18th March 2021 | Shift - 1
MATHEMATICS

JEE I NEET I Foundation

SECTION – A

1. If the functions are defined as $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then what is the common domain of the following functions : f + g, f – g, f/g, g/f, g – f where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \frac{f(x)}{g(x)}$

(1) $0 < x \le 1$
(2) $0 \le x < 1$
(3) $0 \le x \le 1$
(4) $0 < x < 1$

Ans. **(4)**

Sol. $f + g = \sqrt{x} + \sqrt{1-x}$

$\Rightarrow x \ge 0 \ \& \ 1 - x \ge 0 \Rightarrow x \in [0, 1]$

$f - g = \sqrt{x} - \sqrt{1-x}$

$\Rightarrow x \ge 0 \ \& \ 1 - x \ge 0 \Rightarrow x \in [0, 1]$

$f/g = \frac{\sqrt{x}}{\sqrt{1-x}}$

$\Rightarrow x \ge 0 \ \& \ 1 - x > 0 \Rightarrow x \in [0, 1)$

$g/f = \frac{\sqrt{1-x}}{\sqrt{x}}$

$\Rightarrow 1 - x \ge 0 \ \& \ x > 0 \Rightarrow x \in (0, 1]$

$g - f = \sqrt{1-x} - \sqrt{x}$

$\Rightarrow 1 - x \ge 0 \ \& \ x \ge 0 \Rightarrow x \in [0, 1]$

$\Rightarrow x \in (0, 1)$

2. Let α, β, γ be the roots of the equations, $x^3 + ax^2 + bx + c = 0$, (a, b, c $\in$ R and a, b and a, b $\ne$ 0). If the system of the equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$; $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solutions, then the value of $\frac{a^2}{b}$ is

(1) 5
(2) 1
(3) 0
(4) 3

Ans. **(4)**

Sol. $x^3 + ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$

For non-trivial solutions,

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma = 0$

$(\alpha + \beta + \gamma)\left[(\alpha + \beta + \alpha)^2 - 3\left(\sum \alpha\beta\right)\right] = 0$

$(-a)\ [a^2 - 3b] = 0$

$a^2 = 3b \quad (\because a \neq 0)$

$\Rightarrow \dfrac{a^2}{b} = 3$

3. If the equation $a|z|^2 + \overline{\alpha z + \alpha \bar{z}} + d = 0$ represents a circle where a, d are real constants, then which of the following condition is correct?

(1) $|\alpha|^2 - ad \neq 0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $\alpha = 0, a, d \in R^+$

(4) $|\alpha|^2 - ad \geq 0$ and $a \in R$

Ans. **(2)**

Sol. $a|z|^2 + \alpha\bar{z} + \bar{\alpha}z + d = 0$

$z\bar{z} + \left(\dfrac{\alpha}{a}\right)\bar{z} + \left(\dfrac{\bar{\alpha}}{a}\right)z + \dfrac{d}{a} = 0$

Centre $= -\dfrac{\alpha}{a}$

$r = \sqrt{\left|\dfrac{\alpha}{a}\right|^2 - \dfrac{d}{a}}$

$\Rightarrow \left|\dfrac{\alpha}{a}\right|^2 \geq \dfrac{d}{a}$

$\Rightarrow |\alpha|^2 \geq ad$

4. $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+.....+\frac{1}{(201)^2-1}$ is equal to:

(1) $\frac{101}{404}$

(2) $\frac{101}{408}$

(3) $\frac{99}{400}$

(4) $\frac{25}{101}$

Ans. **(4)**

Sol. $S=\sum_{r=1}^{100}\frac{1}{(2r+1)^2-1}=\sum_{r=1}^{100}\frac{1}{(2r+2)\cdot 2(r)}$

$\therefore \quad S=\frac{1}{4}\sum_{r=1}^{100}\left[\frac{1}{r}-\frac{1}{r+1}\right]$

$S=\frac{1}{4}\left(\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+....+\left(\frac{1}{100}-\frac{1}{101}\right)\right)$

$\therefore \quad S=\frac{1}{4}\left[\frac{100}{101}\right]=\frac{25}{101}$

5. The number of integral values of m so that the abscissa of point of intersection of lines 3x + 4y = 9 and y = mx + 1 is also an integer, is:

