JEE MAIN JAN 2020

PAPER WITH SOLUTION

7th January 2020 _ SHIFT - 1

MATHEMATICS

MOTION™
Nurturing potential through education
H.O. : 394, Rajeev Gandhi Nagar, Kota
www.motion.ac.in | ✉: info@motion.ac.in

1. Let $x^k + y^k = a^k$, (a, k > 0) and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is:

यदि $x^k + y^k = a^k$, (a, k > 0) तथा $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$,तो k बराबर है :

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) $\frac{1}{3}$

Sol. **1**

$k.x^{k-1}\ k.y^{k-1}\ \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$

$\frac{dy}{dx} + \left(\frac{x}{y}\right)^{k-1} = 0$

$k - 1 = -\frac{1}{3}$

$k = 1 - \frac{1}{3} = \frac{2}{3}$

2. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix A^{31} is equal to

यदि समीकरण $x^2 + x + 1 = 0$ का एक मूल α है तथा आव्यूह $A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\ 1 & \alpha & \alpha^2\\ 1 & \alpha^2 & \alpha^4\end{bmatrix}$ है, तो आव्युह A^{31} बराबर है

(1) A (2) A^3 (3) A^2 (4) I_3

Sol. **2**

$$A^2 = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & \omega & \omega^2\\ 1 & \omega^2 & \omega\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$$

$\Rightarrow A^4 = 1$

$\Rightarrow A^{31} = A^{28} \times A^3 = A^3$

3. Let α and β be two real roots of the equation $(k + 1)\tan^2x - \sqrt{2}\,.\,\lambda\tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. if $\tan^2(\alpha+\beta) = 50$, then a value of λ is:

माना समीकरण $(k + 1)\tan^2x - \sqrt{2}\,.\,\lambda\tan x = (1 - k)$,$k(\neq -1)$,$\lambda \in R$) के α तथा β दो वास्तविक मूल हैं। यदि $\tan^2(\alpha+\beta) = 50$ है, तो λ का एक मान हैं –

(1) 5 (2) 10 (3) $10\sqrt{2}$ (4) $5\sqrt{2}$

Sol. 2

$(k + 1)\tan^2 x - \sqrt{2}\lambda \tan x + (k - 1) = 0$

$\tan\alpha + \tan\beta = \dfrac{\sqrt{2}\lambda}{k+1}$

$\tan\alpha \times \tan\beta = \dfrac{k-1}{k+1}$

$$\tan(\alpha + \beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$$

$\tan^2(\alpha + \beta) = \dfrac{\lambda^2}{2} = 50$

$\lambda = 10$

4. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}\ (\alpha, \beta \in R)$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

एक सदिश $\vec{a} = a\hat{i} + 2\hat{j} + \beta\hat{k}\ (\alpha, \beta \in R)$ उस समतल में, जिसमें दोनों सदिश $\vec{b} = \hat{i} + \hat{j}$ तथा $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$ स्थित है, स्थित है,। यदि $\vec{a}$ सदिशों $\vec{b}$ और $\vec{c}$ के बीच के कोण को समद्विभाजित करता है तो :

(1) $\vec{a}\cdot\hat{i} + 1 = 0$ (2) $\vec{a}\cdot\hat{k} + 2 = 0$

(3) $\vec{a}\cdot\hat{i} + 3 = 0$ (4) $\vec{a}\cdot\hat{k} + 4 = 0$

Sol. 2

angle bisector can be $\vec{a} = \vec{a} = \lambda(\vec{b} + \vec{c})$ or $\vec{a} = \mu(\vec{b} - \vec{c})$

$$\vec{a} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{i}+\hat{j}+4\hat{k}}{3\sqrt{2}}\right) = \frac{\lambda}{3\sqrt{2}}[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}] = \frac{\lambda}{3\sqrt{2}}[4\hat{i}+2\hat{j}+4\hat{k}]$$

compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\dfrac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$

$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Not in option so now consider $\vec{a} = \mu\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}} - \dfrac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a} = \dfrac{\mu}{3\sqrt{2}}\left(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k}\right)$

$= \dfrac{\mu}{3\sqrt{2}}\left(2\hat{i}+4\hat{j}-4\hat{k}\right)$

compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2} \Rightarrow \vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a}\cdot\hat{k} + 2 = 0$

5. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where z = x + iy, then the point (x, y) lies on a:

यदि $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, जहाँ z = x + iy, तो बिन्दु (x, y) स्थित है :

(1) straight line whose slope is $\frac{3}{2}$.

