JEE MAIN JAN 2020

PAPER WITH SOLUTION

8th January 2020 _ SHIFT - 1

MATHEMATICS

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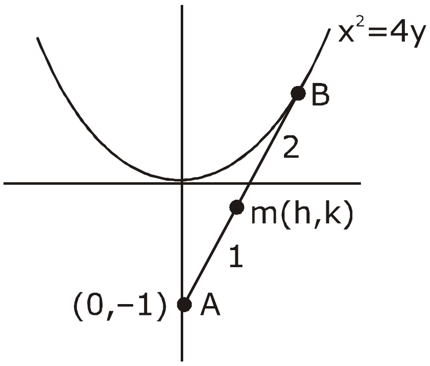
1. The locus of a point which divides the line segment joining the point (0,–1) and a point on the Parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is :

बिन्दु (0,–1) तथा परवलय $x^2 = 4y$ पर स्थित एक बिंदु को मिलाने वाले रेखाखण्ड का 1 : 2 के अनुपात में अंतः विभाजन करने वाले बिन्दु का बिंदुपथ है :

(A) $x^2 - 3y = 2$ (B) $4x^2 - 3y = 2$ (C) $9x^2 - 3y = 2$ (D) $9x^2 - 12y = 8$

Sol. **4**



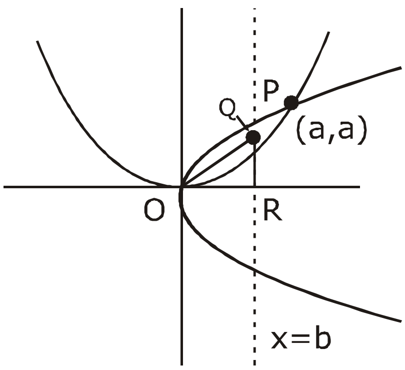
B : (3h, 3k + 2)
lies on $x^2 = 4y$
$(3h)^2 = 4(3k + 2)$
$9x^2 = 12y + 8$

2. For a > 0, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a pont P. Let the line x = b (0 < b < a) intersect the chord OP and the x - axis at points Q and R, respectively. If the line x = b bisects the area bounded by the curves, C_1, and C_2, and the area of $\Delta OQR = \frac{1}{2}$, then 'a' satisfies the equation :

a > 0 के लिए, माना वक्र $C_1 : y^2 = ax$ तथा $C_2 : x^2 = ay$ मूलबिन्दु O तथा एक बिन्दु P पर काटते हैं। माना रेखा x = b (0 < b < a) जीवा OP तथा x-अक्ष को क्रमशः बिन्दुओं Q तथा R पर काटती है। यदि रेखा x=b वक्रों C_1 तथा C_2 द्वारा परिबद्ध क्षेत्र को समद्विभाजित करती है तथा ΔOQR का क्षेत्रफल = $\frac{1}{2}$ है 'a' जिस समीकरण को संतुष्ट करता है वह है :

(A) $x^6 - 12x^3 + 4 = 0$ (B) $x^6 - 6x^3 + 4 = 0$ (C) $x^6 - 12x^3 - 4 = 0$ (D) $x^6 + 6x^3 - 4 = 0$

Sol. **1**



for p solve

$x^2 = ay$ & $y^2 = ax$

$x^4 = a^2 y^2$

$x^4 = a^2 ax \Rightarrow x = a, y = a$

P : (a,a)

area bounded by C_1 & C_2

$$\text{Area} = \frac{16}{3}\frac{a}{4}\cdot\frac{a}{4} = \frac{a^2}{3}$$

Now Q = (b,b)

$$\frac{1}{2}b^2 = \frac{1}{2}$$

b = 1

Now area bounded by

C_1, C_2 & x = 1 is

$$\int_0^1\left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \frac{1}{2}\frac{a^2}{3}$$

$$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$2\sqrt{a} - \frac{1}{a} = \frac{a^2}{2}$$

$$4\sqrt{a} - \frac{2}{a} = a^2$$

$$4\sqrt{a}\,a - 2 = a^3$$

$$(a^3 + 2)^2 = 4a\sqrt{a}$$

$a^6 + 4 + 4a^3 = 16a^3$

$a^6 - 12a^3 + 4 = 0$

3. Which one of the following is a tautology ?

निम्न में से कौन सा कथन एक पुनरूक्ति है ?

