

**JEE
MAIN
4th
Attempt**

MATHEMATICS
27th August 2021 [SHIFT – 1]
QUESTION WITH SOLUTION

JEE | NEET | Foundation

MOTION[®]

29900+
SELECTIONS SINCE 2007

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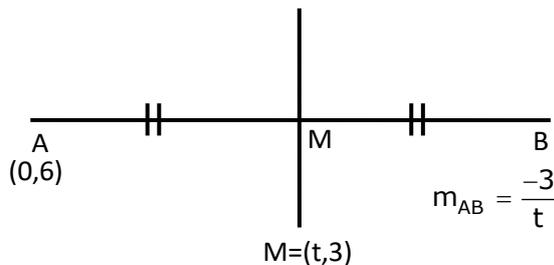
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SECTION – A

1. Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is:
- (1) $3x^2 + 2y - 6 = 0$
 - (2) $2x^2 + 3y - 9 = 0$
 - (3) $2x^2 - 3y + 9 = 0$
 - (4) $3x^2 - 2y - 6 = 0$

Ans. (2)

Sol. A(0, 6) and B(2t, 0)



Perpendicular bisector of AB is

$$(y - 3) = \frac{t}{3}(x - t)$$

$$\text{So, } C = \left(0, 3 - \frac{t^2}{3}\right)$$

Let P be (h, k)

$$h = \frac{t}{2}; k = \left(3 - \frac{t^2}{6}\right)$$

$$\Rightarrow k = 3 - \frac{4h^2}{6} \Rightarrow 2x^2 + 3y - 9 = 0 \text{ option (2)}$$

2. A tangent and a normal are drawn at the point P(2, -4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q (a, b) is a point such that AQBP is a square, then $2a + b$ is equal to:
- (1) -16
 - (2) -20
 - (3) -18
 - (4) -12

Ans. (1)



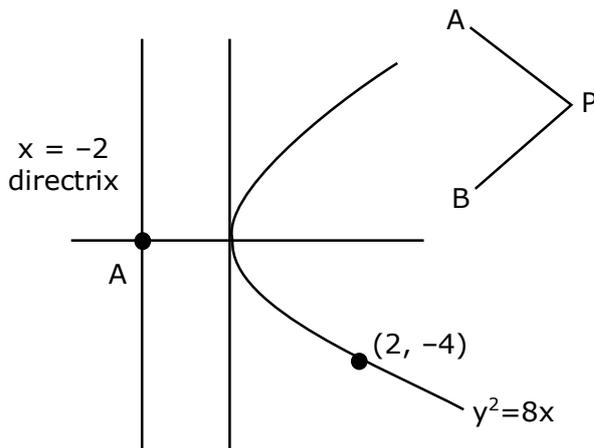
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Sol.



Equation of tangent at $(2, -4)$ ($T = 0$)

$$-4y = 4(x + 2)$$

$$x + y + 2 = 0 \quad \dots(1)$$

equation of normal

$$x - y + \lambda = 0$$

$$\downarrow (2, -4)$$

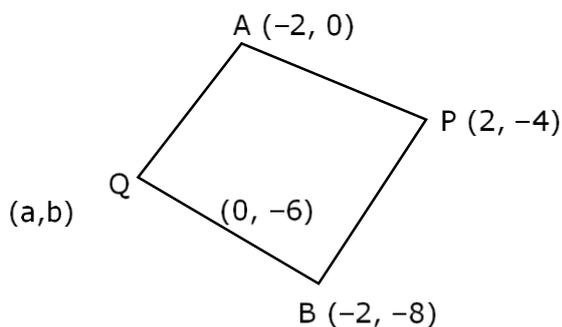
$$\lambda = -6$$

Thus $x - y = 6$ (2) equation of normal

POI of (1) & $x = -2$ is $A(-2, 0)$

POI of (2) & $x = -2$ is $A(-2, 8)$

Given AQB is a sq.



