



**JEE  
MAIN  
MARCH  
2021**

16<sup>th</sup> March 2021 | Shift - 2  
**PHYSICS**

**JEE I NEET I Foundation**

**MOTION™**

**25000+**  
SELECTIONS SINCE 2007

## SECTION – A

1. The least value of  $|z|$  where  $z$  is complex number which satisfies the inequality  $\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1} \log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7} + 9i|, i = \sqrt{-1}$  is equal to :
- (1) 2
  - (2) 3
  - (3) 8
  - (4)  $\sqrt{5}$

**Ans. (2)**

**Sol.** 
$$2^{\frac{(|z|+3)(|z|-1)}{|z|+1}} \geq 2^3 \Rightarrow \frac{(|z|+3)(|z|-1)}{|z|+1} \geq 3$$

$$\Rightarrow |z|^2 + 2|z| - 3 \geq 3|z| + 3$$

$$\Rightarrow |z|^2 - |z| - 6 \geq 0$$

$$(|z|-3)(|z|+2) \geq 0$$

$$|z|_{\min} = 3$$

2. Let  $f: S \rightarrow S$  where  $S = (0, \infty)$  be a twice differentiable function such that  $f(x+1) = xf(x)$ . If  $g: S \rightarrow R$  be defined as  $g(x) = \log_e f(x)$ , then the value of  $|g''(5) - g''(1)|$  is equal to :
- (1)  $\frac{197}{144}$
  - (2)  $\frac{187}{144}$
  - (3)  $\frac{205}{144}$
  - (4) 1

**Ans. (3)**

**Sol.**  $f(x+1) = xf(x)$

$$g(x+1) = \log_e(f(x+1))$$

$$g(x+1) = \log_e x + \log_e f(x)$$

$$g(x+1) - g(x) = \log_e x$$

$$g''(x+1) - g''(x) = -\frac{1}{x^2}$$

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$$g''(2) - g''(1) = -1$$

$$g''(3) - g''(2) = -\frac{1}{4}$$

$$g''(4) - g''(3) = -\frac{1}{9}$$

$$g''(5) - g''(4) = -\frac{1}{16}$$

$$g''(5) - g''(1) = -\left[1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}\right]$$

$$|g''(5) - g''(1)| = \left[\frac{144 + 36 + 16 + 9}{16 \times 9}\right] = \left[\frac{205}{16 \times 9}\right]$$

3. If  $y = y(x)$  is the solution of the differential equation  $\frac{dy}{dx} + (\tan x)y = \sin x, 0 \leq x \leq \frac{\pi}{3}$ , with  $y(0) = 0$ , then  $y\left(\frac{\pi}{4}\right)$  equal to :

(1)  $\log_e 2$

(2)  $\frac{1}{2} \log_e 2$

(3)  $\left(\frac{1}{2\sqrt{2}}\right) \log_e 2$

(4)  $\frac{1}{4} \log_e 2$

**Ans. (3)**

**Sol.** I.f. =  $e^{\int \tan x dx}$

$$= e^{\ln[\sec x]}$$

$$= \sec x$$

Solution of the equation

$$y(\sec x) = \int (\sin x)(\sec x) dx$$

$$\Rightarrow \frac{y}{\cos x} = \ln(\sec x) + c$$

Put  $x = 0, c = 0$

$$\therefore y = \cos x \ln(\sec x)$$

$$\text{put } x = \pi/4$$

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$$y = \frac{1}{\sqrt{2}} \ln \sqrt{2} = \frac{1}{2\sqrt{2}} \ln 2$$

$$y = \frac{\ln 2}{2\sqrt{2}}$$

4. If the foot of the perpendicular from point (4, 3, 8) on the line  $L_1 : \frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$ ,  $\ell \neq 0$  is (3, 5, 7), then the shortest distance between the line  $L_1$  and line  $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$  is equal to :

(1)  $\frac{\sqrt{2}}{3}$

(2)  $\frac{1}{\sqrt{3}}$

(3)  $\frac{1}{2}$

(4)  $\frac{1}{\sqrt{6}}$

**Ans. (4)**

**Sol.** (3, 5, 7) lie on given line  $L_1$

$$\frac{3-a}{\ell} = \frac{3}{3} = \frac{7-b}{4}$$

$$\frac{7-b}{4} = 1 \Rightarrow b = 3$$

$$\frac{3-a}{\ell} = 1 \Rightarrow 3-a = \ell$$

A (4, 3, 8)

B (3, 5, 7)

DR'S of AB = (1, -2, 1)

AB  $\perp$  line  $L_1$

$$(1)(\ell) + (-2)(3) + 4(1) = 0$$

$$\Rightarrow \ell = 2$$

$$a = 1$$

$$a = 1, b = 3, \ell = 2$$

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$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

$$S.D. = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}} = \frac{1}{\sqrt{6}}$$