(A) $(P \wedge (P \to Q)) \to Q$ (B) $Q \to (P \wedge (P \to Q))$ (C) $P \wedge (P \vee Q)$ (D) $P \vee (P \wedge Q)$

Sol. 1

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$	$q \to p \wedge (p \to q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

4. Let

$f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1,$

$|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}(f(x))\right)$ and

$y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to :

माना

$f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1,$

$|x| > 1$ है। यदि $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}(f(x))\right)$ तथा

$y(\sqrt{3}) = \frac{\pi}{6}$ है, तो $y(-\sqrt{3})$ का मान है

(A) $\frac{2\pi}{3}$ (B) $-\frac{\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $\frac{\pi}{3}$

Sol. **BONUS**

$f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$

$f(x) = (\sin(\tan^{-1}x) + \cos(\tan^{-1}x))^2 - 1$

$f(x) = \sin(2\tan^{-1}x) = y$

$\sin^{-1}(f(x)) = \sin^{-1}(\sin(2\tan^{-1}x))$

case - I

$x < -1$

$\sin^{-1}(f(x)) = -\pi - 2\tan^{-1}x$

$$\frac{2dy}{dx} = \frac{d(\sin^{-1}(f(x))}{dx}$$

$2y = -\pi - 2\tan^{-1}x + c_1$

case - II

$x > 1$

$\sin^{-1}(f(x)) = \pi - 2\tan^{-1}x$

$2y = \pi - 2\tan^{-1}x + c_2$

$y(\sqrt{3}) = \frac{\pi}{6}$

$x > 1$

$$2\left(\frac{\pi}{6}\right) = \pi - 2 \times \frac{\pi}{3} + c_2$$

$c_2 = 0$

$y = (-\sqrt{3}) = ?$

$x < -1$

$2y = -\pi - 2\tan^{-1}(-\sqrt{3}) + c_1$

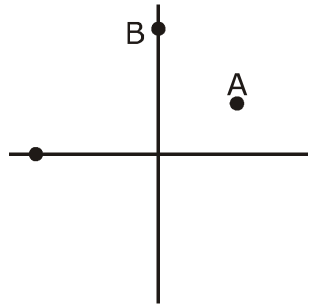
$$y = -\frac{\pi}{6} + \frac{c_1}{2} = \frac{-\pi}{6} + \lambda$$

5. Let two points be A(1,–1) and B(0,2). If a point P(x',y') be such that the area of ΔPAB = 5 sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is :

माना A(1,–1) तथा B(0,2) दो बिन्दु हैं। यदि एक बिन्दु P(x',y') इस प्रकार है कि ΔPAB का क्षेत्रफल= 5 वर्ग इकाई है तथा यह रेखा $3x + y - 4\lambda = 0$ पर स्थित है तो λ का एक मान है :

(A) –3 (B) 3 (C) 1 (D) 4

Sol. **2**



let P : $(a\ ,\ 4\lambda - 3a) \Rightarrow \Delta = \left|\frac{1}{2}\begin{vmatrix} a & 4\lambda - 3a & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix}\right| = 5$

$|a(-1-2) - (4\lambda - 3a)(1 - 0) + 1(2 + 0)| = 10$

$|-3a - 4\lambda + 3a + 2| = 10$

$|2 - 4\lambda| = 10 \Rightarrow 2 - 4\lambda = 10$

$\lambda = -2$

$2 - 4\lambda = -10 \Rightarrow \lambda = 3$

6. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is :

रेखाओं $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ तथा $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ के बीच की न्यूनतम दूरी है :