$$\Rightarrow m_{AQ} \cdot m_{AP} = -1$$

$$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a+2 = b \quad \dots(1)$$

Also PQ must be parallel to x-axis thus

$$\Rightarrow b = -4$$

$$\therefore a = -6$$

$$\text{Thus } 2a + b = -16$$



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3. If α, β are the distinct roots of $x^2 + bx + c = 0$, the $\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + bx + c)}{(x - \beta)^2}$ is equal to:

- (1) $b^2 + 4c$
- (2) $2(b^2 - 4c)$
- (3) $b^2 - 4c$
- (4) $2(b^2 + 4c)$

Ans. (2)

Sol.
$$\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2 + bx + c)}{(x - \beta)^2}$$

$$\Rightarrow \lim_{x \rightarrow \beta} \frac{1 \left(1 + \frac{2(x^2 + bx + c)}{1!} + \frac{2^2(x^2 + bx + c)^2}{2!} + \dots \right) - 1 - 2(x^2 + bx + c)}{(x - \beta)^2}$$

$$\Rightarrow \lim_{x \rightarrow \beta} \frac{2(x^2 + bx + c)^2}{(x - \beta)^2}$$

$$\Rightarrow \lim_{x \rightarrow \beta} \frac{2(x - \alpha)^2 (x - \beta)^2}{(x - \beta)^2}$$

$$\Rightarrow 2(\beta - \alpha)^2 = 2(b^2 - 4c)$$

4. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2 \sin x - 5)x - 2 \cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to:

- (1) $3e^{\pi^2} + 5$
- (2) $7e^{\pi^2} + 5$
- (3) $2e^{\pi^2} + 5$
- (4) $e^{\pi^2} + 5$

Ans. (3)

Sol. $\frac{dy}{dx} - 2xy = 2(2 \sin x - 5)x - 2 \cos x$

$$\text{IF} = e^{-x^2}$$

$$\text{So, } y.e^{-x^2} = \int e^{-x^2} (2x(2 \sin x - 5) - 2 \cos x) dx$$

$$\Rightarrow y.e^{-x^2} = e^{-x^2} (5 - 2 \sin x) + c$$

$$\Rightarrow y = 5 - 2 \sin x + c.e^{x^2}$$

$$\text{Given at } x = 0, y = 7$$



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$$\Rightarrow 7 = 5 + c \Rightarrow c = 2$$

$$\text{So, } y = 5 - 2 \sin x + 2e^{x^2}$$

Now at $x = \pi$,

$$y = 5 + 2e^{\pi^2}$$

5. When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6} - x$ and its opposite face occurs with probability $\frac{1}{6} + x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is:

(1) $\frac{1}{9}$

(2) $\frac{1}{12}$

(3) $\frac{1}{8}$

(4) $\frac{1}{16}$

Ans. (3)

Sol. Probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces

$$= 2 \left[\left(\frac{1}{6} - x \right) \left(\frac{1}{6} + x \right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \right]$$

$$= 2 \left[\left(\frac{1}{6} - x \right) \left(\frac{1}{6} + x \right) + \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} \right] = \frac{13}{96}$$

(given)

$$x = \frac{1}{8}$$

6. A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is:

(1) $\frac{10}{3 + 2\sqrt{3}}$

(2) $\frac{5}{3 + \sqrt{3}}$

(3) $\frac{10}{2 + 3\sqrt{3}}$

(4) $\frac{5}{2 + \sqrt{3}}$



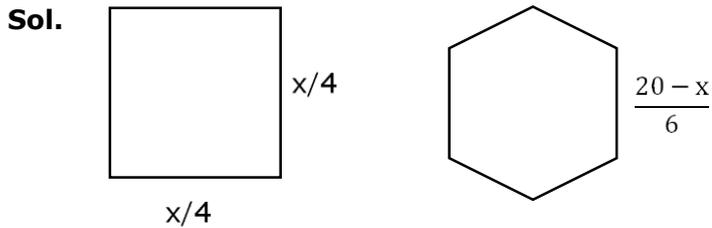
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Ans. (1)