(1) 3

(2) 2

(3) 1

(4) 0

Ans. **(2)**

Sol. 3x + 4(mx + 1) = 9

x(3 + 4m) = 5

$x=\frac{5}{(3+4m)}$

(3 + 4m) = ±1, ±5

4m = – 3 ± 1, –3 ± 5

4m = – 4, –2, –8, 2

m = –1, $-\frac{1}{2}$, – 2, $\frac{1}{2}$

Two integral value of m

6. The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, are:

(1) $\frac{\pi}{6}, \frac{5\pi}{6}$

(2) $\frac{7\pi}{12}, \frac{11\pi}{12}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$

(4) $\frac{\pi}{12}, \frac{\pi}{6}$

Ans. **(2)**

Sol. $R_1 \to R_1 + R_2$

$$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$\therefore\ 2 + 8\sin 2x - 4\sin 2x = 0$

$\Rightarrow \sin 2x = -\frac{1}{2}$ $\qquad \Rightarrow x = \frac{7\pi}{12}, \frac{11\pi}{12}$

7. If $f(x) = \begin{cases} \frac{1}{|x|} & ;\ |x| \geq 1 \\ ax^2 + b & ;\ |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively:

(1) $\frac{5}{2}, -\frac{3}{2}$

(2) $-\frac{1}{2}, \frac{3}{2}$

(3) $\frac{1}{2}, \frac{1}{2}$

(4) $\frac{1}{2}, -\frac{3}{2}$

Ans. **(2)**

Sol. f(x) is continuous at x = 1 $\Rightarrow$ 1 = a + b

f(x) is differentiable at x = 1 $\Rightarrow$ –1 = 2a

$\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$

8. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If with respect to new system, $\vec{a}$ has components p +1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1

(2) –1

(3) $\frac{4}{5}$

(4) $-\frac{5}{4}$

Ans. **(2)**

Sol. $\left|\vec{a}\right|_{old} = \left|\vec{a}\right|_{new}$

$(3p)^2 + 1 = (P+1)^2 + 10$

$9p^2 - p^2 - 2p - 10 = 0$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$4p^2 - 5p + 4p - 5 = 0$

$(4p - 5)(p + 1) = 0$

$p = \frac{5}{4}, -1$

9. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664

(2) 122664

(3) 122234

(4) 22264

Ans. **(1)**

Sol.

1	2	2	3
1	2	3	2
1	3	2	2
3	1	2	2
3	2	1	2
3	2	2	1
2	1	3	2
2	3	1	2
2	2	1	3
2	2	3	1
2	3	2	1
2	1	2	3

2 6 6 6 4

10. Choose the correct statement about two circles whose equations are given below:

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have no meeting point

(2) circles have two meeting points

(3) circles have only one meeting point

(4) circles have same centre

Ans. **(3)**

Sol. Let $S_1 : x^2 + y^2 - 10x - 10y + 41 = 0$

$\Rightarrow (x - 5)^2 + (y - 5)^2 = 9$

Centre $(C_1) = (5, 5)$

Radius $r_1 = 3$

$S_2 : x^2 + y^2 - 22x - 10y + 137 = 0$

$\Rightarrow (x - 11)^2 + (y - 5)^2 = 9$

Centre $(C_2) = (11, 5)$

radius $r_2 = 3$

distance $(C_1 C_2) = \sqrt{(5-11)^2 + (5-5)^2}$

distance $(C_1 C_2) = 6$

$\because r_1 + r_2 = 3 + 3 = 6$

$\therefore$ circles touch externally

Hence, circle have only one meeting point.

11. If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199)$, then the slope of the line passing through (α, β) and origin is:

(1) 510

(2) 550

(3) 540

(4) 530

Ans. **(2)**

Sol. RHS = $\sum_{r=0}^{99} (100 - r)(100 + r)$

$= (100)^3 - \frac{99 \times 100 \times 199}{6} = (100)^3 - (1650)199$

LHS = $(100)^{\alpha} - (199)\beta$

So, $\alpha = 3$, $\beta = 1650$

Slope = $\tan\theta = \frac{\beta}{\alpha}$

$\tan\theta = 550$

12. The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ is equal to:

(1) $3+2\sqrt{3}$

(2) $4+\sqrt{3}$

(3) $2+\sqrt{3}$

(4) $1.5+\sqrt{3}$

Ans. **(4)**

Sol. Let y = 3 + $\cfrac{1}{4+\cfrac{1}{y}}$

$y = 3+\frac{y}{4y+1}$

$\Rightarrow 4y^2+y = 12y+3+y$

$\Rightarrow 4y^2-12y-3=0$

$\Rightarrow y=\frac{12\pm\sqrt{144+48}}{8}$

$\Rightarrow y=\frac{12\pm 8\sqrt{3}}{8}$

$\Rightarrow y=\frac{3\pm 2\sqrt{3}}{2}$

$\Rightarrow y = 1.5\pm\sqrt{3}$

$y = 1.5+\sqrt{3}$.