(A) $\frac{7}{2}\sqrt{30}$ (B) $3\sqrt{30}$ (C) 3 (D) $2\sqrt{30}$

Sol. **2**

$\vec{a} = <3,8,3>$

$\vec{b} = <-3, -7, 6>$

$\vec{p} = <3, -1, 1>$

$\vec{q} = <-3,2, 4>$

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = <-6, 15, 3>$$

$$\text{S.D.} = \left|\frac{(\vec{a}-\vec{b})(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$$= \left|\frac{(6,15,-3).(-6,-15,3)}{\sqrt{36+225+9}}\right| = \left|\frac{-36-225-9}{\sqrt{36+225+9}}\right| = \left|\frac{-270}{\sqrt{270}}\right| = \sqrt{270} = 3\sqrt{30}$$

7. If the equation, $x^2 + bx + 45 = 0$ $(b \in R)$ has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then :

यदि समीकरण $x^2 + bx + 45 = 0$ $(b \in R)$ के संयुग्मी सम्मिश्र मूल है जो $|z + 1| = 2\sqrt{10}$ को संतुष्ट करते हैं, तो

(A) $b^2 - b = 42$ (B) $b^2 + b = 72$ (C) $b^2 + b = 12$ (D) $b^2 - b = 30$

Sol. **4**

$x^2 + bx = 45 = 0$ $(b \in R)$

has roots $\alpha + i\beta$, $\alpha - i\beta$

sum of roots $= -b = 2\alpha$

product of roots $= 45 = \alpha^2 + \beta^2$

$|z + 1| = 2\sqrt{10}$

$(\alpha + 1)^2 + \beta^2 = 40$

$$\left(-\frac{b}{2}+1\right)^2 + 45 - \left(-\frac{b}{2}\right)^2 = 40$$

$$\frac{b^2}{4}+1-b+45 - \frac{b^2}{4} = 40$$

$b = 6$

8. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then :

यदि a, b तथा c क्रमशः ${}^{19}C_p$, ${}^{20}C_q$ तथा ${}^{21}C_r$ के अधिकतम मान हैं, तो :

(A) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$ (B) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$ (C) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$ (D) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$

Sol. **3**

${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$

$a = ({}^{19}C_p)_{max} \Rightarrow a = {}^{19}C_{10} = {}^{19}C_9$

$b = ({}^{20}C_p)_{max} \Rightarrow b = {}^{20}C_{10}$

$c = ({}^{21}C_r)_{max} \Rightarrow c = {}^{21}C_{10} = {}^{21}C_{11}$

$$\text{Now } \frac{a}{b} = \frac{{}^{19}C_9}{{}^{20}C_{10}} = \frac{19!}{9!10!} \times \frac{10!10!}{20!} = \frac{10}{20} = \frac{1}{2}\times\frac{11}{11}$$

$$\frac{b}{c} = \frac{{}^{20}C_{10}}{{}^{21}C_{11}} = \frac{20!}{10!10!}\times\frac{11!10!}{21!} = \frac{11}{21} = \frac{11\times 2}{21\times 2}$$

$$\frac{a}{b} = \frac{11}{22} \ \& \ \frac{b}{c}=\frac{22}{42}$$

$a : b : c : : 11 : 22 : 42$

9. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to :

10 प्रेक्षणों के माध्य तथा मानक विचलन क्रमशः 20 तथा 2 हैं। इन 10 प्रेक्षणों में से प्रत्येक को p से गुणा करने के पश्चात प्रत्येक में से q कम किया गया, जहाँ $p \neq 0$ तथा $q \neq 0$ है। यदि नए माध्य तथा मानक विचलन के मान अपने मूल मानों के आधे हैं, तो q का मान है :

(A) −10 (B) − 5 (C) 10 (D) −20

Sol. 4

If each observation is multiplied with p & then q is subtracted

New mean $\bar{x}_1 = p\bar{x} - q$

$\Rightarrow 10 = p(20) - q$ ...(A)

and new standard deviations.

