

$\beta = 13$

21. If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in R)$ are coplanar and $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$, then the value of λ is

यदि सदिश $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$ तथा $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}$ $(a \in R)$ सहतलीय हैं तथा $3(\vec{p}.\vec{q})^2 - \lambda|\vec{r} \times \vec{q}|^2 = 0$ है, तो λ का मान है.............।

Sol. **1**

$$[\vec{p} \ \vec{q} \ \vec{r}] = \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & -1 & 0 \\ a & a+1 & a \\ 0 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 3a + 1 = 0 \Rightarrow a = -1/3$

$$\vec{p} = \frac{1}{3}\left(2\hat{i} - \hat{j} - \hat{k}\right), \; \vec{q} = \frac{1}{3}\left(-\hat{i} + 2\hat{j} - \hat{k}\right), \; \vec{r} = \frac{1}{3}\left(-\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$\vec{p}.\vec{q} = -\frac{1}{3} \;\&\; \vec{r} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{vmatrix} = \frac{1}{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$= 3 \times \frac{1}{9} - \lambda\left(\sqrt{\frac{3}{9}}\right)^2 = 0 \quad \Rightarrow \lambda = 1$$

22. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is

$(1 + x + x^2)^{10}$ के प्रसार में x^4 का गुणाक है .... ।

Sol. **615**

$$= \frac{10!}{p!q!r!}(1)^p (x)^q (x^2)^r$$

$$= q + 2r = 4, \; p + q + r = 10$$

p	q	R
8	0	2
6	4	0
7	2	1

$$= \frac{10!}{8!2!} + \frac{10!}{6!4!} + \frac{10!}{7!2!}$$

$$= 45 + 210 + 360 = 615$$

23. If for $x \geq 0, y = y(x)$ is the solution of the differential equation, $(x+1)dy = \left((x+1)^2 + y - 3\right)dx, y(2) = 0,$, then y(3) is equal to.........

यदि $x \geq 0$ के लिए $y = y(x)$, अवकल समीकरण $(x+1)dy = \left((x+1)^2 + y - 3\right)dx, y(2) = 0,$ का हल है, तो y(3) का मान है .... ।

Sol. **3**

$$(x+1)dy = \left[(x+1)^2 + (y-3)\right]dx$$

$$\frac{dy}{dx} - \frac{y}{x+1} = x + 1 - \frac{3}{x+1}$$

$$\Rightarrow \text{I.F.} = e^{-\int \frac{dx}{x+1}} = \frac{1}{x+1}$$

$$\Rightarrow \frac{y}{x+1} = \int \frac{x^2+2x-2}{(x+1)^2}\,dx$$

$$\Rightarrow \frac{y}{x+1} = x + \frac{3}{x+1} + C$$

Equation passes through (2,0) then, C = -3

$$= \frac{y}{x+1} = x + \frac{3}{x+1} - 3$$

Put x = 3 then, y = 3

24. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$ is ....
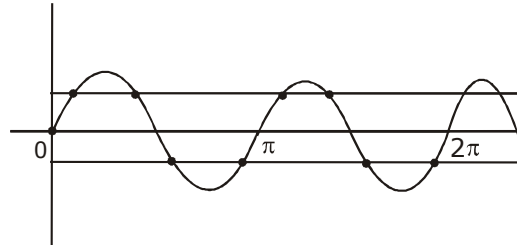
समीकरण $\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$ के अंतराल $[0, 2\pi]$ में भिन्न हलों की संख्या है ....

Sol. 8

$$\log_{\frac{1}{2}}|\sin x| = 2 - \log_{\frac{1}{2}}|\cos x|$$

$$= |\sin x||\cos x| = \frac{1}{4}$$

$$= \sin 2x = \pm \frac{1}{2}$$

25. The projection of the line segment joining the points (1, -1,3) and (2, -4, 11) on the line joining the points (-1,2,3) and (3, -2, 10) is ....
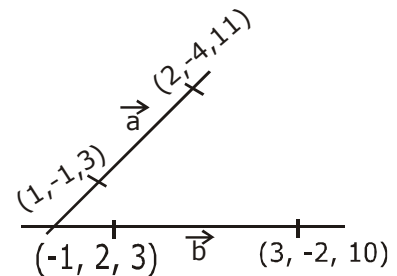
बिन्दुओं (1, -1,3) तथा (2, -4, 11) को मिलाने वाले रेखाखण्ड का बिन्दुओं (-1,2,3) तथा (3, -2, 10) को मिलाने वाली रेखा पर प्रक्षेप है .... ।

Sol. 8

$$\vec{a} = 1, -3, 8$$

$$\vec{b} = 4, -4, 7$$

$$= \frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{4+12+56}{\sqrt{16+16+49}} = 8$$