5. If  $(x, y, z)$  be an arbitrary point lying on a plane P which passes through the points  $(42, 0, 0)$ ,  $(0, 42, 0)$  and  $(0, 0, 42)$ , then the value of the expression

$$3 + \frac{x-11}{(y-19)^2(z-12)^2} + \frac{y-19}{(x-11)^2(z-12)^2} + \frac{z-12}{(z-11)^2(y-19)^2} - \frac{x+y+z}{14(x-11)(y-19)(z-12)}$$
 is equal

to :

- (1) 3
- (2) 0
- (3) 39
- (4) -45

**Ans.** (1)

**Sol.** equation of plane  $x + y + z = 42$

Let pt. on plane  $x = 10, y = 21, z = 11$

$$3 + \frac{(-1)}{(4)(1)} + \frac{(2)}{(1)(1)} + \frac{(-1)}{(1)(4)} - \frac{42}{14(-1)(2)(-1)}$$

$$3 - \frac{1}{4} + 2 - \frac{1}{4} - \frac{3}{2} = 3$$

6. Consider the integral

$$I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx$$

Where  $[x]$  denotes the greatest integer less than or equal to  $x$ . Then the value of  $I$  is equal to :

- (1)  $45(e-1)$
- (2)  $45(e+1)$
- (3)  $9(e-1)$
- (4)  $9(e+1)$

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**Ans. (1)**

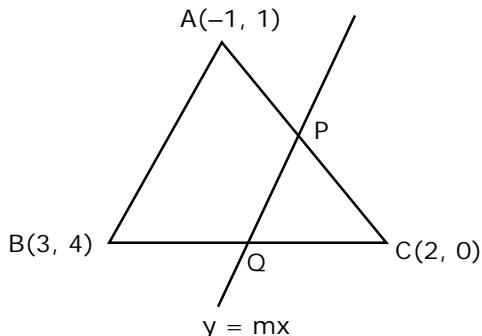
$$\begin{aligned}
 \text{Sol. } I &= \int_0^{10} [x] \cdot e^{[x]+1-x} dx \\
 &= \int_1^2 e^{2-x} dx + \int_2^3 2 \cdot e^{3-x} dx + \int_3^4 3 \cdot e^{4-x} dx + \dots + \int_9^{10} 9e^{10-x} dx \\
 &= -\{(1-e) + 2(1-e) + 3(1-e) + \dots + 9(1-e)\} \\
 &= 45(e-1)
 \end{aligned}$$

7. Let A (-1, 1), B (3, 4) and C(2, 0) be given three points. A line  $y = mx$ ,  $m > 0$ , intersects lines AC and BC at point P and Q respectively. Let  $A_1$  and  $A_2$  be the areas of  $\triangle ABC$  and  $\triangle PQC$  respectively, such that  $A_1 = 3A_2$ , then the value of m is equal to :

- (1)  $\frac{4}{15}$
- (2) 1
- (3) 2
- (4) 3

**Ans. (2)**

**Sol.**



$$A_1 = \Delta ABC = \frac{1}{2} \begin{vmatrix} -1 & 1 & 1 \\ 2 & 0 & 1 \\ 3 & 4 & 1 \end{vmatrix}$$

$$A_1 = \frac{13}{2}$$

$$\text{Equation of line AC is } y - 1 = -\frac{1}{3}(x + 1)$$

$$\text{solve it with line } y = mx, \text{ we get } P\left(\frac{2}{3m+1}, \frac{2m}{3m+1}\right)$$

$$\text{Equation of line BC is } y - 0 = 4(x - 2)$$

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Solve it with line  $y = mx$ , we get  $Q\left(\frac{-8}{m-4}, \frac{-8m}{m-4}\right)$

$$A_2 = \text{Area of } \triangle PQC = \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{vmatrix} = \frac{A_1}{3} = \frac{13}{6}$$

$$= \frac{1}{2} \left( 2 \left( \frac{2m}{3m+1} + \frac{8m}{m-4} \right) - 1 \left( \frac{-16m}{(3m+1)(m-4)} + \frac{16m}{(3m+1)(m-4)} \right) \right)$$

$$= \pm \frac{13}{6}$$

$$\frac{26m^2}{3m^2 - 11m - 4} = \pm \frac{13}{6}$$

$$\Rightarrow 12m^2 = \pm (3m^2 - 11m - 4)$$

taking +ve sign

$$9m^2 + 11m + 4 = 0 \text{ (Rejected as } m \text{ is imaginary)}$$

taking -ve sing

$$15m^2 - 11m - 4 = 0$$

$$m = 1, -\frac{4}{15}$$

8. Let  $f$  be a real valued function, defined on  $R - \{-1, 1\}$  and given by

$$f(x) = 3 \log_e \left| \frac{x-1}{x+1} \right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function  $f(x)$  is increasing ?

