

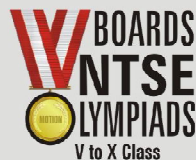
हमारा विश्वास... हर एक विद्यार्थी है सवास

**JEE
MAIN
JAN
2020**

PAPER WITH SOLUTION

8th January 2020 _ SHIFT - 1

MATHEMATICS



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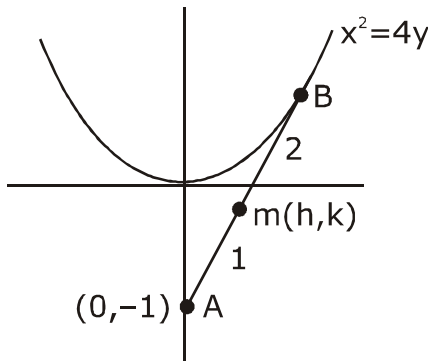
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1. The locus of a point which divides the line segment joining the point $(0, -1)$ and a point on the Parabola, $x^2 = 4y$, internally in the ratio $1 : 2$, is :

बिन्दु $(0, -1)$ तथा परवलय $x^2 = 4y$ पर स्थित एक बिन्दु को मिलाने वाले रेखाखण्ड का $1 : 2$ के अनुपात में अंतः विभाजन करने वाले बिन्दु का बिंदुपथ है :

(1) $x^2 - 3y = 2$ (2) $4x^2 - 3y = 2$ (3) $9x^2 - 3y = 2$ (4) $9x^2 - 12y = 8$

Sol. 4



B : $(3h, 3k + 2)$
lies on $x^2 = 4y$
 $(3h)^2 = 4(3k + 2)$
 $9x^2 = 12y + 8$

2. For $a > 0$, let the curves $C_1 : y^2 = ax$ and $C_2 : x^2 = ay$ intersect at origin O and a point P. Let the line $x = b$ ($0 < b < a$) intersect the chord OP and the x-axis at points Q and R, respectively. If

the line $x = b$ bisects the area bounded by the curves, C_1 , and C_2 , and the area of $\Delta OQR = \frac{1}{2}$, then

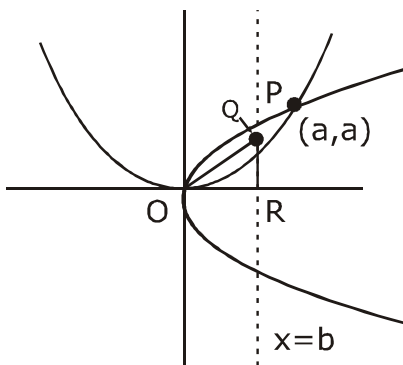
'a' satisfies the equation :

$a > 0$ के लिए, माना वक्र $C_1 : y^2 = ax$ तथा $C_2 : x^2 = ay$ मूलबिन्दु O तथा एक बिन्दु P पर काटते हैं। माना रेखा $x = b$ ($0 < b < a$) जीवा OP तथा x-अक्ष को क्रमशः बिन्दुओं Q तथा R पर काटती है। यदि रेखा $x=b$ वक्रों C_1 तथा C_2 द्वारा परिबद्ध

क्षेत्र को समद्विभाजित करती है तथा ΔOQR का क्षेत्रफल $= \frac{1}{2}$ है 'a' जिस समीकरण को संतुष्ट करता है वह है :

(1) $x^6 - 12x^3 + 4 = 0$ (2) $x^6 - 6x^3 + 4 = 0$ (3) $x^6 - 12x^3 - 4 = 0$ (4) $x^6 + 6x^3 - 4 = 0$

Sol. 1



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for p solve

$$x^2 = ay \text{ \& } y^2 = ax$$

$$x^4 = a^2 y^2$$

$$x^4 = a^2 ax \Rightarrow x = a, y = a$$

$$P : (a, a)$$

area bounded by C_1 & C_2

$$\text{Area} = \frac{16}{3} \cdot \frac{a}{4} \cdot \frac{a}{4} = \frac{a^2}{3}$$

Now $Q = (b, b)$

$$\frac{1}{2}b^2 = \frac{1}{2}$$

$$b = 1$$

Now area bounded by

C_1, C_2 & $x = 1$ is

$$\int_0^1 \left(\sqrt{ax} - \frac{x^2}{a} \right) dx = \frac{1}{2} \frac{a^2}{3}$$

$$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$2\sqrt{a} - \frac{1}{a} = \frac{a^2}{2}$$

$$4\sqrt{a} - \frac{2}{a} = a^2$$

$$4\sqrt{a}a - 2 = a^3$$

$$(a^3 + 2)^2 = 4a\sqrt{a}$$

$$a^6 + 4 + 4a^3 = 16a^3$$

$$a^6 - 12a^3 + 4 = 0$$

3. Which one of the following is a tautology ?

निम्न में से कौन सा कथन एक पुनरुक्ति है ?