(2) circle whose center is $\left(-\frac{1}{2},-\frac{3}{2}\right)$.

(3) circle whose diameter is $\frac{\sqrt{5}}{2}$

(4) straight line whose slope is $-\frac{2}{3}$.

(1) एक सरल रेखा पर, जिसका ढाल $\frac{3}{2}$ है।

(2) एक वत पर, जिसका केन्द्र बिन्दु $\left(-\frac{1}{2},-\frac{3}{2}\right)$.

(3) एक वत्त पर, जिसका व्यास $\frac{\sqrt{5}}{2}$ है।

(4) एक सरल रेखा पर, जिसका ढाल $-\frac{2}{3}$ है.

Sol. **3**

z = x + iy

$$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1)+iy}{2(x+iy)+i} = \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$$

$$\text{Re}\left(\frac{z+1}{2z+i}\right) = \frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$$

$\Rightarrow 2x^2 + 2y^2 - 2x + y = 4x^2 + 4y^2 + 4y + 1$

$\Rightarrow 2x^2 + 2y^2 + 2x + 3y + 1 = 0$

$\Rightarrow x^2 + y^2 - x + \frac{3}{2}y + \frac{1}{2} = 0$

Cricle with center $\left(-\frac{1}{2},-\frac{3}{4}\right)$

$r = \sqrt{\frac{1}{4}+\frac{9}{16}-\frac{1}{2}} = \sqrt{\frac{4+9-8}{16}} = \frac{\sqrt{5}}{4}$

6. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is:

छः अंकों वाली सभी संख्याओं की कुल संख्या जिनमें केवल तथा सभी पाँच अंक 1, 3, 5, 7 और 9 ही हों, है:

(1) $\frac{1}{2}(6!)$ (2) 6! (3) $\frac{5}{2}(6!)$ (4) 5^6

Sol. **3**

1, 3, 5, 7, 9

For digit to repeat we have ${}^5C_1$ choices

and six digits can be arranged in $\frac{\lfloor 6}{\lfloor 2}$ ways.

Hence total such numbers = $\frac{5\lfloor 6}{\lfloor 2} = \frac{5.6!}{2}$

7. If y = mx + 4 is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to:

यदि y = mx + 4 दोनो परवलयों, $y^2 = 4x$ तथा $x^2 = 2by$ को स्पर्श करती है, तो b बराबर है :

(1) 128 (2) -32 (3) -128 (4) -64

Sol. **3**

$y = mx + 4$ .....(i)

$y^2 = 4x$ tangent $y = mx + \frac{a}{m} \Rightarrow y = mx + \frac{1}{m}$ ....(ii)

from (i) and (ii)

$4 = \frac{1}{m} \Rightarrow m = \frac{1}{4}$

So line $y = \frac{1}{4}x + 4$ is also tangent to parabola $x^2 = 2by$, so solve

$x^2 = 2b\left(\frac{x+16}{4}\right) \Rightarrow 2x^2 - bx - 16b = 0 \Rightarrow d = 0$

$\Rightarrow b^2 - 4 \times 2 \times (-16b) = 0 \Rightarrow b^2 + 32 \times 4b = 0$

b = -128, b = 0 (not possible)

8. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

यदि एक दीर्घवत्त की नाभियों के बीच की दूरी 6 है तथा इसकी नियताओं के बीच की दूरी 12 है, तो इसकी नाभिलम्ब जीवा की लम्बाई है :