13. The integral $\int\frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to:

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

(2) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(4) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

Ans. **(2)**

Sol. $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

Put $(2x-1)^2 + 5 = t^2$

$2(2x-1)\,dx = 2tdt$

$\Rightarrow \int \frac{\cos t}{t} \times \frac{t}{2} dx = \frac{1}{2}\sin t + C$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5} + C$

14. The differential equations satisfied by the system of parabolas $y^2 = 4a(x+a)$ is:

(1) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

Ans. **(2)**

Sol. $y^2 = 4a(x + a)$ ......(1)

$2yy' = 4a$

$\therefore yy' = 2a$

$\therefore$ by(1) $y^2 = 2yy'\left(x + \frac{yy'}{2}\right)$

$y^2 = 2yy'x + (yy')2$

$\Rightarrow y(y')^2 + 2xy' - y = 0$

(as $y \neq 0$)

15. The real valued function $f(x) = \frac{\cos ec^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to:

(1) all non- integers except the interval [–1, 1]

(2) all integers except 0, –1, 1

(3) all reals except integers

(4) all reals except the interval [–1, 1]

Ans. **(1)**

Sol. $f(x) = \dfrac{\operatorname{cosec}^{-1} x}{\sqrt{x-[x]}}$

$x \in (-\infty, -1] \cup [1, \infty)$

& $\{x\} \neq 0$

$x \neq$ Integer

$\Rightarrow x \in (-\infty, -1) \cup (1, \infty)$ – all integers

16. If $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x - \tan^{-1} x}{3x^3}$ is equal to L, then the value of (6L + 1) is:

(1) $\dfrac{1}{2}$

(2) 2

(3) $\dfrac{1}{6}$

(4) 6

Ans. **(2)**

Sol. $L = \lim\limits_{x \to 0} \dfrac{\left(x + \dfrac{x^3}{6} + \ldots\right) - \left(x - \dfrac{x^3}{3} \ldots\right)}{3x^3}$

$L = \dfrac{1}{3}\left(\dfrac{1}{6} + \dfrac{1}{3}\right) = \dfrac{1}{6}$

$\Rightarrow 6L + 1 = 6 \cdot \dfrac{1}{6} + 1 = 2$

17. For all four circles M, N, O and P, following four equations are given:

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a:

(1) Rectangle

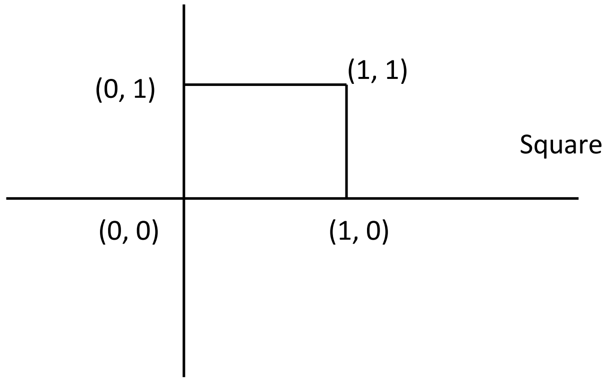
(2) Square

(3) Parallelogram

(4) Rhombus

Ans. **(2)**

Sol. $C_M = (0, 0)$
$C_N = (1, 0)$
$C_O = (1, 1)$
$C_P = (0, 1)$



18. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + \ldots. + a_{37}$ is equal to:
(1) $2^{20}(2^{20} + 21)$
(2) $2^{19}(2^{20} + 21)$
(3) $2^{20}(2^{20} - 21)$
(4) $2^{19}(2^{20} - 21)$

Ans. **(4)**

Sol. Put x = 1, –1and subtract
$4^{20} - 2^{20} = (a_0 + a_1 + \ldots.. + a_{40}) - (a_0 - a_1 + \ldots\ldots.)$
$\Rightarrow 4^{20} - 2^{20} = 2\ (a_1 + a_3 + \ldots.. + a_{39})$
$\Rightarrow a_1 + a_3 + \ldots.. + a_{37} = 2^{39} - 2^{19} - a_{39}$
a_{39} = coeff of x^{39} in $(1 + x + 2x^2)^{20} = {}^{20}C_1\ 2^{19}$
$\Rightarrow a_1 + a_3 + \ldots\ldots + a_{37} = 2^{39} - 2^{19} - 20(2^{19})$
$= 2^{39} - 21\ (2^{19}) = 2^{19}(2^{20} - 21)$

19. Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. It Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to:
(1) 0
(2) 1
(3) 3
(4) 2

Ans. **(4)**

Sol. $t_r(A + 2B) \equiv t_r(A) + 2\,t_r(B) = -1$ .....(1)

and $t_r(2A - B) \equiv 2t_r(A) - t_r(B) = 3$ .....(2)

on solving (1) and (2) we get

$t_r(A) = 1, \qquad t_r(B) = -1$

$\therefore \qquad t_r(A) - t_r(B) = 1 + 1 = 2$

20. The equations of one of the straight lines which passes through the point (1, 3) and makes an angle $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y + 1 = 3\sqrt{2}\,x$ is:

(1) $5\sqrt{2}\,x + 4y - \left(15 + 4\sqrt{2}\right) = 0$

(2) $4\sqrt{2}\,x - 5y - \left(5 + 4\sqrt{2}\right) = 0$

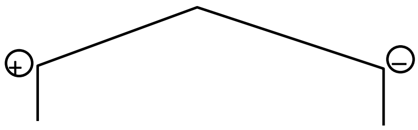
(3) $4\sqrt{2}\,x + 5y - 4\sqrt{2} = 0$

(4) $4\sqrt{2}\,x + 5y - \left(15 + 4\sqrt{2}\right) = 0$

Ans. **(4)**

Sol. $\tan\left(\tan^{-1}\sqrt{2}\right) = \left|\dfrac{m - 3\sqrt{2}}{1 + 3m\sqrt{2}}\right|$

$$\sqrt{2} = \left|\frac{m - 3\sqrt{2}}{1 + 3m\sqrt{2}}\right|$$



$6m + \sqrt{2} = m - 3\sqrt{2}$	$-6m - \sqrt{2} = m - 3\sqrt{2}$
$5m = -4\sqrt{2}$	$2\sqrt{2} = 7m$
$m = -\dfrac{4\sqrt{2}}{5}$	$m = \dfrac{2\sqrt{2}}{7}$

SECTION – B

1. The numbers of times al digit 3 will be written when listing the integers from 1 to 1000 is ______.

Ans. **(300)**

Sol.
$$\begin{array}{ccccccccccc} \underline{\boxed{3}} & \underline{10} & \underline{10} & + & \underline{9} & \underline{\boxed{3}} & \underline{10} & + & \underline{9} & \underline{10} & \underline{\boxed{3}} \\ \uparrow & \uparrow & \uparrow & & & \uparrow & & & \uparrow & \uparrow & \uparrow \end{array}$$

$\Rightarrow$ 100 + 90 + 90

$\Rightarrow$ 280

$$\left(\begin{array}{cc} \underline{\ \ } & \underline{10} \\ \uparrow & \uparrow \end{array}\right) + \left(\begin{array}{cc} \underline{9} & \underline{\ \ } \\ \uparrow & \uparrow_3 \end{array}\right) \Rightarrow \boxed{19}$$

$3 \rightarrow 1$

280 + 19 + 1 = 300

2. The equation of the planes parallel to the plane x – 2y + 2z – 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d = 0. If (b – d) = K(c – a), then the positive value of K is ______.

Ans. **(4)**

Sol. $x - 2y + 2z + \lambda = 0$

Now given

$$d = \frac{|1-4+6+\lambda|}{\sqrt{9}} = 1$$

$$|\lambda + 3| = 3$$

$$\lambda + 3 = \pm 3 \Rightarrow \lambda = 0, -6$$

So planes are: $x - 2y + 2z - 6 = 0$

$x - 2y + 2z = 0$

$b - d = -2 + 6 = 4$

$c - a = 2 - 1 = 1$

$$\Rightarrow \frac{b-d}{c-a} = k$$

$$\Rightarrow k = 4$$

3. Let f(x) and g(x) be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x^2)\,dx$ is ______.

Ans. **(512)**

Sol. $I = 2\int_0^4 f(x^2)\,dx$ ....(1)

$\Rightarrow I = 2\int_0^4 f((4-x)^2)\,dx$ ....(2)

Adding equation (1) & (2)