$\sigma_2 = |p|\sigma_1 \Rightarrow 1 = |p|(B) \Rightarrow |p| = \frac{1}{2} \Rightarrow p = \pm\frac{1}{2}$

If $p = \frac{1}{2}$

then q = 0 (from equation (A))

If $p = -\frac{1}{2}$

q = – 20

10. Let A and B two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ?

माना A तथा B दो ऐसी स्वतंत्र घटनाएँ है कि $P(A) = \frac{1}{3}$ तथा $P(B) = \frac{1}{6}$ हैं, तो निम्न में से कौन सा सत्य है ?

(A) $P(A'/B') = \frac{1}{3}$ (B) $P(A/B) = \frac{2}{3}$ (C) $P(A/(A \cup B)) = \frac{1}{4}$ (D) $P(A/B') = \frac{1}{3}$

Sol. 4

$P(A) = \frac{1}{3}$ & $P(B) = \frac{1}{6}$

(A) $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1-\left(\frac{1}{3}+\frac{1}{6}-\frac{1}{3}.\frac{1}{6}\right)}{1-\frac{1}{6}} = \frac{1-\left(\frac{4}{9}\right)}{\frac{5}{6}} = \frac{5.6}{9.5} = \frac{2}{3}$

(B) $P(A/B) = \frac{\frac{1}{3}.\frac{1}{6}}{\frac{1}{6}} = \frac{1}{3}$

(C) $P(A/A \cup B) = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{3}{5}$

(D) $P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3}-\frac{1}{3}.\frac{1}{6}}{1-\frac{1}{6}} = \frac{\frac{6-1}{18}}{\frac{6-1}{6}} = \frac{1}{3}$

11. Let y = y(x) be a solution of the differential equation,

$\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$, $|x| < 1$

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to :

माना y = y(x), अवकल समीकरण

$\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0$, $|x| < 1$ का एक

हल है। यदि $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ है, तो $y\left(\frac{-1}{\sqrt{2}}\right)$ बराबर है :

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}$

Sol. **BONUS**

$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$

$\sin^{-1}y + \sin^{-1}x = \lambda$

$\frac{\pi}{3} + \frac{\pi}{6} = \lambda \Rightarrow \lambda = \frac{\pi}{2}$

Now $\sin^{-1}(y) - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sin^{-1}y = \frac{3\pi}{4}$ (Not possible)

12. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval [3,4], where $\alpha \in R$, then f"(c) is equal to :

यदि c एक बिन्दं है जिस पर, अन्तराल [3,4] में, फलन $f(x) = \log_e\left(\frac{x^2+\alpha}{7x}\right)$ पर रोले प्रमेय लागू होता है, जहाँ $\alpha \in R$ है, तो f"(c) बराबर है :

(A) $-\frac{1}{12}$ (B) $\frac{\sqrt{3}}{7}$ (C) $\frac{-1}{24}$ (D) $\frac{1}{12}$

Sol. **4**

f(C) = f(D) for Rolle's

$\Rightarrow \ln\left(\frac{9+\alpha}{21}\right) = \ln\left(\frac{16+\alpha}{28}\right)$

$= \frac{9+\alpha}{21} = \frac{16+\alpha}{28}$

$36 + 4\alpha = 48 + 3\alpha$

$\alpha = 12$

$f'(x) = \frac{x^2-12}{x\left(x^2+12\right)}$

$f'(c) = 0 \Rightarrow x = \pm\sqrt{12} \Rightarrow c = \sqrt{12}$

$f''(c) = \frac{1}{12}$

13. Let f: R $\rightarrow$ R be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P., then the minimum value of f(x) is :

माना f: R $\rightarrow$ R इस प्रकार है कि सभी $x \in R$ के लिए $(2^{1+x} + 2^{1-x})$, f(x) तथा $(3^x + 3^{-x})$ एक समांतर रेणी में है, तो f(x) का न्यूनतम मान है :

(A) 2 (B) 4 (C) 0 (D) 3

Sol. **4**

$(2^{1+x} + 2^{1-x})$, f(x), $(3^x + 3^{-x})$ are in A.P.

$$2f(x) = 2.2^x + \frac{2}{2^x} + 3^x + \frac{1}{3^x}$$

$$2f(x) = 2\left(2^x + \frac{1}{2^x}\right) + \left(3^x + \frac{1}{3^x}\right)$$

$2f(x) \geq 6$

$f(x) \geq 3$

14. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent ?