(1) $3\sqrt{2}$ (2) $2\sqrt{3}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\sqrt{3}$

Sol. **1**

$2ae = 6$ and $\frac{2a}{e} = 12$

$\Rightarrow ae = 3$ and $\frac{a}{e} = 6$

$\Rightarrow a^2 = 18$

$\Rightarrow b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

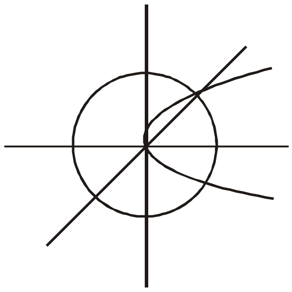
$\Rightarrow$ L.R. = $\frac{2b^2}{a} = \frac{2 \times 9}{3\sqrt{2}} = 3\sqrt{2}$

9. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is:
वृत्त $x^2 + y^2 = 2$ द्वारा परिबद्ध क्षेत्र का वह क्षेत्रफल जो परवलय $y^2 = x$ तथा सरल रेखा $y = x$ द्वारा परिबद्ध क्षेत्र में नही है, है

(1) $\frac{1}{3}(6\pi - 1)$ (2) $\frac{1}{6}(12\pi - 1)$ (3) $\frac{1}{3}(12\pi - 1)$ (4) $\frac{1}{6}(24\pi - 1)$

Sol. **2**

Total area - enclosed area



$$2\pi - \int_0^1 \left[\sqrt{x} - x\right]dx$$

$$2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$2\pi - \left(\frac{2}{3} - \frac{1}{2}\right) \Rightarrow 2\pi - \left(\frac{1}{6}\right) \Rightarrow \frac{12\pi - 1}{6}$$

10. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value of k when k consecutive heads are obtained for k = 3, 4, 5, otherwise X takes the value -1. Then the expected value of X, is:
एक अनभिनत सिक्के को पाँच बार उछाला जाता हैं। माना, एक चर X को, k = 3, 4, 5 के लिए, मान k दिया जाता है जब सिक्के पर क्रमागत k चित आएं तथा अन्य सभी स्थितियों में X का मान −1 है, तो X का अपेक्षित मान है।

(1) $\frac{1}{8}$ (2) $-\frac{3}{16}$ (3) $\frac{3}{16}$ (4) $-\frac{1}{8}$

Sol. **1**

K	0	1	2	3	4	5
P(k)	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = no. of times head occur consecutively
now expected value is

$$= \sum xP(k) = (-1)\times\frac{1}{32} + (-1)\times\frac{12}{32} + (-1)\times\frac{11}{32} + 3\times\frac{5}{32} + 4\times\frac{2}{32} + 5\times\frac{1}{32} = \frac{1}{8}$$

11. Let the function, $f:[-7, 0] \rightarrow R$ be continuous on [-7,0] and differentiable on (-7, 0). If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7,0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval:\
माना फलन $f:[-7, 0] \rightarrow R$, [-7,0] पर संतत है तथा (-7, 0) पर अवकलनीय है। यदि $f(-7) = -3$ और सभी $x \in (-7,0)$ के लिए, $f'(x) \le 2$ है, तो ऐसे सभी फलनों f के लिए $f(-1) + f(0)$ जिस अन्तराल में है, वह है :
(1) $(-\infty, 20]$ (2) [-3, 11] (3) $(-\infty, 11]$ (4) [-6, 20]

Sol. **1**

Lets use LMVT for $x \in [-7, -1]$

$$\frac{f(-1) - f(-7)}{(-1+7)} \le 2$$

$$\frac{f(-1)+3}{6} \le 2 \Rightarrow f(-1) \le 9$$

Also use LMVT for $x \in [-7,0]$

$$\frac{f(0)-f(-7)}{(0+7)} \le 2$$

$$\frac{f(0)+3}{7} \le 2 \Rightarrow f(0) \le 11$$

$\therefore f(0) + f(-1) \le 20$

12. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots + 49^2 + 49 + 1$, is:

सबसे बड़ी धन पूर्णांक संख्या k, जिसके लिए $49^k + 1$ योगफल $49^{125} + 49^{124} + \ldots + 49^2 + 49 + 1$, का एक गुणनखंड है, है :

(1) 32 (2) 63 (3) 60 (4) 65

Sol. **2**

$$\frac{(49)^{126}-1}{48} = \frac{\left((49)^{63}+1\right)\left(49^{63}-1\right)}{48}$$

13. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to:

यदि $g(x) = x^2 + x - 1$ तथा $(gof)(x) = 4x^2 - 10x + 5$, तो $f\left(\frac{5}{4}\right)$ बराबर है :

(1) $-\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{1}{2}$

Sol. **1**

$g(f(x)) = f^2(x) + f(x) - 1$

$$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10.\frac{5}{4} + 5 = \frac{-5}{4}$$

$$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$-\frac{5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$$

$$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$$

$$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$$

$$f\left(\frac{5}{4}\right) = -\frac{1}{2}$$

14. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:
यदि एक समतल P तीन बिन्दुओं (2, 1, 0), (4, 1, 1) और (5, 0, 1) से होकर जाता है, तथा कोई और बिन्दु R (2, 1, 6) है, तो समतल P में R का प्रतिबिम्ब (image) है :
(1) (6, 5, - 2)   (2) (4, 3, 2)   (3) (6, 5, 2)   (4) (3, 4, -2)

Sol. **1**

Plane is x + y - 2z = 3 $\Rightarrow \frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \frac{-2(2+1-12-3)}{6}$

$\Rightarrow$ (x, y, z) = (6, 5, -2)

15. If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers, then $\frac{1}{a+b}\int_a^b x\left(f(x)+f(x+1)\right)dx$ is equal to:
यदि सभी x के लिए, $f(a + b + 1 - x) = f(x)$ है, जबकि a तथा b स्थिर (fixed) धन वास्तविक संख्याएँ हैं, तो $\frac{1}{a+b}\int_a^b x\left(f(x)+f(x+1)\right)dx$ बराबर है :

(1) $\int_{a-1}^{b-1} f(x)dx$ (2) $\int_{a+1}^{b+1} f(x)dx$ (3) $\int_{a-1}^{b-1} f(x+1)dx$ (4) $\int_{a+1}^{b+1} f(x+1)dx$

Sol. **2 & 3**

$$I = \frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx \quad .....(i)$$

$x \to a + b - x$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(x+1)+f(x)]dx \quad ....(ii)$$

[$\because$ put $x \to x + 1$ in given equation]
(i) + (ii)

$$2I = \int_a^b [f(x+1)+f(x)]dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x)dx$$

$$\int_a^b f(a+b+1-x)dx + \int_a^b f(x)dx$$

$$2I = 2\int_a^b f(x)dx$$

$$I = \int_a^b f(x)dx$$

substitute x = z + 1

$$I = \int_{a-1}^{b-1} f(z+1)dz$$

3 Ans.

OR

$$I = \frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx \qquad .....(i)$$

$x \to a + b - x$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$$

$$I = \frac{1}{(a+b)}\int_a^b (a+b-x)[f(x+1)+f(x)]dx \qquad ....(ii)$$

[$\because$ put $x \to x + 1$ in given equation]

(i) + (ii)

$$2I = \int_a^b [f(x+1)+f(x)]dx$$

$$2I = \int_a^b f(a+b-x)dx + \int_a^b f(x+1)dx$$

$$2I = \int_a^b f(x+1)dx + \int_a^b f(x+1)dx$$

$$I = \int_a^b f(x+1)dx$$

$$I = \int_{a+1}^{b+1} f(x)dx$$

Ans. 2

16. If the system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0

2x + 4cy + cz = 0,

where a, b, c $\in$R are non-zero distinct; has a non-zero solution, then:

(1) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. (2) a + b + c = 0

(3) a,b,c are in A.P. (4) a, b, c are in G.P.