$2I = 2\int_0^4 \left[f(x)^2 + f(4-x)^2\right]dx$ ....(3)

Now using $f(x^2) + g(4 - x) = 4x^3$ ....(4)

$x \to 4 - x$

$f((4 - x)^2) + g(x) = 4(4 - x)^3$ ....(5)

Adding equation (4) & (5)

$f(x^2) + f(4 - x^2) + g(x) + g(4 - x) = 4(x^3 + (4 - x)^3]$

$\Rightarrow f(x^2) + f(4 - x^2) = 4(x^3 + (4 - x)^3]$

Now, $I = 4\int_0^4 \left(x^3 + (4-x)^3\right)dx = 512$

4. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is ______.

Ans. **(35)**

Sol. $x_1 + x_2 + ... + x_{25} = 25 \times 40 = 1000$

$$\frac{x_1 + x_2 + ... + x_{25} - 60 + a}{25} = 39$$

$100 - 60 + a = 25 \times 39$

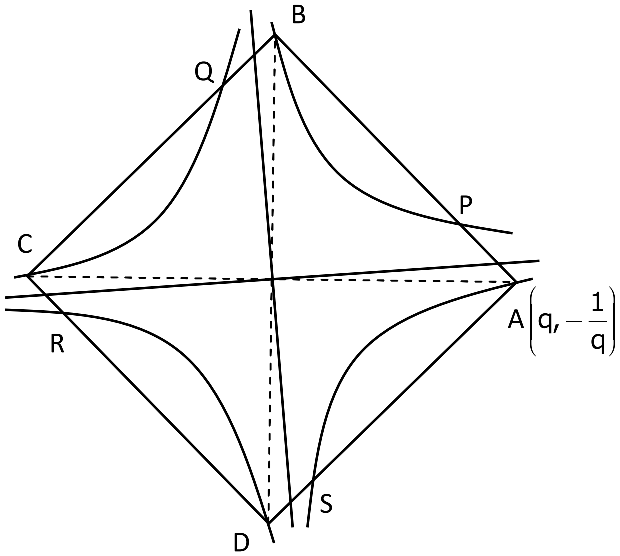
$a = -940 + 975$

$a = 35$

5. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is ______.

Ans. **(80)**

Sol.



OA $\perp$ OB

$$\Rightarrow \left(\frac{1}{P^2}\right)\left(-\frac{1}{q^2}\right) = -1$$

$$\Rightarrow p^2q^2 = 1$$

$$P\left(\frac{p+q}{2}, \frac{\frac{1}{p}-\frac{1}{q}}{2}\right) \text{lies}$$

On $x^2 y^2 = 1$

$$\Rightarrow (p+q)^2\left(\frac{1}{p}-\frac{1}{q}\right)^2 = 16$$

$$\Rightarrow (p+q)^2 (p-q)^2 = 16$$

$$\Rightarrow (p^2 - q^2)^2 = 16$$

$$\Rightarrow P^2 - \frac{1}{P^2} = \pm 4$$

$$\Rightarrow p^4 \pm 4p^2 - 1 = 0$$

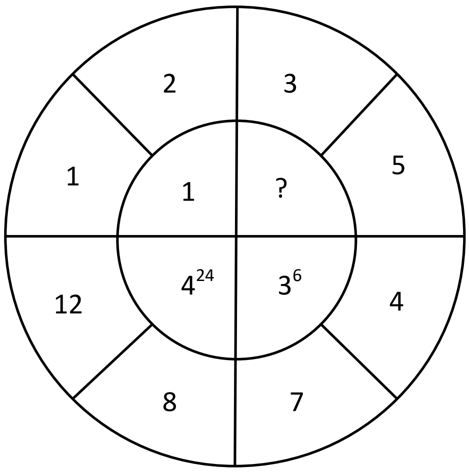
$$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$$

$$\Rightarrow p^2 = 2+\sqrt{5} \text{ or } -2+\sqrt{5}$$

$$OB^2 = p^2 + \frac{1}{p^2} = 2+\sqrt{5}+\frac{1}{2+\sqrt{5}} \text{ or } -2+\sqrt{5}+\frac{1}{-2+\sqrt{5}} = 2\sqrt{5}$$

$$\text{Area} = 4\left(\frac{1}{2}\right)(OA)(OB) = 2(OB)^2 = 4\sqrt{5}$$

6. The missing value in the following figure is ______.



Ans. **(4)**

Sol. 4^{24} has base 4 (= 12 – 8)

36 has base 3 (= 7 – 4)

(?) will have base 2 (= 5 – 3)

Power 24 = 6 × 4 = (no. of divisor of 12) × (no. of divisor of 8)

Power 6 = 2 × 3 = (no. of divisor of 7) × (no. of divisor of 4)

(?) will have power = (no. of divisor of 3) × (no. of divisor of 5) = 2 × 2 = 4

7. The numbers of solutions of the equation $|\cot x| = \cot x + \dfrac{1}{\sin x}$ in the interval $[0, 2\pi]$ is ______.