- (1) $(P \wedge (P \rightarrow Q)) \rightarrow Q$ (2) $Q \rightarrow (P \wedge (P \rightarrow Q))$ (3) $P \wedge (P \vee Q)$ (4) $P \vee (P \wedge Q)$

Sol. 1

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	$q \rightarrow p \wedge (p \rightarrow q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

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score 200-240

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score above 240

4. Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$,
 $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx}(\sin^{-1}(f(x)))$ and

$y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to :

माना

$$f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1,$$

$$|x| > 1 \text{ है। यदि } \frac{dy}{dx} = \frac{1}{2} \frac{d}{dx}(\sin^{-1}(f(x))) \text{ तथा}$$

$y(\sqrt{3}) = \frac{\pi}{6}$ है, तो $y(-\sqrt{3})$ का मान है

- (1) $\frac{2\pi}{3}$ (2) $-\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{3}$

- Sol. 2**
 $2y = \sin^{-1}f(x) + C = \sin^{-1}(\sin(2\tan^{-1}x)) + C$

$$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$$

$$\frac{\pi}{3} = \frac{\pi}{3} + C \therefore C = 0$$

$$\text{for } x = -\sqrt{3}, 2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0 \Rightarrow 2y = \frac{-\pi}{3}$$

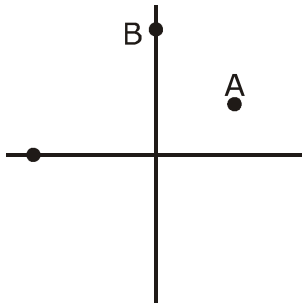
$$y = -\frac{\pi}{6}$$

5. Let two points be A(1,-1) and B(0,2). If a point P(x',y') be such that the area of $\Delta PAB = 5$ sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is :

माना A(1,-1) तथा B(0,2) दो बिन्दु हैं। यदि एक बिन्दु P(x',y') इस प्रकार है कि ΔPAB का क्षेत्रफल = 5 वर्ग इकाई है तथा यह रेखा $3x + y - 4\lambda = 0$ पर स्थित है तो λ का एक मान है :

- (1) -3 (2) 3 (3) 1 (4) 4

- Sol. 2**



$$\text{let } P : (a, 4\lambda - 3a) \Rightarrow \Delta = \begin{vmatrix} a & 4\lambda - 3a & 1 \\ \frac{1}{2} & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 5$$

$$\begin{aligned} |a(-1-2) - (4\lambda - 3a)(1-0) + 1(2+0)| &= 10 \\ |-3a - 4\lambda + 3a + 2| &= 10 \\ |2 - 4\lambda| &= 10 \Rightarrow 2 - 4\lambda = 10 \\ \lambda &= -2 \\ 2 - 4\lambda &= -10 \Rightarrow \lambda = 3 \end{aligned}$$

6. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is :

रेखाओं $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ तथा $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ के बीच की न्यूनतम दूरी है :

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$ (3) 3 (4) $2\sqrt{30}$

Sol. 2

$$\vec{a} = \langle 3, 8, 3 \rangle$$

$$\vec{b} = \langle -3, -7, 6 \rangle$$

$$\vec{p} = \langle 3, -1, 1 \rangle$$

$$\vec{q} = \langle -3, 2, 4 \rangle$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = \langle -6, 15, 3 \rangle$$

$$\text{S.D.} = \frac{|(\vec{a} - \vec{b}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$$

$$= \frac{|(6, 15, -3) \cdot (-6, -15, 3)|}{\sqrt{36 + 225 + 9}} = \frac{|-36 - 225 - 9|}{\sqrt{36 + 225 + 9}} = \frac{|-270|}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$$

7. If the equation, $x^2 + bx + 45 = 0$ ($b \in \mathbb{R}$) has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then :

यदि समीकरण $x^2 + bx + 45 = 0$ ($b \in \mathbb{R}$) के संयुग्मी सम्मिश्र मूल हैं जो $|z + 1| = 2\sqrt{10}$ को संतुष्ट करते हैं, तो