Let the wire is cut into two pieces of length x and $20-x$.

$$\begin{aligned} \text{Area of square} &= \left(\frac{x}{4}\right)^2 & \text{Area of regular hexagon} \\ &= 6 \times \frac{\sqrt{3}}{4} \left(\frac{20-x}{6}\right)^2 \end{aligned}$$

$$\text{Total area} = A(x) = \frac{x^2}{16} + \frac{3\sqrt{3}}{2} \frac{(20-x)^2}{36}$$

$$A'(x) = \frac{2x}{16} + \frac{3\sqrt{3} \times 2}{2 \times 36} (20-x)(-1)$$

$$A'(x) = 0 \text{ at } x = \frac{40\sqrt{3}}{3+2\sqrt{3}}$$

$$\begin{aligned} \text{Length of side of regular Hexagon} &= \frac{1}{6}(20-x) \\ &= \frac{1}{6} \left(20 - \frac{4\sqrt{3}}{3+2\sqrt{3}} \right) \\ &= \frac{10}{2+2\sqrt{3}} \end{aligned}$$

7. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is:

- (1) $-x + 2y + 2z - 3 = 0$
- (2) $3x - 4z + 3 = 0$
- (3) $3x - y - 5z + 2 = 0$
- (4) $4x - y - 5z + 2 = 0$

Ans. (4)

Sol. Required equation of plane

$$P_1 + \lambda P_2 = 0$$

$$(x - y - z - 1) + \lambda(2x + y - 3z + 4) = 0$$

Given that its dist. From origin is $\frac{2}{\sqrt{21}}$



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$$\text{Thus } \frac{|4\lambda - 1|}{\sqrt{(2\lambda + 1)^2 + (\lambda - 1)^2 + (-3\lambda - 1)^2}} = \frac{\sqrt{2}}{\sqrt{21}}$$

$$\Rightarrow 21(4\lambda - 1)^2 = 2(14\lambda^2 + 8\lambda + 3)$$

$$\Rightarrow 336\lambda^2 - 168\lambda + 21 = 28\lambda^2 + 16\lambda + 6$$

$$\Rightarrow 308\lambda^2 - 184\lambda + 15 = 0$$

$$\Rightarrow 308\lambda^2 - 154\lambda - 30\lambda + 15 = 0$$

$$\Rightarrow (2\lambda - 1)(154\lambda - 15) = 0$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ or } \frac{15}{154}$$

for $\lambda = \frac{1}{2}$ reqd. plane is

$$4x - y - 5z + 2 = 0$$

8. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is:

(1) equivalent to $q \rightarrow \sim r$

(2) equivalent to $p \rightarrow \sim r$

(3) a fallacy

(4) a tautology

Ans. (4)

Sol. $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$
 $= (p \wedge (\sim p \vee q) \vee (\sim q \vee r)) \rightarrow r$
 $= ((p \wedge q) \wedge (\sim p \vee r)) \rightarrow r$
 $= (p \wedge q \wedge r) \rightarrow r$
 $= (p \wedge q \wedge r) \vee r$
 $= (\sim p) \vee (\sim q) \vee (\sim r) \vee r$
 \Rightarrow tautology

9. If for $x, y \in \mathbb{R}, x > 0, y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \dots$ upto ∞ terms and $\frac{2 + 4 + 6 + \dots + 2y}{3 + 6 + 9 + \dots + 3y}$

$= \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to:

(1) $(10^6, 6)$

(2) $(10^6, 9)$

(3) $(10^2, 3)$

(4) $(10^4, 6)$

Ans. (2)

Sol. $\frac{2(1 + 2 + 3 + \dots + y)}{3(1 + 2 + 3 + \dots + y)} = \frac{4}{\log_{10} x}$