Ans. **(1)**

Sol. Case I : $x \in \left[0, \dfrac{\pi}{2}\right] \cup \left[\pi, \dfrac{3\pi}{2}\right]$

$$\cot x = \cot x + \frac{1}{\sin x} \quad \Rightarrow \text{not possible}$$

Case II : $x \in \left[\dfrac{\pi}{2}, \pi\right] \cup \left[\dfrac{3\pi}{2}, 2\pi\right]$

$$-\cot x = \cot x + \frac{1}{\sin x}$$

$$\Rightarrow \frac{-2\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\Rightarrow \cos x = \frac{-1}{2}$$

$$\Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

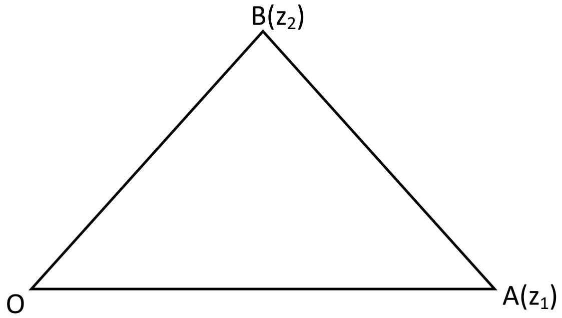
= 1

8. Let z_1, z_2 be the roots of the equations $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is ______.

Ans. **(6)**

Sol. In equilateral Δ,

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$



$$z_1^2 + z_2^2 = z_1z_2 \qquad (\because z_3 = 0)$$

$$(z_1 + z_2)^2 = 3z_1z_2$$

$$a^2 = 36$$

$$|a| = 6$$

9. Let the plane $ax + by + cz + d = 0$ bisect the line joining the points (4, –3, 1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is ______.

Ans. **(28)**

Sol. normal of plane = $\overrightarrow{PQ}$

$$= -2\hat{i} + 6\hat{j} - 6\hat{k}$$

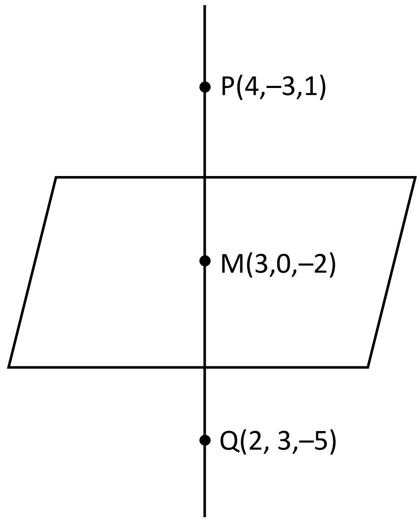
a = –2, b = 6, c = –6

& equation of plane is

$-2x + 6y - 6z + d = 0$

$\Downarrow$ M(3, 0, –2)

d = –6



Now equation of plane is

$-2x + 6y - 6z - 6 = 0$

$x - 3y + 3z + 3 = 0$

$\Rightarrow (a^2 + b^2 + c^2 + d^2)_{min} = 1^2 + 9 + 9 + 9 = 28$

10. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, $(x \geq 0)$, $f(0) = 0$ and $f(1) = \frac{1}{k}$, then the value of K is ______.

Ans. **(4)**

Sol. $\int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = \int \frac{5x^8 + 7x^6}{x^{14}\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$

$$\int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$$

put $2 + \frac{1}{x^5} + \frac{1}{x^7} = t$

$\Rightarrow -\left(\frac{5}{x^6} + \frac{7}{x^8}\right) dx = dt$

$\int \frac{-dt}{t^2} = \frac{1}{t} + c$

$\Rightarrow f(x) = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} + C = \frac{x^7}{2x^7 + 1 + x^2} + C$

$f(0) = 0 \Rightarrow C = 0$

$f(x) = \frac{1}{4} = \frac{1}{k}$

$\Rightarrow k = 4$