(1)  $(-\infty, -1) \cup \left[ \frac{1}{2}, \infty \right) - \{1\}$

(2)  $\left[ -1, \frac{1}{2} \right]$

(3)  $(-\infty, \infty) - \{-1, 1\}$

(4)  $\left[ -\infty, \frac{1}{2} \right] - \{-1\}$

**Ans. (1)**

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**Sol.**  $f'(x) = \left( \frac{x+1}{x-1} \times \frac{x+1-(x-1)}{(x+1)^2} \right) 3 + \frac{2}{(x-1)^2} = \frac{6}{(x-1)(x+1)} + \frac{2}{(x-1)^2}$

$$= \frac{2}{(x-1)} \left( \frac{3}{x+1} + \frac{1}{x-1} \right) = \frac{4(2x-1)}{(x+1)(x-1)^2}$$

$\boxed{+}$	$-$	$\boxed{+}$
$-1$	$\frac{1}{2}$	

$$x \in (-\infty, -1) \cup \left[ \frac{1}{2}, \infty \right) - \{1\}$$

9. Let the lengths of intercepts on x-axis and y-axis made by the circle  $x^2 + y^2 + ax + 2ay + c = 0$ , ( $a < 0$ ) be  $2\sqrt{2}$  and  $2\sqrt{5}$ , respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line  $x + 2y = 0$ , is equal to :

- (1)  $\sqrt{10}$
- (2)  $\sqrt{6}$
- (3)  $\sqrt{11}$
- (4)  $\sqrt{7}$

**Ans. (2)**

**Sol.**  $2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$

$$\sqrt{a^2 - 4c} = 2\sqrt{2}$$

$$a^2 - 4c = 8 \quad \dots (1)$$

$$2\sqrt{a^2 - c} = 2\sqrt{5}$$

$$a^2 - c = 5 \quad \dots (2)$$

$$(2) - (1)$$

$$3c = -3a \Rightarrow c = -1$$

$$a^2 = 4 \Rightarrow a = -2$$

$$x^2 + y^2 - 2x - 4y - 1 = 0$$

$$\text{Equation of tangent } 2x - y + \lambda = 0$$

$$\therefore p = r$$

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$$\left| \frac{2 - 2 + \lambda}{\sqrt{5}} \right| = \sqrt{6}$$

$$\Rightarrow \lambda = \pm \sqrt{30}$$

$$\therefore \text{tangent } 2x - y \pm \sqrt{30} = 0$$

$$\text{Distance from origin} = \frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$$

10. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1)  $\frac{4}{9}$

(2)  $\frac{9}{56}$

(3)  $\frac{3}{7}$

(4)  $\frac{11}{27}$

**Ans. (1)**

**Sol.** Total case =  $6|6$

$$\text{Fav. case} = (0, 1, 2, 3, 4, 5) + (0, 1, 2, 4, 5, 6) + (1, 2, 3, 4, 5, 6)$$

$$= 5|5 + 5|5 + |6$$

$$= 1920$$

$$\text{Probability} = \frac{1920}{6|6} = \frac{4}{9}$$

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11. Let  $\alpha \in \mathbb{R}$  be such that the function  $f(x) = \begin{cases} \frac{\cos^{-1}(1 - \{x\}^2)\sin^{-1}(1 - \{x\})}{\{x\} - \{x\}^3} & x \neq 0 \\ \alpha & x = 0 \end{cases}$  is

Continuous at  $x = 0$ , where  $\{x\} = x - [x]$ ,  $[x]$  is the greatest integer less than or equal to  $x$ .

Then :

- (1)  $\alpha = \frac{\pi}{4}$
- (2) No such  $\alpha$  exists
- (3)  $\alpha = 0$
- (4)  $\alpha = \frac{\pi}{\sqrt{2}}$

**Ans. (2)**

**Sol.** RHL =  $\lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1 - x^2)\sin^{-1}(1 - x)}{x(1 - x^2)} = \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1 - x^2)}{x}$

$$= \frac{\pi}{2} \lim_{x \rightarrow 0^+} \frac{-1}{\sqrt{1 - (1 - x^2)^2}}(-2x) \quad (\text{L}' \text{ Hospital Rule})$$

$$= \pi \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2x^2 - x^4}} = \pi \lim_{x \rightarrow 0^+} \frac{1}{\sqrt{2 - x^2}} = \frac{\pi}{\sqrt{2}}$$

LHL =  $\lim_{x \rightarrow 0^-} \frac{\cos^{-1}(1 - (1 + x)^2)\sin^{-1}(-x)}{(1 + x) - (1 + x)^3} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{(1 + x)[(1 + x)^2 - 1]} = \frac{\pi}{2} \lim_{x \rightarrow 0^-} \frac{\sin^{-1} x}{x^2 + 2x}$

$$= \frac{\pi}{2} \left(\frac{1}{2}\right) = \frac{\pi}{4}$$

As LHL  $\neq$  RHL so  $f(x)$  is not continuous at  $x = 0$

12. The maximum value of  $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$ ,  $x \in \mathbb{R}$  is :
- (1)  $\sqrt{7}$
  - (2)  $\sqrt{5}$
  - (3) 5
  - (4)  $\frac{3}{4}$