(1) $b^2 - b = 42$ (2) $b^2 + b = 72$ (3) $b^2 + b = 12$ (4) $b^2 - b = 30$

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Sol. 4

$x^2 + bx = 45 = 0$ ($b \in \mathbb{R}$)
has roots $\alpha + i\beta$, $\alpha - i\beta$
sum of roots = $-b = 2\alpha$
product of roots = $45 = \alpha^2 + \beta^2$

$$|z + 1| = 2\sqrt{10}$$

$$(\alpha + 1)^2 + \beta^2 = 40$$

$$\left(-\frac{b}{2} + 1\right)^2 + 45 - \left(-\frac{b}{2}\right)^2 = 40$$

$$\frac{b^2}{4} + 1 - b + 45 - \frac{b^2}{4} = 40$$

$$b = 6$$

8. If a , b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then :
यदि a , b तथा c क्रमशः ${}^{19}C_p$, ${}^{20}C_q$ तथा ${}^{21}C_r$ के अधिकतम मान हैं, तो :

$$(1) \frac{a}{10} = \frac{b}{11} = \frac{c}{21} \quad (2) \frac{a}{11} = \frac{b}{22} = \frac{c}{21} \quad (3) \frac{a}{11} = \frac{b}{22} = \frac{c}{42} \quad (4) \frac{a}{10} = \frac{b}{11} = \frac{c}{42}$$

Sol. 3

${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$
 $a = ({}^{19}C_p)_{\max} \Rightarrow a = {}^{19}C_{10} = {}^{19}C_9$
 $b = ({}^{20}C_p)_{\max} \Rightarrow b = {}^{20}C_{10}$
 $c = ({}^{21}C_r)_{\max} \Rightarrow c = {}^{21}C_{10} = {}^{21}C_{11}$

$$\text{Now } \frac{a}{b} = \frac{{}^{19}C_9}{{}^{20}C_{10}} = \frac{19!}{9!10!} \times \frac{10!10!}{20!} = \frac{10}{20} = \frac{1}{2} \times \frac{11}{11}$$

$$\frac{b}{c} = \frac{{}^{20}C_{10}}{{}^{21}C_{11}} = \frac{20!}{10!10!} \times \frac{11!10!}{21!} = \frac{11}{21} = \frac{11 \times 2}{21 \times 2}$$

$$\frac{a}{b} = \frac{11}{22} \quad \& \quad \frac{b}{c} = \frac{22}{42}$$

$$a : b : c :: 11 : 22 : 42$$

9. The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q , where $p \neq 0$ and $q \neq 0$. If the new mean and new s.d. become half of their original values, then q is equal to :

10 प्रेक्षणों के माध्य तथा मानक विचलन क्रमशः 20 तथा 2 हैं। इन 10 प्रेक्षणों में से प्रत्येक को p से गुणा करने के पश्चात प्रत्येक में से q कम किया गया, जहाँ $p \neq 0$ तथा $q \neq 0$ है। यदि नए माध्य तथा मानक विचलन के मान अपने मूल मानों के आधे हैं, तो q का मान है :

$$(1) -10 \quad (2) -5 \quad (3) 10 \quad (4) -20$$

Sol. 4

If each observation is multiplied with p & then q is subtracted

$$\text{New mean } \bar{x}_1 = p\bar{x} - q$$

$$\Rightarrow 10 = p(20) - q \quad \dots(1)$$

and new standard deviations.

$$\sigma_2 = |p|\sigma_1 \Rightarrow 1 = |p|(2) \Rightarrow |p| = \frac{1}{2} \Rightarrow p = \pm \frac{1}{2}$$

If $p = \frac{1}{2}$

then $q = 0$ (from equation (1))

If $p = -\frac{1}{2}$

$q = -20$

- 10.** Let A and B two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{6}$. Then, which of the following is TRUE ?

माना A तथा B दो ऐसी स्वतंत्र घटनाएँ है कि $P(A) = \frac{1}{3}$ तथा $P(B) = \frac{1}{6}$ हैं, तो निम्न में से कौन सा सत्य है ?

(1) $P(A'/B') = \frac{1}{3}$ (2) $P(A/B) = \frac{2}{3}$ (3) $P(A / (A \cup B)) = \frac{1}{4}$ (4) $P(A/B') = \frac{1}{3}$

Sol. 4

$P(A) = \frac{1}{3}$ & $P(B) = \frac{1}{6}$

$$(1) P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - \left(\frac{1}{3} + \frac{1}{6} - \frac{1}{3} \cdot \frac{1}{6}\right)}{1 - \frac{1}{6}} = \frac{1 - \left(\frac{4}{9}\right)}{\frac{5}{6}} = \frac{5 \cdot 6}{9 \cdot 5} = \frac{2}{3}$$

$$(2) P(A/B) = \frac{\frac{1}{3} \cdot \frac{1}{6}}{\frac{1}{6}} = \frac{1}{3}$$

$$(3) P(A / A \cup B) = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{3}{5}$$

$$(4) P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3} - \frac{1}{3} \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{6 - 1}{6 - 1} = \frac{1}{3}$$

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score 160-200 | score 200-240 | score above 240

11. Let $y = y(x)$ be a solution of the differential equation,

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$$

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to :