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$$\Rightarrow \log_{10} x = 6 \Rightarrow x = 10^6$$

Now,

$$y = (\log_{10} x) + \left(\log_{10} x^{\frac{1}{3}}\right) + \left(\log_{10} x^{\frac{1}{9}}\right) + \dots$$

$$= \left(1 + \frac{1}{3} + \frac{1}{9} + \dots\right) \log_{10} x$$

$$= \left(\frac{1}{1 - \frac{1}{3}}\right) \log_{10} x = 9$$

$$\text{So, } (x, y) = (10^6, 9)$$

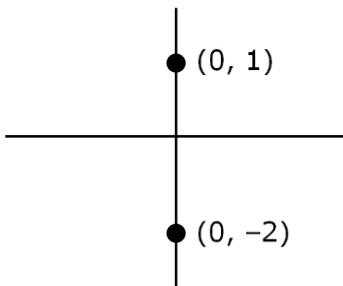
10. If $S = \left\{z \in \mathbb{C} : \frac{z-i}{z+2i} \in \mathbb{R}\right\}$, then:

- (1) S is a circle in the complex plane
- (2) S contains only one element
- (3) S is a straight line in the complex plane
- (4) S contains exactly two elements

Ans. (3)

Sol. Given $\frac{z-i}{z+2i} \in \mathbb{R}$

Then $\arg\left(\frac{z-i}{z+2i}\right)$ is 0 or π



\Rightarrow S is straight line in complex

11. $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$ is equal to:

- (1) 10
- (2) 8
- (3) 6
- (4) 5

Ans. (4)



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Sol. Let $I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x - 22)^2} dx \dots (1)$$

We know

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx \text{ (king)}$$

$$\text{So } I = \int_6^{16} \frac{\log_e (22 - x)^2}{\log_e (22 - x)^2 + \log_e (22 - (22 - x))^2} dx$$

$$I = \int_6^{16} \frac{\log_e (22 - x)^2}{\log_e x^2 + \log_e (22 - x)^2} dx \dots (2)$$

$$(1) + (2)$$

$$2I = \int_6^{16} 1 dx = 10$$

$$I = 5$$

12. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to:

(1) $x \left(\frac{1+x}{1-x} \right) + \log_e (1-x)$

(2) $x \left(\frac{1-x}{1+x} \right) + \log_e (1-x)$

(3) $\frac{1-x}{1+x} + \log_e (1-x)$

(4) $\frac{1+x}{1-x} + \log_e (1-x)$

Ans. (1)

Sol. Let $t = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$

$$= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \dots$$

$$= 2(x^2 + x^3 + x^4 + \dots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$$

$$= \frac{2x^2}{1-x} - (\ln(1-x) - x)$$

$$\Rightarrow t = \frac{2x^2}{1-x} + x - \ln(1-x)$$



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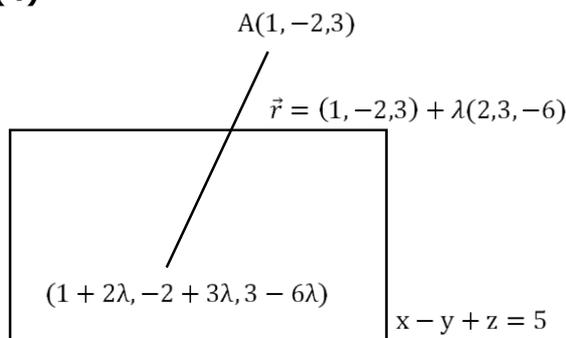
$$\Rightarrow t = \frac{x(1+x)}{1-x} - \ln(1-x)$$

13. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is:

- (1) 2
- (2) 5
- (3) 3
- (4) 1

Ans. (4)

Sol.