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**Ans.** (2)

**Sol.**  $C_1 \rightarrow C_1 + C_2$

$$\begin{vmatrix} 2 & 1 + \cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$$= (-1)[2\sin 2x - \cos 2x] = \cos 2x - 2\sin 2x$$

$$\text{maximum value} = \sqrt{5}$$

- 13.** Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let  $\alpha$  be the number of triangles having these points from different sides as vertices and  $\beta$  be the number of quadrilaterals having these points from different sides as vertices. Then  $(\beta - \alpha)$  is equal to:

- (1) 1890
- (2) 795
- (3) 717
- (4) 1173

**Ans.** (3)

**Sol.**  $\alpha = {}^6C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^7C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^9C_1 + {}^5C_1 {}^6C_1 {}^7C_1 = 378 + 315 + 270 + 210 = 1173$   
 $\beta = {}^5C_1 {}^6C_1 {}^7C_1 {}^9C_1 = 1890$   
 $\Rightarrow \beta - \alpha = 1890 - 1173 = 717$

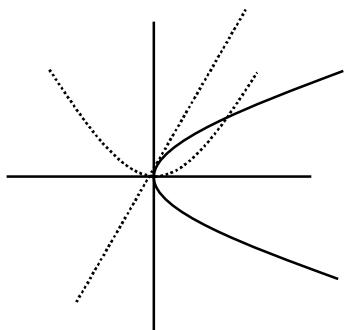
- 14.** Let C be the locus of the mirror image of a point on the parabola  $y^2 = 4x$  with respect to the line  $y = x$ . Then the equation of tangent to C at P(2, 1) is :

- (1)  $2x + y = 5$
- (2)  $x + 2y = 4$
- (3)  $x + 3y = 5$
- (4)  $x - y = 1$

**Ans.** (4)

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**Sol.** Image of  $y^2 = 4x$  w.r.t.  $y = x$  is  $x^2 = 4y$

tangent from  $(2, 1)$

$$xx_1 = 2(y + y_1)$$

$$2x = 2(y + 1)$$

$$x = y + 1$$

- 15.** Given that the inverse trigonometric functions take principal values only. Then, the number of real values of  $x$  which satisfy  $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$  is equal to :

(1) 1

(2) 2

(3) 3

(4) 0

**Ans.** (3)

**Sol.** Taking sine both sides

$$\frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} = x$$

$$\Rightarrow 3x\sqrt{25 - 16x^2} = 25x - 4x\sqrt{25 - 9x^2}$$

$$\Rightarrow x = 0 \text{ or } 3\sqrt{25 - 16x^2} = 25 - 4\sqrt{25 - 9x^2}$$

$$\Rightarrow 9(25 - 16x^2) = 625 - 200\sqrt{25 - 9x^2} + 16(25 - 9x^2)$$

$$\Rightarrow 200\sqrt{25 - 9x^2} = 800$$

$$\Rightarrow \sqrt{25 - 9x^2} = 4$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x \pm 1$$

$\therefore$  Total number of solution = 3

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16. Let  $C_1$  be the curve obtained by solution of differential equation  $2xy \frac{dy}{dx} = y^2 - x^2, x > 0$  Let the curve  $C_2$  be the solution of  $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$ , If both the curves pass through (1, 1) then the area enclosed by the curves  $C_1$  and  $C_2$  is equal to :

(1)  $\frac{\pi}{2} - 1$

(2)  $\frac{\pi}{4} + 1$

(3)  $\pi - 1$

(4)  $\pi + 1$

**Ans.** (1)

**Sol.**

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Put  $y = vx$

$$v + x \frac{dv}{dx} = \frac{v^2 x^2 - x^2}{2vx^2} = \frac{v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} = -\frac{(v^2 + 1)}{2v}$$

$$\Rightarrow \frac{2v}{v^2 + 1} dv = -\frac{dx}{x}$$

$$\ell n(v^2 + 1) = -\ell nx + \ell nc \Rightarrow v^2 + 1 = \frac{c}{x}$$

$$\Rightarrow \frac{y^2}{x^2} + 1 = \frac{c}{x} \Rightarrow x^2 + y^2 = cx$$

If pass through (1, 1)

$$\therefore x^2 + y^2 - 2x = 0$$

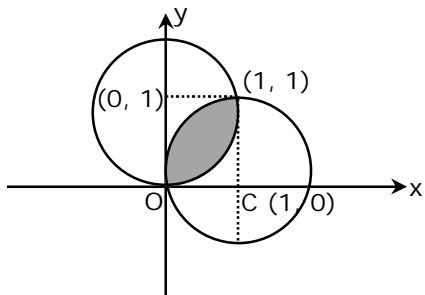
Similarly for second differential equation  $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Equation of curve is  $x^2 + y^2 - 2y = 0$