यदि निम्न रैखिक समीकरण निकाय

2x + 2ay + az = 0

2x + 3by + bz = 0
2x + 4cy + cz = 0,
जहाँ a, b, तथा c विभिन्न शून्येतर वास्तविक संख्याएँ है का एक शून्येतर हल है, तो :–

(1) $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ समान्तर श्रेढ़ी में है। (2) a + b + c = 0
(3) a,b,c समान्तर श्रेढ़ी में है। (4) a, b, c गुणोत्तर श्रेढ़ी में है।

Sol. **1**

For non - trivial solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

(3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0
- bc + 2ac - ab = 0
a, b, c in H.P.

$\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

17. If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is:

यदि $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}, \alpha \in \left(\frac{3\pi}{4}, \pi\right)$, है, तो $\alpha = \frac{5\pi}{6}$ पर $\frac{dy}{d\alpha}$ का मान है :

(1) $\frac{4}{3}$ (2) -4 (3) 4 (4) $-\frac{1}{4}$

Sol. **3**

$y = \sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{1}{\sin^2\alpha}} = \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha} = |1+\cot\alpha| = -1 - 1\cot\alpha$

$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha \quad \Rightarrow \left(\frac{dy}{d\alpha}\right)_{at\,\alpha = \frac{5\pi}{6}}$ will be = 4

18. If y = y(x) is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that y(0) = 0, then y(1) is equal to:

यदि अवकलन समीकरण, $e^y\left(\frac{dy}{dx} \quad 1\right) = e^x$, जबकि y(0) = 0, का हल y=y(x) है, तो y(1) बराबर है:

(1) $1 + \log_e 2$ (2) $\log_e 2$ (3) $2 + \log_e 2$ (4) 2e

Sol. **1**

$e^y = t$

$e^y \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - t = e^x$

IF = $e^{\int -1dx} = e^{-x}$

$t(e^{-x}) = \int e^x . e^{-x} dx$

$e^{y-x} = x + c$

Put x = 0, y = 0 then C = 1

$e^{y-x} = x + 1$

y = x - log (x - 1)

at x = 1 , y = 1 + $\log_e(2)$

19. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to:

तर्कसंगत कथन $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ निम्न कथनों में से किसके तुल्य है ?

(1) p (2) q (3) ~p (4) ~q

Sol. **3**

p	q	p → q	~p	q→~p	(p→q) ∧(p→~q)
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Ciearly $(p \rightarrow q) \wedge (q \rightarrow \sim p)$ is equivalent to ~p

20. Five numbers are in A.P. whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

पाँच संख्याएँ समान्तर श्रेढ़ी में है, जिनका योगफल 25 तथा गुणनफल 2520 है। यदि इन पाँच संख्याओं में से एक $-\frac{1}{2}$ है, तो इनमें सबसे बड़ी संख्या है

(1) 16 (2) 7 (3) $\frac{21}{2}$ (4) 27

Sol. **1**

Let terms be a - 2d, a, a - d, a + d, a + 2d

sum = 25 ⇒ 5a = 25 ⇒ a = 5

product = 2520

(5 - 2d) (5 - d) + 5 (5 + d) (5 + 2d) = 2520

$\Rightarrow (25-4d^2) (25 - d^2 = 504$

$\Rightarrow 625 - 100d^2 - 25d^2 + 4d^4 = 504$

$\Rightarrow 4d^4 - 125d^2 + 121 = 0$

$\Rightarrow 4d^4 - 121d^2 - 4d^2 + 121 = 0$

$\Rightarrow (d^2 - 1)(4d^2 - 121) = 0$

$\Rightarrow d = \pm 1, \qquad d = \pm \frac{11}{2}$

$d = \pm 1,$ does not give $\frac{-1}{2}$ as a term

$\therefore d = \frac{11}{2}$

$\therefore$ largest term = 5 + 2d = 5 + 11 = 16

21. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + ...+x^{2n})(1 - x + x^2 - x^3 + ...x^{2n})$ is 61, then n is equal to ________.