निम्न में से किस क्रमित युग्म (μ, δ) के लिए रैखिक समीकरण निकाय

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

असंगत है ?

(A) (1,0) (B) (4,6) (C) (3,4) (D) (4,3)

Sol. **4**

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix}$$

Δ 1(16–20) – 2(12– 20) + 3(12– 16)

= – 4 + 16 – 12 = 0

$\Delta_1 \neq 0$, $\Delta_2 \neq 0$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 3 \\ \mu & 4 & 5 \\ \delta & 4 & 4 \end{vmatrix} \neq 0$$

$= -4 - 2(4\mu - 5\delta) + 3(4\mu - 4\delta) \neq 0$

$\Rightarrow -4 - 8\mu + 10\delta + 12\mu - - 12\delta \neq 0$

$4\mu - 2\delta \neq 4$

$2\mu - \delta \neq 2$

Check option

15. If $\int \frac{\cos x dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}$ = f(x)(1+sin⁶x)$^{1/\lambda}$ + c where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

यदि $\int \frac{\cos x dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}}$ = f(x)(1+sin⁶x)$^{1/\lambda}$ + c है, जहाँ c एक समाकलन अचर है, तो $\lambda f\left(\frac{\pi}{3}\right)$ का मान है :

(A) $-\frac{9}{8}$ (B) -2 (C) $\frac{9}{8}$ (D) 2

Sol. **2**

sinx = t

$$\int \frac{dt}{t^3\left(1+t^6\right)^{2/3}} 7$$

$$= \int \frac{dt}{t^7\left(\frac{1}{t^6}+1\right)^{2/3}}$$

$$\frac{1}{t^6}+1 = z^3$$

$$= \int \frac{3z^2 dz}{-6\ z^2} = -\frac{1}{2}\int dz = \frac{-1}{2}\left(1+\frac{1}{\sin^6 x}\right)^{1/3} + C$$

$$= \frac{-1}{2\sin^2 x}\left(1+\sin^6 x\right)^{1/3} + C$$

$$f(x) = -\frac{1}{2}\text{cosec}^2 x \ \ \& \ \lambda = 3$$

$$\lambda f\left(\frac{\pi}{3}\right) = 3.\left(-\frac{1}{2}.\frac{4}{3}\right) = -2$$

16. Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v} = \hat{i}+\hat{j}+3\hat{k}$ and $\vec{w} = 2\hat{i}+\hat{j}+\hat{k}$ be 1 cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then cosθ can be :

माना एक समानतर षट्फलक, जिसके एक ही शीर्ष से होकर जाने वाले किनारे $\vec{u} = \hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v} = \hat{i}+\hat{j}+3\hat{k}$ तथा $\vec{w} = 2\hat{i}+\hat{j}+\hat{k}$ द्वारा प्रदत हैं, का आयतन 1 घन ईकाई है। यदि किनारों $\vec{u}$ तथा $\vec{w}$ के बीच का कोण θ है तो cosθ हो सकता है :

(A) $\frac{5}{7}$ (B) $\frac{7}{6\sqrt{6}}$ (C) $\frac{5}{3\sqrt{3}}$ (D) $\frac{7}{6\sqrt{3}}$

Sol. **4**

$\vec{u} = \langle 1, 1, \lambda \rangle$

$\vec{v} = \langle 1, 1, 3 \rangle$

$\vec{w} = \langle 2, 1, 1 \rangle$

volume = $[\vec{u}\ \vec{v}\ \vec{w}]$ = 1 = $\left| \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \right|$