$$(1 + 2\lambda) + 2 - 3\lambda + 3 - 6\lambda = 5$$

$$\Rightarrow 6 - 7\lambda = 5 \Rightarrow \lambda = \frac{1}{7}$$

$$\text{So, } P = \left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7} \right)$$

$$AP = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$$

$$AP = \sqrt{\left(\frac{4}{49}\right) + \frac{9}{49} + \frac{36}{49}} = 1$$

14. If $x^2 + 9y^2 - 4x + 3 = 0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals:

- (1) $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$
- (2) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$
- (3) $[1, 3]$ and $[1, 3]$
- (4) $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Ans. (1)

Sol. $x^2 + 9y^2 - 4x + 3 = 0$

$$(x^2 - 4x) + (9y^2) + 3 = 0$$



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$$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$$

$$(x - 2)^2 + (3y^2) = 1$$

$$\frac{(x - 2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1 \text{ (equation of an ellipse).}$$

As it is equation of an ellipse, x & y can vary inside the ellipse.

$$\text{So, } x - 2 \in [-1, 1] \text{ and } y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$$

$$x \in [1, 3] \quad y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$$

- 15.** Let $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$, where A, B, C are angles of a triangle ABC . If the lengths of the sides opposite these angles are a, b, c respectively, then:
- (1) c^2, a^2, b^2 are in A.P.
 - (2) $b^2 - a^2 = a^2 + c^2$
 - (3) b^2, c^2, a^2 are in A.P.
 - (4) a^2, b^2, c^2 are in A.P.

Ans. (3)

Sol. $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$

As A, B, C are angles of triangle

$$A + B + C = \pi$$

$$A = \pi - (B + C)$$

$$\text{So, } \sin A = \sin(B + C) \dots (1)$$

$$\text{Similarly, } \sin B = \sin(A + C) \dots$$

From (1) and (2)

$$\frac{\sin(B + C)}{\sin(A + C)} = \frac{\sin(A - C)}{\sin(C - B)}$$

$$\sin(C + B) \cdot \sin(C - B) = \sin(A - C) \sin(A + C)$$

$$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$$



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$$\{\because \sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y\}$$

$$2\sin^2 C = \sin^2 A + \sin^2 B$$

By sine rule

$$2c^2 = a^2 + b^2$$

$\Rightarrow b^2, c^2$ and a^2 are in A.P.

16. $\sum_{k=0}^{20} \binom{20}{k}^2$ is equal to:

(1) ${}^{41}C_{20}$

(2) ${}^{40}C_{20}$

(3) ${}^{40}C_{21}$

(4) ${}^{40}C_{19}$

Ans. (2)

Sol. $\sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k}$

Sum of suffix is const. so summation will be ${}^{40}C_{20}$.

17. If $U_n = \left(1 + \frac{1}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right)^2 \dots \left(1 + \frac{n^2}{n^2}\right)^n$, then $\lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$ is equal to:

(1) $\frac{4}{e^2}$

(2) $\frac{4}{e}$

(3) $\frac{16}{e^2}$

(4) $\frac{e^2}{16}$

Ans. (4)

Sol. $U_n = \prod_{r=1}^n \left(1 + \frac{r^2}{n^2}\right)^r$

$$L = \lim_{n \rightarrow \infty} (U_n)^{\frac{-4}{n^2}}$$

$$\log L = \lim_{n \rightarrow \infty} \frac{-4}{n^2} \sum_{r=1}^n \log \left(1 + \frac{r^2}{n^2}\right)^r$$



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$$\Rightarrow \log L = \lim_{n \rightarrow \infty} \sum_{r=1}^n -\frac{4r}{n} \cdot \frac{1}{n} \log \left(1 + \frac{r^2}{n^2} \right)$$

$$\Rightarrow \log L \Rightarrow -4 \int_0^1 x \log(1+x^2) dx$$

put $1+x^2 = t$

Now, $2x dx = dt$

$$= -2 \int_1^2 \log(t) dt = -2 [t \log t - t]_1^2$$

$$\Rightarrow \log L = -2(2 \log 2 - 1)$$

$$\therefore L = e^{-2(2 \log 2 - 1)}$$

$$= e^{-2 \left(\log \left(\frac{4}{e} \right) \right)}$$

$$= e^{\log \left(\frac{4}{e} \right)^{-2}}$$

$$= \left(\frac{e}{4} \right)^2 = \frac{e^2}{16}$$

18. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is:

- (1) 1
- (2) -1
- (3) $-\frac{1}{2}$
- (4) $\frac{1}{2}$

Ans. (4)

Sol. Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$$A^4 + 3IA = 2I$$

$$\Rightarrow A^4 = 2I - 3A$$

Also characteristic equation of A is



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$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 0 - \lambda & 2 \\ k & -1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda + \lambda^2 - 2k = 0$$

$$\Rightarrow A + A^2 = 2KI$$

$$\Rightarrow A^2 = 2KI - A$$

$$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$$

$$\text{Put } A^2 = 2KI - A$$

$$\text{and } A^4 = 2I - 3A$$

$$2I - 3A = 4K^2I + 2KI - A - 4AK$$

$$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$$

$$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$$

$$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$$

$$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$$

$$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$$

$$\Rightarrow K = \frac{1}{2}$$

19. Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$. Then:

(1) $x^2 + 2xf'(x) + 4 = 0$

(2) $x^3 - 3xf'(x) - 4 = 0$

(3) $x^3 + xf'(x) + 12 = 0$

(4) $x^2 + 2xf'(x) - 12 = 0$

Ans. (2)

Sol. $y + \frac{xdy}{dx} = x^2$ (given)



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$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x$$

$$\text{If } = e^{\int \frac{1}{x} dx} = x$$

Solution of DE

$$\Rightarrow y \cdot x = \int x \cdot dx$$

$$\Rightarrow xy = \frac{x^3}{3} + \frac{c}{3}$$

Passes through $(-2, 2)$, so

$$-12 = -8 + c \Rightarrow c = -4$$

$$\therefore 3xy = x^3 - 4$$

$$\text{i.e. } 3x \cdot f(x) = x^3 - 4$$

20. If $(\sin^{-1} x)^2 - (\cos^{-1} x)^2 = a$; $0 < x < 1, a \neq 0$, then the value of $2x^2 - 1$ is:

(1) $\cos\left(\frac{2a}{\pi}\right)$

(2) $\sin\left(\frac{4a}{\pi}\right)$

(3) $\cos\left(\frac{4a}{\pi}\right)$

(4) $\sin\left(\frac{2a}{\pi}\right)$

Ans. (4)

Sol. Given

$$a = (\sin^{-1} x)^2 - (\cos^{-1} x)^2$$

$$= (\sin^{-1} x + \cos^{-1} x)(\sin^{-1} x - \cos^{-1} x)$$

$$= \frac{\pi}{2} \left(\frac{\pi}{2} - 2 \cos^{-1} x \right)$$

$$\Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} - \frac{2a}{\pi}$$



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$$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$$

option (2)

Section B

1. Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and \vec{a} is perpendicular to \vec{b} . Then the greatest amongst the values of $|\vec{a}|^2$ is _____.

Ans. (90)

Sol. since, $\vec{a} \cdot \vec{b} = 0$

$$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16$$

Also,

$$|\vec{b} \times \vec{c}|^2 = 75 \Rightarrow (10 + \beta^2)14 - (5 - 3\beta)^2 = 75$$

$$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$$

$$\Rightarrow \beta = -4, -2$$

$$\Rightarrow \alpha = 4, 8$$

$$\Rightarrow |\vec{a}|_{\max}^2 = (26 + \alpha^2)_{\max} = 90$$

2. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _____.