Now required area is

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$$\begin{aligned}
 &= \left( \frac{1}{4} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1 \right) \times 2 \\
 &= \left( \frac{\pi}{2} - 1 \right) \text{ sq. units}
 \end{aligned}$$

17. Let  $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$  and  $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ . If  $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}, \vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$  and  $\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1, \alpha \in \mathbb{R}$ , then the value of  $\alpha + |\vec{r}|^2$  is equal to :

- (1) 11
- (2) 15
- (3) 9
- (4) 13

**Ans. (2)**

**Sol.**  $\vec{r} \times \vec{a} = -\vec{r} \times \vec{b}$

$$\vec{r} \times (\vec{a} + \vec{b}) = 0 \quad (\vec{a} + \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k})$$

$$\vec{r} \parallel (\vec{a} + \vec{b})$$

$$\vec{r} = \lambda (\vec{a} + \vec{b})$$

$$\therefore \vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$$

$$\lambda [3\hat{i} - \hat{j} + 2\hat{k}] \cdot [2\hat{i} + 5\hat{j} - \alpha\hat{k}] = -1$$

$$\Rightarrow \lambda(6 - 5 - 2\alpha) = -1$$

$$\lambda(1 - 2\alpha) = -1 \quad \dots(1)$$

$$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\lambda(3\hat{i} - \hat{j} + 2\hat{k}) \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$$

$$\Rightarrow \lambda[3\alpha - 2 + 2] = 3 \Rightarrow \lambda\alpha = 1 \quad \dots(2)$$

(1) & (2)

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$$\lambda \left[ 1 - \frac{2}{\lambda} \right] = -1$$

$$\lambda - 2 = -1 \Rightarrow \lambda = 1 \quad \alpha = 1$$

$$\vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$\alpha + |\vec{r}|^2 \Rightarrow 1 + 14 = 15$$

- 18.** Let  $P(x) = x^2 + bx + c$  be a quadratic polynomial with real coefficients such that  $\int_0^1 P(x) dx = 1$  and  $P(x)$  leaves remainder 5 when it is divided by  $(x - 2)$ . Then the value of  $9(b+c)$  is equal to :  
 (1) 7  
 (2) 11  
 (3) 15  
 (4) 9

**Ans. (1)**

**Sol.**  $(x - 2)Q(x) + 5 = x^2 + bx + c$

Put  $x = 2$

$$5 = 2b + c + 4 \quad \dots(1)$$

$$\int_0^1 (x^2 + bx + c) dx = 1$$

$$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$$

$$\frac{b}{2} + c = \frac{2}{3} \quad \dots(2)$$

Solve (1) & (2)

$$b = \frac{2}{9}$$

$$c = \frac{5}{9}$$

$$9(b+c) = 7$$

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19. If the points of intersections of the ellipse  $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$  and the circle  $x^2 + y^2 = 4b$ ,  $b > 4$  lie on the curve  $y^2 = 3x^2$ , then b is equal to :

- (1) 5
- (2) 6
- (3) 12
- (4) 10

**Ans.** (3)

**Sol.**  $\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \quad \dots(1)$

$$x^2 + y^2 = 4b \quad \dots(2)$$

$$y^2 = 3x^2 \quad \dots(3)$$

From eq (2) and (3)  $x^2 = b$  and  $y^2 = 3b$

$$\text{From equation (1)} \quad \frac{b}{16} + \frac{3b}{b^2} = 1$$

$$\Rightarrow b^2 + 48 = 16b$$

$$\Rightarrow b = 12$$

20. Let  $A = \{2, 3, 4, 5, \dots, 30\}$  and ' $\simeq$ ' be an equivalence relation on  $A \times A$ , defined by  $(a, b) \simeq (c, d)$ , if and only if  $ad = bc$ . Then the number of ordered pairs which satisfy this equivalence relation with ordered pair  $(4, 3)$  is equal to :

- (1) 7
- (2) 5
- (3) 6
- (4) 8

**Ans.** (1)

**Sol.**  $ad = bc$

$$(a, b) R (4, 3) \Rightarrow 3a = 4b$$

$$a = \frac{4}{3}b$$

b must be multiple of 3

$$b = \{3, 6, 9, \dots, 30\}$$

$$(a, b) = (4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)$$

$$\Rightarrow 7 \text{ ordered pair}$$

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## SECTION – B

1. Let  $\vec{c}$  be a vector perpendicular to the vectors  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ . If  $\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 8$  then the value of  $\vec{c} \cdot (\vec{a} \times \vec{b})$  is equal to .....