λ = 2 or λ = 4

For λ = 2

$\cos\theta = \dfrac{2+1+2}{\sqrt{6}\sqrt{6}} = \dfrac{5}{6}$

for λ = 4

$\cos\theta = \dfrac{2+1+4}{\sqrt{6}\sqrt{18}} = \dfrac{7}{6\sqrt{3}}$

17. The inverse function of $f(x) = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1,1)$, is

$f(x) = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1,1)$ का व्युत्क्रम फलन है______

(A) $\dfrac{1}{4}(\log_8 e)\log_e\left(\dfrac{1+x}{1-x}\right)$ (B) $\dfrac{1}{4}\log_e\left(\dfrac{1+x}{1-x}\right)$

(C) $\dfrac{1}{4}(\log_8 e)\log_e\left(\dfrac{1-x}{1+x}\right)$ (D) $\dfrac{1}{4}\log_e\left(\dfrac{1-x}{1+x}\right)$

Sol. **1**

$f(x) = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1,1) \Rightarrow y = \dfrac{8^{4x} - 1}{8^{4x} + 1}$

$8^{4x}y + y = 8^{4x} - 1 \quad \Rightarrow 8^{4x}(y - 1\} = -1 - y$

$8^{4x} = \dfrac{y+1}{1-y} \Rightarrow 4x = \log_8\left(\dfrac{y+1}{1-y}\right)$

$x = \dfrac{1}{4}\log_8\left(\dfrac{1+y}{1-y}\right) \Rightarrow f^{-1}(x) = \dfrac{1}{4}\log_8\left(\dfrac{1+x}{1-x}\right)$

18. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to :

$\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ बराबर है :

(A) $\frac{1}{e^2}$ (B) e^2 (C) e (D) $\frac{1}{e}$

Sol. 1

$\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ (1^∞)

e^L

$L = \lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}-1\right)\frac{1}{x^2}$

$= \lim_{x\to 0}\frac{-4}{7x^2+2} = -2 \Rightarrow e^{-2}$

19. Let the line y = mx and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0,\beta)$, then β is equal to

माना रेखा y = mx तथा दीर्घवत्त $2x^2 + y^2 = 1$ प्रथम चतुर्थांश में स्थित एक बिन्दु P पर काटते है। यदि इस दीर्घवत का P पर अभिलंब, निर्देशांक अक्षो को क्रमशः $\left(-\frac{1}{3\sqrt{2}},0\right)$ तथा $(0,\beta)$ पर मिलता है, तो β का मान है

(A) $\frac{2}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{2\sqrt{2}}{3}$

Sol. 2

Let P be (x_1, y_1)

Equation of normal at P is $\frac{x}{2x_1}-\frac{y}{y_1}=-\frac{1}{2}$

It passes through $\left(-\frac{1}{3\sqrt{2}},0\right) \Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

So $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in Ist quadrant)

so $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

20. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true ?

(A) $f'(0) = -\frac{\pi}{2}$

(B) f is not differentiable at x = 0

(C) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(D) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

माना $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ है, तब निम्न में से कौनसा सत्य है ?

(A) $f'(0) = -\frac{\pi}{2}$

(B) x=0 पर f अवकलनीय नही है

(C) f', $\left(-\frac{\pi}{2}, 0\right)$ में ह्रासमान तथा $\left(0, \frac{\pi}{2}\right)$ में वर्धमान है

(D) f', $\left(-\frac{\pi}{2}, 0\right)$ में वर्धमान तथा $\left(0, \frac{\pi}{2}\right)$ ह्रासमान है।

Sol. **3**

$$f'(x) = x(\pi - \cos^{-1}(\sin|x|)) = x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right) = x\left(\frac{\pi}{2} + |x|\right)$$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{\pi}{2} + 2x & x \geq 0 \\ \frac{\pi}{2} - 2x & x < 0 \end{cases}$$

f'(x) is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(-\frac{\pi}{2}, 0\right)$