यदि गुणनफल $(1 + x + x^2 + ...+x^{2n})(1 - x + x^2 - x^3 + ...x^{2n})$ में, x के सभी समघातों वाले गुणांको का योगफल 61 है, तो n बराबर है ________.

Sol. **30**

Sol. **30**

Let $(1 - x + x^2.....)(1 + x + x^2......) = a_0 + a_1x + a_2x^2 + ...$

Put x = 1

$1(2n + 1) = a_0 + a_1 + a_2 + ...a_{2n}$ .....(i)

put x = -1

$(2n + 1) \times 1 = a_0 - a_1 + a_2 + ...a_{2n}$ ....(ii)

Form (i) and (ii)

$4n + 2 = 2(a_0 + a_2 + ...)$

$= 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30$

22. Let S be the set of points where the function, $f(x) = |2-|x-3||, x \in R$ is not differentiable. Then $\sum_{x \in S} f(f(x))$ is equal to______.

यदि S उन सभी बिन्दुओं का समुच्चय है, जिनके लिए फलन, $f(x) = |2-|x-3||, x \in R$ अवकलनीय नही है, तो $\sum_{x \in S} f(f(x))$ बराबर है______.

Sol. **3**

$\because$ f(x) is non differentiable at x = 1, 3, 5

$\sum f(f(x)) = f(f(1)) + f(f(3)) + f(f(5))$

= 1 + 1 + 1

= 3

23. Let A(1, 0), B(6, 2) and $C\left(\frac{3}{2}, 6\right)$ be the vertices of a triangle ABC. If P is a Point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6}, -\frac{1}{3}\right)$, is_________.

माना A(1, 0), B(6, 2) तथा $C\left(\frac{3}{2}, 6\right)$, एक त्रिभुज ABC के शीर्ष बिन्दु है। यदि एक बिन्दु P, ΔABC के अन्दर इस प्रकार है, कि त्रिभुजों APC, APB और BPC के क्षेत्रफल बराबर हैं, तो रेखाखण्ड PQ, जबकि बिन्दु $Q\left(-\frac{7}{6}, -\frac{1}{3}\right)$ है, की लम्ब ई है_________.

Sol. **5**

P will be centriod of ΔABC

$$P = \left(\frac{17}{6}, \frac{8}{3}\right) \Rightarrow PQ = \sqrt{\left(\frac{24}{6}\right)^2 + 3^2}$$

$= 5$

24. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to_____.

यदि प्रथम n प्राकत संख्याओं का प्रसरण 10 है और प्रथम m सम–प्राकत संख्याओं का प्रसरण 16 है, तो m+n बराबर है_____.

Sol. **18**

$$\text{var}(1, 2, \ldots\ldots n) = 10 \Rightarrow \frac{1^2+2^2+\ldots\ldots\ldots+n^2}{n} - \left(\frac{1+2+\ldots\ldots\ldots+n}{n}\right)^2 = 10$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$$

$\Rightarrow n^2 - 1 = 120$ $\Rightarrow n = 11$

$\text{var}(2, 4, 6\ldots\ldots\ldots,2m) = 16 \Rightarrow \text{var}(1, 2,\ldots\ldots m) = 4$

$\Rightarrow m^2 - 1 = 48 \Rightarrow m = 7 \Rightarrow m + n = 18$

25. $\lim\limits_{x\to 2} \dfrac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ is equal to_______.

$\lim\limits_{x\to 2} \dfrac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$ बराबर है_______.

Sol. **36**

Put $3^{\frac{x}{2}} = t$

$$\Rightarrow \lim_{t\to 3} \frac{t^2 + \frac{27}{t^2} - 12}{-\frac{3}{t^2} + \frac{1}{t}} = \lim_{t\to 3} \frac{(t^2-3)(t^2-9)}{(-3+t)} = \lim_{t\to 3}(3+t)(t^2-3) = 36$$