JEE MAIN 4th Attempt

MATHEMATICS

26th August 2021 [SHIFT – 2]

QUESTION WITH SOLUTION

हो चुकी है ऑफलाइन क्लासरुम की शुरुआत
अपने सपने को करो साकार, कोटा कोचिंग के साथ

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SECTION - A

1. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+4\hat{k}) = 16$ and $\vec{r}.(-\hat{i}+\hat{j}+\hat{k}) = 6$.

Then which of the following points does NOT lie on P ?

(1)(3, 3, 2) (2) (−8, 8, 6)

(3) (4, 2, 2) (4) (6, −6, 2)

Ans. **(3)**

Sol. $(x + y + 4z - 16) + \lambda(-x + y + z - 6) = 0$

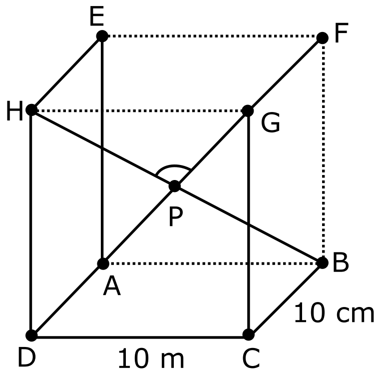
Passes through (1,2,3)

$-1 + \lambda(-2) \Rightarrow \lambda = -\dfrac{1}{2}$

$2(x + y + 4z - 16) - (-x + y + z - 6) = 0$

$3x + y + 7z - 26 = 0$

2. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\dfrac{1}{5}$, then the height of the hall (in meters) is :



(1) $5\sqrt{3}$ (2) 5

(3) $2\sqrt{10}$ (4) $5\sqrt{2}$

Ans. **(4)**

Sol. $A(\hat{j})$. $B(10\hat{i})$

$\mathbf{H}(h\hat{j}+10\hat{k})$

$\mathbf{G}(10\hat{i}+h\hat{j}+10\hat{k})$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$

$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$

$\cos\theta = \dfrac{\overrightarrow{AG}\ \overrightarrow{BH}}{\left|\overrightarrow{AG}\right|\left|\overrightarrow{BH}\right|}$

$\dfrac{1}{5} = \dfrac{h^2}{h^2 + 200}$

$4h^2 = 200 \Rightarrow h = 5\sqrt{2}$

3. Consider the two statements:
(S1) : $(p \to q) \vee (\sim q \to p)$ is a tautology
(S2): $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.
Then:
(1) only (S1) is true.
(2) only (S2) is true.
(3) both (S1) and (S2) are true.
(4) both (S1) and (S2) are false

Ans. **(3)**

Sol. $S_1 : (\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$
$S_1 = q \vee (\sim p \vee p) = q \vee t = t$ = tautology
$S_2 : (p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$
= fallacy

4. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

(1) A^5
(2) A^6
(3) $A^6 - A$
(4) $A^5 - A$

Ans. **(3)**

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left[-\frac{1}{2}, \infty\right) - \{0\}$
(2) $\left(-\frac{1}{2}, \infty\right) - \{0\}$
(3) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$
(4) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$

Ans. **(1)**

Sol. $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$$

$$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$$

$$x \in \left[-\frac{1}{2}, 0\right) - \{0\}$$

6. The value of

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is :

(1) $\frac{1}{8}$ (2) $\frac{1}{8\sqrt{2}}$

(3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{4}$

Ans. **(1)**

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$

$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$

7. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}$ (p + iq), then p and q are roots of the equation :

(1) $x^2 + (\sqrt{3} + 1)x + \sqrt{3} = 0$ (2) $x^2 + (\sqrt{3} - 1)x - \sqrt{3} = 0$

(3) $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ (4) $x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

Ans. **(4)**

Sol. $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p+iq)$

$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p+iq)$

$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$p = -1,\ q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

8. If $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$, then the value of tan p is:

(1) $\frac{51}{50}$ (2) $\frac{101}{102}$

(3) 100 (4) $\frac{50}{51}$

Ans. (4)