**Ans. (28)**

**Sol.**  $\vec{c} \cdot (\vec{a} \times \vec{b}) = [\vec{c} \vec{a} \vec{b}]$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (3, -2, 1)$$

$$\vec{c} \perp \vec{a}, \vec{c} \perp \vec{b} \Rightarrow \vec{c} \parallel \vec{a} \times \vec{b}$$

$$\vec{c} = \lambda (\vec{a} \times \vec{b})$$

$$\Rightarrow \vec{c} = \lambda (3\hat{i} - 2\hat{j} + \hat{k})$$

$$\vec{c}(\hat{i} + \hat{j} + 3\hat{k}) = 8$$

$$\Rightarrow 3\lambda - 2\lambda + 3\lambda = 8$$

$$\Rightarrow 4\lambda = 8 \Rightarrow \lambda = 2$$

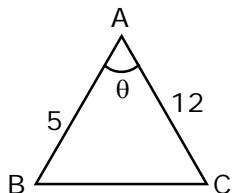
$$\vec{c} = 6\hat{i} - 4\hat{j} + 2\hat{k}$$

$$\vec{c} \cdot (\vec{a} \times \vec{b}) = [\vec{c} \vec{a} \vec{b}] = \begin{vmatrix} 6 & -4 & 2 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow 18 + 8 + 2 = 28$$

2. In  $\triangle ABC$ , the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of  $\triangle ABC$  is  $30 \text{ cm}^2$  and R and r are respectively the radii of circumcircle and incircle of  $\triangle ABC$ , then the value of  $2R + r$  (in cm) is equal to .....

**Ans. (15)**



**Sol.**

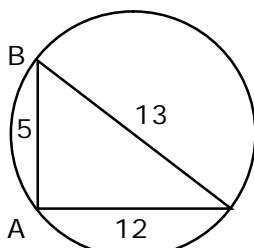
$$\text{Area} = \frac{1}{2}(5)(12)\sin\theta = 30$$

$$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$\Delta$  is right angle  $\Delta$

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$$r = (s-a) \tan \frac{A}{2}$$

$$r = (s-a)$$

$$r = (s-a) \quad (a = 2R)$$

$$2R + r = s$$

$$2R + r = \frac{30}{2} = 15$$

3. Consider the statistics of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is  $\frac{17}{9}$ , then the value of n is equal to .....

**Ans. (5)**

**Sol.** For group-1 :  $\frac{\sum x_i}{10} = 2 \Rightarrow \sum x_i = 20$

$$\frac{\sum x_i}{10} - (2)^2 = 2 \Rightarrow \sum x_i^2 = 60$$

For group-2 :  $\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$

$$\frac{\sum y_i^2}{n} - 3^2 = 1 \Rightarrow \sum y_i^2 = 10n$$

Now, combined variance

$$\sigma^2 = \frac{\sum (x_i^2 + y_i^2)}{10+n} - \left( \frac{\sum (x_i + y_i)}{10+n} \right)^2$$

$$\Rightarrow \frac{17}{9} = \frac{60+10n}{10+n} - \frac{(20+3n)^2}{(10+n)^2}$$

$$\Rightarrow 17(n^2 + 20n + 100) = 9(n^2 + 40n + 200)$$

$$\Rightarrow 8n^2 - 20n - 100 = 0$$

$$\Rightarrow 2n^2 - 5n - 25 = 0 \Rightarrow n = 5$$

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4. Let

$$S_n(x) = \log_{\frac{1}{a^2}} x + \log_{\frac{1}{a^3}} x + \log_{\frac{1}{a^6}} x + \log_{\frac{1}{a^{11}}} x + \log_{\frac{1}{a^{18}}} x + \dots \text{ up to } n\text{-terms,}$$

Where  $a > 1$ . If  $S_{24}(x) = 1093$  and  $S_{12}(2x) = 265$ , the value of  $a$  is equal to .....

**Ans. (16)**

**Sol.**  $S_n(x) = \log_a x^2 + \log_a x^3 + \log_a x^6 + \log_a x^{11}$

$$S_n(x) = 2 \log_a x + 3 \log_a x + 6 \log_a x + 11 \log_a x + \dots$$

$$S_n(x) = \log_a x (2 + 3 + 6 + 11 + \dots)$$

$$S_r = 2 + 3 + 6 + 11$$

$$\text{General term } T_r = r^2 - 2r + 3$$

$$S_n(x) = \sum_{r=1}^n \log_a x (r^2 - 2r + 3)$$

$$S_{24}(x) = \sum_{r=1}^{24} \log_a x (r^2 - 2r + 3)$$

$$S_{24}(x) = \log_a \sum_{r=1}^{24} (r^2 - 2r + 3)$$

$$1093 = 4372 \log_a x$$

$$\log_a x = \frac{1}{4}$$

$$x = a^{1/4}$$

$$S_{12}(2x) = \log_a (2x) \sum_{r=1}^{12} (r^2 - 2r + 3)$$

$$265 = 530 \log_a (2x)$$

$$\log_a (2x) = \frac{1}{2}$$

$$2x = a^{1/2}$$

After solving (i) and (ii), we get

$$a^{1/4} = 2$$

$$a = 16$$

5. Let  $n$  be a positive integer. Let  $A = \sum_{k=0}^n (-1)^k nC_k \left[ \left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$  If  $63A = 1$

–  $\frac{1}{2^{30}}$ , then  $n$  is equal to .....