Ans. (4)

Sol. $3x^4 + 4x^3 - 12x^2 + 4 = 0$

So, Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$$\therefore f'(x) = 12x(x^2 + x - 2)$$

$$= 12x(x + 2)(x - 1)$$

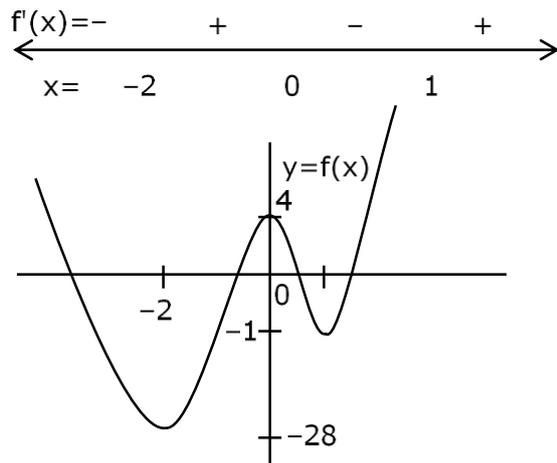


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3. Let n be an odd natural number such that the variance of $1, 2, 3, 4, \dots, n$ is $1(4)$. Then n is equal to _____.

Ans. (13)

Sol. $\frac{n^2 - 1}{12} = 14 \Rightarrow n = 13$

(4) If the system of linear equations

$$2x + y - z = 3$$

$$x - y - z = \alpha$$

$$3x + 3y + \beta z = 3$$

Has infinitely many solutions, then $\alpha + \beta - \alpha\beta$ is equal to _____.

Ans. (5)

Sol. $2 \times (i) - (ii) - (iii)$ gives:

$$-(1 + \beta)z = 3 - \alpha$$

For infinitely many solutions.

$$\beta + 1 = 0 = 3 - \alpha \Rightarrow (\alpha, \beta) = (3, -1)$$

$$\text{Hence, } \alpha + \beta - \alpha\beta = 5$$

5. If $A = \{x \in \mathbb{R} : |x - 2| > 1\}$, $B = \{x \in \mathbb{R} : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbb{R} : |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _____.

Ans. (256)

Sol. $A = (-\infty, 1) \cup (3, \infty)$

$$B = (-\infty, -2) \cup (2, \infty)$$

$$C = (-\infty, 2] \cup [6, \infty)$$

$$\text{So, } A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$$

$$Z \cap (A \cap B \cap C)^c = \{-2, -1, 0, 1, 2, 3, 4, 5\}$$

$$\text{Hence no. of its subsets} = 2^8 = 256$$



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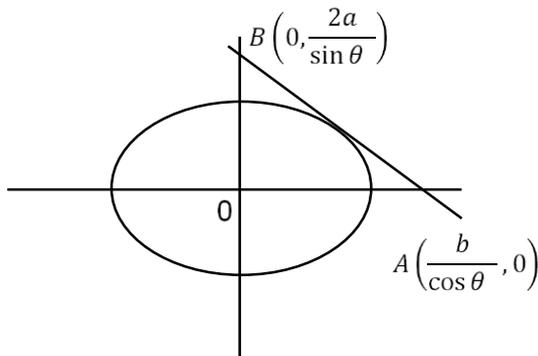


6. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab , then k is equal to _____.

Ans. (2)

Sol. Tangent

$$\frac{x \cos \theta}{b} + \frac{y \sin \theta}{2a} = 1$$



$$\begin{aligned} \text{So, area } (\Delta OAB) &= \frac{1}{2} \times \frac{b}{\cos \theta} \times \frac{2a}{\sin \theta} \\ &= \frac{2ab}{\sin 2\theta} \geq 2ab \\ \Rightarrow k &= 2 \end{aligned}$$

7. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _____.

Ans. (100)

5	a	b	b	a	5
---	---	---	---	---	---

Sol.