Sol. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$\tan^{-1}(101) - \tan^{-1} 1 \Rightarrow \tan^{-1}\frac{50}{51}$

9. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}\,dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in R$, $5\alpha + 6\beta = 0$, and [x] denotes the greatest integer less than or equal to x, then the value of $(\alpha + \beta)^2$ is equal to :

(1) 36 (2) 100

(3) 16 (4) 25

Ans. (4)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}}\,dx$

$\int_0^1 \frac{x}{e^x}dx + \int_1^2 \frac{x+1}{e^{x-1}}dx + \int_2^3 \frac{x+2}{e^{x-2}}dx + \ldots\ldots \int_4^5 \frac{x+4}{e^{x-4}}dx$

$\downarrow \quad \downarrow \quad \downarrow$

$x = t + 1 \quad x = z + 2 \quad x = y + 4$

$\int_0^1 \frac{t+2}{e^t}dt + \int_0^1 \frac{z+4}{e^z}dz + \ldots\ldots + \int_0^1 \frac{y+4}{e^y}dy$

$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dt = 5\int_0^1 \frac{x+4}{e^x}dx$

$\Rightarrow 5\int_0^1 (x+4)e^{-x}\,dx$

$\Rightarrow 5e^{-x}(-x-5)]_0^1 \Rightarrow -\frac{30}{e} + 25$

$\alpha = -30$

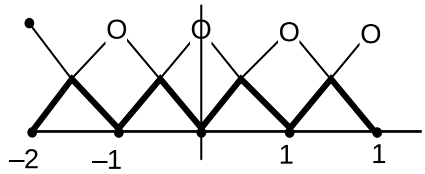
$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

10. Let [t] denote the greatest integer less than or equal to t.
Let f(x) = x – [x], g(x) =1 – x + [x], and h(x) = min {f(x), g(x)}, x ∈[–2, 2]. Then h is :
(1) not continuous at exactly four points in [–2, 2]
(2) not continuous at exactly three points in [–2, 2]
(3) continuous in [–2,2] but not differentiable at exactly three points in (–2, 2)
(4) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

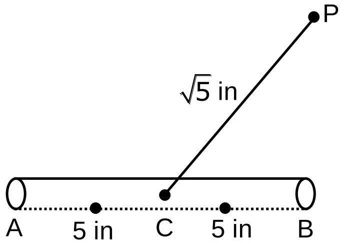
Ans. **(4)**

Sol. min{x – [x], 1 – x + [x]}
h(x) = min {x –[x], 1–(x–[x])}



⇒ always continuous in [–2,2]
but non differentiable at 7 points

11. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and ∠PCB = $\tan^{-1}(2)$.
The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

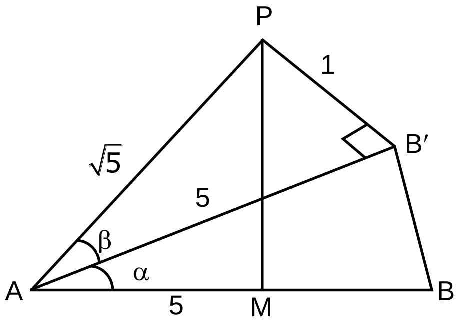


(1) $\tan^{-1}\left(\frac{4}{3}\right)$ (2) $\tan^{-1}\left(\frac{3}{4}\right)$

(3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $\tan^{-1}(1)$

Ans. **(2)**

Sol.



From figure

$\sin\beta = \frac{1}{\sqrt{5}}$

$\therefore \tan\beta = \frac{1}{2}$

$\tan(\alpha + \beta) = 2$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta} = 2$

$\frac{\tan\alpha + \frac{1}{2}}{1 - \tan\alpha\left(\frac{1}{2}\right)} = 2$

$\tan\alpha = \frac{3}{4}$

$\boxed{\alpha = \tan^{-1}\left(\frac{3}{4}\right)}$

12. A circle C touches the line x = 2y at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $\sqrt{285}$ (2) $4\sqrt{15}$