**Ans. (6)**

**Sol.**  $A = \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n + \left(\frac{1}{32}\right)^n$

$$= \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \frac{1}{2^{5n}}$$

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$$= \frac{1}{2^n} \left[ \frac{1 - \left( \frac{1}{2^n} \right)^5}{1 - \frac{1}{2^n}} \right]$$

$$A = \frac{2^{5n} - 1}{2^{5n}(2^n - 1)}$$

$$63A = \frac{63(2^{5n} - 1)}{2^{5n}(2^n - 1)}$$

$$\frac{63}{2^n - 1} \left( 1 - \frac{1}{2^{5n}} \right) = 63A = \left( 1 - \frac{1}{2^{30}} \right)$$

$$= \frac{63}{2^n - 1} \left( 1 - \frac{1}{2^{5n}} \right) = \left( 1 - \frac{1}{2^{30}} \right)$$

$$n = 6$$

6. Let  $f: R \rightarrow R$  and  $g: R \rightarrow R$  be defined as

$$f(x) = \begin{cases} x + a, & x < 0 \\ |x - 1|, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x + 1, & x < 0 \\ (x - 1)^2 + b, & x \geq 0 \end{cases}$$

Where  $a, b$  are non-negative real numbers. If  $(gof)(x)$  is continuous for all  $x \in R$ , then  $a + b$  is equal to .....

**Ans. (1)**

$$\begin{aligned} \text{Sol. } g[f(x)] &= \begin{cases} f(x) + 1 & f(x) < 0 \\ (f(x) - 1)^2 + b & f(x) \geq 0 \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x + a < 0 \& x < 0 \\ |x - 1| + 1 & |x - 1| < 0 \& x \geq 0 \\ (x + a - 1)^2 + b & x + a \geq 0 \& x < 0 \\ (|x - 1| - 1)^2 + b & |x - 1| \geq 0 \& x \geq 0 \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x \in (-\infty, -a) \& x \in (-\infty, 0) \\ |x - 1| + 1 & x \in \emptyset \\ (x + a - 1)^2 + b & x \in [-a, \infty) \& x \in (-\infty, 0) \\ (|x - 1| - 1)^2 + b & x \in R \& x \in [0, \infty) \end{cases} \\ g[f(x)] &= \begin{cases} x + a + 1 & x \in (-\infty, -a) \\ (x + a - 1)^2 + b & x \in [-a, 0] \\ (|x - 1| - 1)^2 + b & x \in [0, \infty) \end{cases} \end{aligned}$$

$g(f(x))$  is continuous

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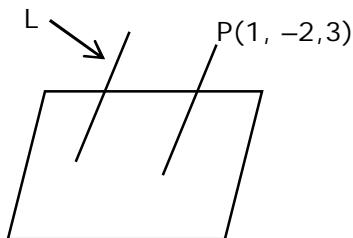
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$$\begin{aligned} \text{at } x = -a & \quad \text{at } x = 0 \\ 1 = b + 1 & \quad (a-1)^2 + b = b \\ b = 0 & \quad \& \quad a = 1 \\ \Rightarrow a + b = 1 & \end{aligned}$$

7. If the distance of the point  $(1, -2, 3)$  from the plane  $x + 2y - 3z + 10 = 0$  measured parallel to the line,

$$\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1} \text{ is } \sqrt{\frac{7}{2}}, \text{ then the value of } |m| \text{ is equal to .....}$$

**Ans. (2)**



**Sol.**

$$\frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$$

Pt. Q  $(3\lambda + 1, -m\lambda - 2, \lambda + 3)$  lie on plane

$$(3\lambda + 1) + 2(-m\lambda - 2) - 3(\lambda + 3) + 10 = 0$$

$$\Rightarrow 3\lambda - 2m\lambda - 3\lambda + 1 - 4 - 9 + 10 = 0$$

$$\Rightarrow -2m\lambda = 2$$

$$m\lambda = -1 \Rightarrow \lambda = -\frac{1}{m}$$

$$Q\left[-\frac{3}{m} + 1, -1, -\frac{1}{m} + 3\right]$$

$$PQ = \sqrt{\frac{7}{2}}$$

$$\sqrt{\left(-\frac{3}{m}\right)^2 + 1 + \left(-\frac{1}{m}\right)^2} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow \frac{10 + m^2}{m^2} = \frac{7}{2}$$

$$\Rightarrow 20 + 2m^2 = 7m^2$$

$$m^2 = 4 \Rightarrow |m| = 2$$

8. Let  $\frac{1}{16}, a$  and  $b$  be in G.P. and  $\frac{1}{a}, \frac{1}{b}, 6$  be in A.P. where  $a, b > 0$ . Then  $72(a+b)$  is equal to .....