21. An urn contains 5 red marbles, 4 black marbles andd 3 white marbles, then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is :

एक कलश में 5 लाल मार्बल, 4 काले मार्बल तथा 3 सफेद मार्बल हैं, तो इसमें से 4 मार्बल इस प्रकार निकालते ताकि उनमें से अधिक से अधिक तीन लाल रंग के हों, के तरीकों की संख्या है

Sol. **490**

5R + 4B + 3W

$$\begin{pmatrix} 0 & R \\ 4 & U \end{pmatrix} + \begin{pmatrix} 1 & R \\ 3 & U \end{pmatrix} + \begin{pmatrix} 2 & R \\ 2 & U \end{pmatrix} + \begin{pmatrix} 3 & R \\ 1 & U \end{pmatrix}$$

$= {}^7C_4 + {}^5C_1.{}^7C_3 + {}^5C_2.\ {}^7C_2 + {}^5C_3{}^7C_1 = 490$

22. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ....

'a' का वह न्यूनतम धनात्मक मान, जिसके लिए समीकरण $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ के वास्तविक मूल है, है ....

Sol. **8**

$2x^2 + (a - 10)x + \frac{33}{2} = 2a$

$D \geq 0$

$\Rightarrow (a - 10)^2 - 4.2.\left(\frac{33}{2} - 2a\right) \geq 0 \Rightarrow a^2 + 100 - 20a - 132 + 16a \geq 0$

$a^2 - 4a - 32 \geq 0 \Rightarrow (a - 8)(a + 4) \geq 0$

$a \leq -4 \cup a \geq 8$

23. The sum $\sum_{k=1}^{20}(1+2+3+....+k)$ is

योगफल $\sum_{k=1}^{20}(1+2+3+....+k)$ है

Sol. **1540**

$\sum_{k=1}^{20}\frac{k(k+1)}{2} = \frac{1}{2}\left(\sum_{k=1}^{20}k^2 + \sum_{k=1}^{20}k\right)$ use formula $= 1540$

24. The number of all 3 × 3 matrices A, with enteries from the set {–1,0,1} such that the sum of the diagonal elements of AA^T is 3, is ....

ऐसे सभी 3 × 3 आव्यूहों A की संख्या, जिनके अवयव समुच्चय {–1,0,1} से हैं तथा AA^T के विकर्ण के अवयवों का योगफल 3 है, है...........

Sol. **672**

Let

$A = A = [a_{ij}]_{3\times 3}$

$tr(AA^T) = 3$

$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + ... + a_{33}^2 = 3$

possible cases

$$\left.\begin{array}{ll} 0,0,0,0,0,0,1,1,1 & \to 1 \\ 0,0,0,0,0,0,1,1,-1 & \to 1 \\ 0,0,0,0,0,0,1,1,-1 & \to 3 \\ 0,0,0,0,0,0,-1,1,-1 & \to 3 \end{array}\right\} {}^{9}C_6 \times 8 = 84\times 8 = 672$$

25. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y - axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to .....

माना वक्र $y^2 - 3x^2 + y + 10 = 0$ के बिन्दु P पर खींचा गया अभिलब, y - अक्ष को $\left(0, \frac{3}{2}\right)$ पर काटता है। यदि P पर वक्र की स्पर्श रेखा का ढाल m है, तो $|m|$ बराबर है...........

Sol. **4**

$P \equiv (x_1, y_1)$

$$2yy' - 6x + y' = 0 \Rightarrow y' = \left(\frac{6x_1}{1+2y_1}\right)$$

$$\left(\frac{\frac{3}{2} - y_1}{-x_1}\right) = -\left(\frac{1+2y_1}{6x_1}\right)$$

$9 - 6y_1 = 1 + 2y_1 \quad \Rightarrow y_1 = 1$

$\therefore x_1 = \pm 2$

$\therefore$ slope of tangent $= \left(\frac{\pm 12}{3}\right)$

$= \pm 4$

$\therefore |m| = 4$