(3) 15 (4) $7\sqrt{5}$

Ans. **(4)**

Sol. $(x - 2)^2 + (y - 1)^2 + \lambda(x - 2y) = 0$

$C : x^2 + y^2 + x(\lambda - 4) + y(-2-2\lambda) + 5 = 0$

$C_1 : x^2 + y^2 + 2y - 5 = 0$

$S_1 - S_2 = 0$ (Equation of PQ)

$(\lambda - 4)x - (2\lambda + 4)y + 10 = 0$ Passes through (0, –1)

$\Rightarrow \quad \lambda = -7$

$C : x^2 + y^2 - 11x + 12y + 5 = 0$

$= \frac{\sqrt{245}}{4}$

Diameter $= 7\sqrt{5}$

13. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(x \ge 5 | x > 2)$ is :

(1) $\frac{25}{36}$ (2) $\frac{11}{36}$

(3) $\frac{125}{216}$ (4) $\frac{5}{6}$

Ans. **(1)**

Sol. $P(x \ge 5 \mid x > 2) = \frac{P(x \ge 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 . \frac{1}{6} + \left(\frac{5}{6}\right)^5 . \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 . \frac{1}{6} + \left(\frac{5}{6}\right)^3 . \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\dfrac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1-\frac{5}{6}}}{\dfrac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1-\frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

14. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$$x + y + z = 5$$
$$x + 2y + 3z = \mu$$
$$x + 3y + \lambda z = 1$$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(2) $p = \frac{1}{6}$ and $q = \frac{1}{36}$

(3) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

(4) $p = \frac{5}{6}$ and $q = \frac{1}{36}$

Ans. **(1)**

Sol. $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$

$p = \frac{5}{6}$

$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

15. The local maximum value of the function

$f(x) = \left(\frac{2}{x}\right)^{x^2}$, $x > 0$ is :

(1) $(e)^{\frac{2}{e}}$

(2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$

(4) 1

Ans. **(1)**

Sol. $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

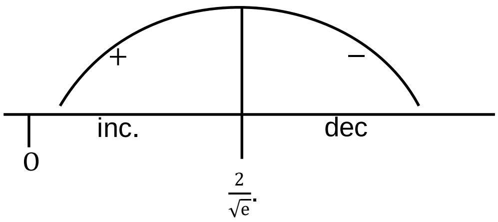
$\ell nf(x) = x^2 (\ell n2 - \ell nx)$

$f'(x) = f(x) \{-x + (\ell n2 - \ell nx)2x\}$

$f'(x) = \underset{+}{f(x)}.\underset{+}{x}\underbrace{(2\ell n2 - 2\ell nx - 1)}_{g(x)}$

$g(x) = 2\ell n^2 - 2\ell n\, x - 1$

$= \ell n\frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$



$LM = \frac{2}{\sqrt{e}}$

Local maximum value $= \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

16. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$. Then y(1) is equal to:

(1) 0 (2) 2

(3) $\log_e(2e)$ (4) $\log_e 2$

Ans. **(4)**

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$

$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow$ Put $e^{-y} = z$

$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$

$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$

$Xe^{-y} = \frac{1}{2}\log_e x + x$, passes through (e, 1)

$\Rightarrow c = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

17. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$ is :

(1) $x^3(x - 2) = y^2$
(2) $x^2(x - 2) = y^3$
(3) $y^2(x - 2) = x^3$
(4) $y^3(x - 2) = x^2$

Ans. (3)

Sol. $T = S_1$

$xh - yk = h^2 - k^2$

$$y = \frac{xh}{2k} - \frac{(h^2 - k^2)}{k}$$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

$$\left(\frac{k^2 - h^2}{k}\right) = \frac{2k}{h}$$

$2y^2 = x(y^2 - x^2)$

$y^2(x - 2) = x^3$

18. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1 + \sin^2 x}{1 + \pi^{\sin x}}\right) dx$ is :

(1) $\frac{3\pi}{2}$
(2) $\frac{\pi}{2}$
(3) $\frac{3\pi}{4}$
(4) $\frac{5\pi}{4}$

Ans. (3)