**Ans. (14)**

$$\text{Sol. } a^2 = \frac{b}{16} \text{ and } \frac{2}{b} = \frac{1}{a} + 6$$

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Solving, we get  $a = \frac{1}{12}$  or  $a = -\frac{1}{4}$  [rejected]

$$\text{if } a = \frac{1}{12} \Rightarrow b = \frac{1}{9}$$

$$\therefore 72(a+b) = 72\left(\frac{1}{12} + \frac{1}{9}\right) = 14$$

9. Let  $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$  and  $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$  be two  $2 \times 1$  matrices with real entries such that  $A = XB$ , where  $x = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ , and  $k \in \mathbb{R}$ . If  $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$  and  $(k^2 + 1)b_2^2 \neq -2b_1b_2$  then the value of  $k$  is .....

**Ans.** (1)

**Sol.**

$$XB = A$$

$$\frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$b_1 - b_2 = \sqrt{3}a_1 \Rightarrow 3a_1^2 = b_1^2 + b_2^2 - 2b_1b_2$$

$$b_1 + kb_2 = \sqrt{3}a_2 \Rightarrow 3a_2^2 = b_1^2 + k^2b_2^2 + 2kb_1b_2$$

$$3(a_1^2 + a_2^2) = 2b_1^2 + (k^2 + 1)b_2^2 + 2b_1b_2(k - 1)$$

$$(k^2 - 1)b_2^2 + 2b_1b_2(k - 1)$$

$$(k - 1)((k + 1)b_2^2 + 2b_1b_2) = 0$$

$$\Rightarrow k = 1$$

10. For real number  $\alpha, \beta, \gamma$  and  $\delta$ , if

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx \\ = \alpha \log_e\left(\tan^{-1}\left(\frac{x^2 + 1}{x}\right)\right) + \beta \tan^{-1}\left(\frac{\gamma(x^2 - 1)}{x}\right) + \delta \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$$

Where  $C$  is an arbitrary constant, then the value of  $10(\alpha + \beta\gamma + \delta)$  is equal to .....

**Ans.** (6)

$$\int \frac{x^2 - 1}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx + \int \frac{1}{x^4 + 3x^2 + 1} dx$$

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$$\int \frac{1 - \frac{1}{x^2}}{\left[ \left( x + \frac{1}{x} \right)^2 + 1 \right] \tan^{-1} \left( x + \frac{1}{x} \right)} dx + \int \frac{dx}{x^4 + 3x^2 + 1}$$

$\downarrow$                                      $\downarrow$   
 $I_1$                                      $I_2$

$$\tan^{-1} \left( x + \frac{1}{x} \right) = t$$

$$I_1 = \int \frac{dt}{t}$$

$$I_1 = \ln(t) = \ln \left| \tan^{-1} \left( x + \frac{1}{x} \right) \right|$$

Now

$$I_2 = \int \frac{dx}{x^4 + 3x^2 + 1}$$

$$= \frac{1}{2} \int \frac{(x^2 + 1) - (x^2 - 1)}{x^4 + 3x^2 + 1} dx$$

$$= \frac{1}{2} \left[ \int \frac{1 + \frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx - \int \frac{\left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx \right]$$

$$\frac{1}{2} \left[ \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx \right]$$

$$= \quad \quad \quad \downarrow \quad \quad \quad \downarrow$$

$$x - \frac{1}{x} = u \quad \quad \quad x + \frac{1}{x} = v$$

$$= \frac{1}{2} \left[ \int \frac{du}{u^2 + (\sqrt{5})^2} - \int \frac{dv}{v^2 + 1} \right]$$

$$I_2 = \frac{1}{2\sqrt{5}} \tan^{-1} \left( \frac{x - \frac{1}{x}}{\sqrt{5}} \right) - \frac{1}{2} \tan^{-1} \left( x + \frac{1}{x} \right)$$

$$I = I_1 + I_2 = \ln \left| \tan^{-1} \left( x + \frac{1}{x} \right) \right| + \frac{1}{2\sqrt{5}} \ln \left( \frac{x^2 - 1}{\sqrt{5}x} \right) - \frac{1}{2} \tan^{-1} \left( \frac{x^2 + 1}{x} \right) + C$$

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$$\alpha = 1, \beta = \frac{1}{2\sqrt{5}}, \lambda = \frac{1}{\sqrt{5}}, \delta = -\frac{1}{2}$$

$$\begin{aligned}10(\alpha + \beta\lambda + \delta) &= 10\left[1 + \frac{1}{10} - \frac{1}{2}\right] \\&= 10\left(\frac{1}{10} + \frac{1}{2}\right) \\&= 1 + 5 = 6\end{aligned}$$

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