It is always divisible by 5 and 1(1)
So, required number = $10 \times 10 = 100$

8. If $\int \frac{dx}{(x^2 + x + 1)^2} = a \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + b \left(\frac{2x+1}{x^2 + x + 1} \right) + C$, $x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3} a + b)$ is equal to _____.

Ans. (15)

Sol.
$$I = \int \frac{dx}{\left[\left(x + \frac{1}{2} \right)^2 + \frac{3}{4} \right]^2}$$

$$\int \frac{dt}{\left(t^2 + \frac{3}{4} \right)^2} \quad \left(\text{Put } x + \frac{1}{2} = t \right)$$



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$$\begin{aligned}
 &= \frac{\sqrt{3}}{2} \int \frac{\sec^2 \theta d\theta}{\frac{9}{16} \sec^4 \theta} \quad \left(\text{Put } t = \frac{\sqrt{3}}{2} \tan \theta \right) \\
 &= \frac{4\sqrt{3}}{9} \int (1 + \cos 2\theta) d\theta \\
 &= \frac{4\sqrt{3}}{9} \left[\theta + \frac{\sin 2\theta}{2} \right] + c \\
 &= \frac{4\sqrt{3}}{9} \left[\tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + \frac{\sqrt{3}(2x+1)}{3+(2x+1)^2} \right] + c \\
 &= \frac{4\sqrt{3}}{9} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + \frac{1}{3} \left(\frac{2x+1}{x^2+x+1} \right) + c
 \end{aligned}$$

Hence, $9(\sqrt{3}a + b) = 15$

9. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2 - 1) \frac{d^2y}{dx^2} + \alpha x \frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _____.

Ans. (17)

Sol. $y^{1/4} + \frac{1}{y^{1/4}} = 2x$

$$\Rightarrow \left(y^{1/4} \right)^2 - 2xy^{1/4} + 1 = 0$$

$$\Rightarrow y^{1/4} = x + \sqrt{x^2 - 1} \quad \text{Or } x - \sqrt{x^2 - 1}$$

So, $\frac{1}{4} \frac{1}{y^{3/4}} \frac{dy}{dx} = 1 + \frac{x}{\sqrt{x^2 - 1}}$

$$\Rightarrow \frac{1}{4} \frac{1}{y^{3/4}} \frac{dy}{dx} = \frac{y^{1/4}}{\sqrt{x^2 - 1}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{4y}{\sqrt{x^2 - 1}} \dots (1)$$

Hence, $\frac{d^2y}{dx^2} = 4 \frac{(\sqrt{x^2 - 1})y' - \frac{yx}{\sqrt{x^2 - 1}}}{x^2 - 1}$

$$\Rightarrow (x^2 - 1)y'' = 4 \frac{(x^2 - 1)y' - xy}{\sqrt{x^2 - 1}}$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left(\sqrt{x^2 - 1}y' - \frac{xy}{\sqrt{x^2 - 1}} \right)$$

$$\Rightarrow (x^2 - 1)y'' = 4 \left(4y - \frac{xy'}{4} \right) \text{ (from I)}$$

$$\Rightarrow (x^2 - 1)y'' + xy' - 16y = 0$$



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So, $|\alpha - \beta| = 17$

- 10.** Let the equation $x^2 + y^2 + px + (1-p)y + 5 = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2 \text{ and } q \text{ is an integer}\}$ is _____.

Ans. (61)

Sol. $r = \sqrt{\frac{p^2}{4} + \frac{(1-p)^2}{4}} - 5 = \frac{\sqrt{2p^2 - 2p - 19}}{2}$

Since, $r \in (0, 5]$

So, $0 < 2p^2 - 2p - 19 \leq 100$

$\Rightarrow p \in \left[\frac{1 - \sqrt{239}}{2}, \frac{1 - \sqrt{39}}{2} \right) \cup \left(\frac{1 + \sqrt{39}}{2}, \frac{1 + \sqrt{239}}{2} \right]$

So, number of integral values of p^2 is 6(1)



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