Sol. $$I = \int_0^{\pi/2} \frac{(1 + \sin^2 x)}{(1 + \pi^{\sin x})} + \frac{\pi^{\sin x}(1 + \sin^2 x)}{(1 + \pi^{\sin x})} dx$$

$$I = \int_0^{\pi/2} (1 + \sin^2 x) dx$$

$$I = \frac{\pi}{2} + \frac{\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$$

19. $\lim_{x \to 2} \left(\sum_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}\right)$ is equal to :

(1) $\frac{7}{36}$
(2) $\frac{5}{24}$
(3) $\frac{1}{5}$
(4) $\frac{9}{44}$

Ans. (4)

Sol. $$S = \lim_{x \to 2} \sum_{n=1}^{9} \frac{x}{n(n+1)x^2 + 2(2n+1)x + 4}$$

$$S = \sum_{n=1}^{9} \frac{x}{4(n^2 + 3n + 2)} = \frac{1}{2} \sum_{n=1}^{9} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{11}\right) = \frac{9}{44}$$

20. The point P $(-2\sqrt{6}, \sqrt{3})$ lies on the hyerpbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyerpbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to :

(1) $6\sqrt{3}$ (2) $4\sqrt{3}$

(3) 6 (4) $3\sqrt{6}$

Ans. (1)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

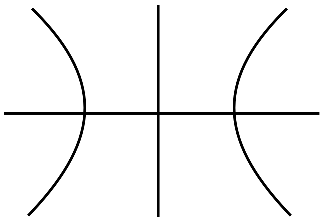
$\Rightarrow \quad \frac{24}{a^2}-\frac{3}{b^2}=1$.......(i)

$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4}-1\right) \Rightarrow 4b^2 = a^2$

Put in (i) $\Rightarrow \frac{6}{b^2}-\frac{3}{b^2}=1 \Rightarrow b=\sqrt{3}$

$\Rightarrow a = \sqrt{12}$

$\frac{x^2}{12}-\frac{y^2}{3}=1$



Tangent at P :

$\frac{-x}{\sqrt{6}}-\frac{y}{\sqrt{3}}=1 \Rightarrow Q(0, \sqrt{3})$

Slope of T = $-\frac{1}{\sqrt{2}}$

Normal at P :

$y-\sqrt{3}=\sqrt{2}(x+2\sqrt{6})$

$\Rightarrow \quad R = (0, 5\sqrt{3})$

$QR = 6\sqrt{3}$

Section B

1. Let $\begin{pmatrix} n \\ k \end{pmatrix}$ denote ${}^{n}C_{k}$ and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \begin{pmatrix} n \\ k \end{pmatrix}, \text{ if } 0 \le k \le n \\ 0, \text{otherwise} \end{cases}$$

If $A_k = \sum_{i=0}^{9} \begin{pmatrix} 9 \\ i \end{pmatrix} \begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8} \begin{pmatrix} 8 \\ i \end{pmatrix} \begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$ and $A_4 - A_3 = 190$ p, then p is equal to ________

Ans. **(49)**

Sol. $A_k = \sum_{i=0}^{9} {}^{9}C_i \, {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^{8}C_i \, {}^{13}C_{k-i}$

$A_k = {}^{21}C_k + {}^{21}C_k = 2.\,{}^{21}C_k$

$A_4 - A_3 = 2\,({}^{21}C_4 - {}^{21}C_3) = 2(5985 - 1330)$

$190p = 2\,(5985 - 1330) \Rightarrow p = 49$

2. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer is

Ans. **(6)**

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{(2)^{\frac{n+2}{2}} i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This positive integer for n = 6

3. If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to ______________.

Ans. **(5)**

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = (2 - \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

$\frac{\vec{a}.\vec{b}}{|\vec{b}|} = 1, \vec{a}.\vec{b} = 12 - \lambda$

$(\vec{a}.\vec{b}) = |\vec{b}|^2$

$\lambda^2 - 24\lambda + 144 = \lambda^2 - 4\lambda + 4 + 40$

$20\lambda = 100 \Rightarrow \lambda = 5.$

4. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is __________.

Ans. **(7744)**

Sol. 209, 220, 231, ….., 495

Sum = $\frac{27}{2}(209 + 495) = 9504$

Number containing 1 at unit place $\underline{2}\ \underline{3}\ \underline{1}$, $\underline{3}\ \underline{4}\ \underline{1}$, $\underline{4}\ \underline{5}\ \underline{1}$

Number containing 1 at 10th place $\underline{3}\ \underline{1}\ \underline{9}$, $\underline{4}\ \underline{1}\ \underline{8}$

Required = 9501 – (231 + 341 + 451 + 319 + 418) = 7744

5. Let A be a 3 × 3 real matrix. If det (2Adj (2Adj (Adj (2A)))) = 2^{41}, then the value of det (A^2) equals _________.

Ans. **(4)**

Sol. adj (2A) = 2^2 adjA

$\Rightarrow$ adj(adj (2A)) = adj(4 adjA) = 16 adj (adj A)

= 16 |A|A

$\Rightarrow$ adj (32 |A| A) = $(32\,|A|)^2$ adj A

$12(32|A|)^2\ |\text{adj } A| = 2^3\,(32|A|)^6\ |\text{adj } A|$

$2^3.2^{30}\ |A|^6\ .\ |A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = 4$

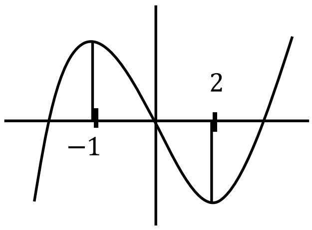
6. Let a and b respectively be the points of local maximum and local minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$.

If A is the total area of the region bounded by y = f(x), the x-axis and the lines x = a and x = b, then 4 A is equal to __________

Ans. **(114)**

Sol. $f'(x) = 6x^2 - 6x - 12 = 6(x - 2)(x + 1)$

Point = (2,–20) & (–1,7)



$$A = \int_{-1}^{0}(2x^3 - 3x^2 - 12x)\,dx + \int_{0}^{2}(12x + 3x^2 - 2x^3)\,dx$$

$$A = \left(\frac{x^4}{2} - x^3 - 6x^2\right)_{-1}^{0} + \left(6x^2 + x^3 - \frac{x^4}{2}\right)_{0}^{2}$$

4A = 114

7. Let $a_1, a_2 \ldots\ldots a_{10}$ be an AP with common difference –3 and $b_1, b_2 \ldots\ldots b_{10}$ be a GP with common ratio 2. Let $c_k = a_k + b_k$, k = 1, 2, ..., 10. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to ________.

Ans. **(2021)**

Sol. $c_2 = a_2 + b_2 = a_1 - 3 + 2b_1 = 12$

$a_1 + 2b_1 = 15$ ....(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ....(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$$\sum_{k=1}^{10} C_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \frac{10}{2}(2 \times 11 + 9 \times (-3)) + \frac{2(2^{10} - 1)}{2 - 1}$$

$= 5(22 - 27) + 2(1023)$

$= 2046 - 25 = 2021$

8. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to ___________.

Ans. **(18)**

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ .... (1)

$\alpha^2 - \alpha + 2\lambda = 0$ .... (2)

Equation (1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in Equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda = 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\therefore \frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3} \times 3}{\frac{1}{9}} = 18$$

9. Let Q be the foot of the perpendicular from the point P(7, −2, 13) on the plane containing the lines , $\frac{x+1}{6}=\frac{y-1}{7}=\frac{z-3}{8}$ and $\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}$.

Then $(PQ)^2$, is equal to ________

Ans. **(96)**

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

$x - 2y + z = 0$

$PQ = \left|\frac{7+4+13}{\sqrt{6}}\right| = 4\sqrt{6}$

$PQ^2 = 96$

10. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x − y is __________.

Ans. **(12)**

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x+y=10$

$Var(x) = 10 = \frac{3^2+7^2+x^2+y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x, y) = (9, 1)$

Four numbers are 21, 9, 10, 8

$Mean = \frac{48}{4} = 12$

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