माना $y = y(x)$, अवकल समीकरण

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1 \text{ का एक}$$

हल है। यदि $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ है, तो $y\left(\frac{-1}{\sqrt{2}}\right)$ बराबर है :

- (1) $\frac{1}{\sqrt{2}}$ (2) $-\frac{1}{\sqrt{2}}$ (3) $\frac{-\sqrt{3}}{2}$ (4) $\frac{\sqrt{3}}{2}$

Sol. 1

$$\int \frac{dy}{\sqrt{1-y^2}} = -\int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}y + \sin^{-1}x = \lambda$$

$$\frac{\pi}{3} + \frac{\pi}{6} = \lambda \Rightarrow \lambda = \frac{\pi}{2}$$

$$\text{Now } \sin^{-1}(y) + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sin^{-1}y = \frac{\pi}{4} \Rightarrow y = \frac{1}{\sqrt{2}}$$

12. If c is a point at which Rolle's theorem holds for the function, $f(x) = \log_e \left(\frac{x^2 + \alpha}{7x} \right)$ in the interval $[3,4]$, where $\alpha \in R$, then $f''(c)$ is equal to :

यदि c एक बिन्दु है जिस पर, अन्तराल $[3,4]$ में, फलन $f(x) = \log_e \left(\frac{x^2 + \alpha}{7x} \right)$ पर रोले प्रमेय लागू होता है, जहाँ $\alpha \in R$ है,

तो $f''(c)$ बराबर है :

- (1) $-\frac{1}{12}$ (2) $\frac{\sqrt{3}}{7}$ (3) $\frac{-1}{24}$ (4) $\frac{1}{12}$

Sol. 4

$f(3) = f(4)$ for Rolle's

$$\Rightarrow \ln \left(\frac{9 + \alpha}{21} \right) = \ln \left(\frac{16 + \alpha}{28} \right)$$

$$= \frac{9 + \alpha}{21} = \frac{16 + \alpha}{28}$$

$$36 + 4\alpha = 48 + 3\alpha$$

$$\alpha = 12$$

$$f'(x) = \frac{x^2 - 12}{x(x^2 + 12)}$$

$$f'(c) = 0 \Rightarrow x = \pm\sqrt{12} \Rightarrow c = \sqrt{12}$$

$$f''(c) = \frac{1}{12}$$

- 13.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that for all $x \in \mathbb{R}$ ($2^{1+x} + 2^{1-x}$), $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is :

माना $f: \mathbb{R} \rightarrow \mathbb{R}$ इस प्रकार है कि सभी $x \in \mathbb{R}$ के लिए $(2^{1+x} + 2^{1-x})$, $f(x)$ तथा $(3^x + 3^{-x})$ एक समांतर रेणी में है, तो $f(x)$ का न्यूनतम मान है :

- (1) 2 (2) 4 (3) 0 (4) 3

Sol. 4

$(2^{1+x} + 2^{1-x})$, $f(x)$, $(3^x + 3^{-x})$ are in A.P.

$$2f(x) = 2 \cdot 2^x + \frac{2}{2^x} + 3^x + \frac{1}{3^x}$$

$$2f(x) = 2\left(2^x + \frac{1}{2^x}\right) + \left(3^x + \frac{1}{3^x}\right)$$

$$2f(x) \geq 6$$

$$f(x) \geq 3$$

- 14.** For which of the following ordered pairs (μ, δ) , the system of linear equations

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta$$

is inconsistent ?

निम्न में से किस क्रमित युग्म (μ, δ) के लिए रैखिक समीकरण निकाय

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta$$

असंगत है ?

- (1) (1,0) (2) (4,6) (3) (3,4) (4) (4,3)

Sol. 4

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix}$$

$$\Delta = 1(16-20) - 2(12-20) + 3(12-16)$$

$$= -4 + 16 - 12 = 0$$

$$\Delta_1 \neq 0, \Delta_2 \neq 0$$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 3 \\ \mu & 4 & 5 \\ \delta & 4 & 4 \end{vmatrix} \neq 0$$

$$= -4 - 2(4\mu - 5\delta) + 3(4\mu - 4\delta) \neq 0$$

$$\Rightarrow -4 - 8\mu + 10\delta + 12\mu - 12\delta \neq 0$$

$$4\mu - 2\delta \neq 4$$

$$2\mu - \delta \neq 2$$

Check option

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score above 240

15. If $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

यदि $\int \frac{\cos x dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$ है, जहाँ c एक समाकलन अचर है, तो $\lambda f\left(\frac{\pi}{3}\right)$ का मान है :

- (1) $-\frac{9}{8}$ (2) -2 (3) $\frac{9}{8}$ (4) 2

Sol. 2

$$\sin x = t$$

$$\int \frac{dt}{t^3 (1 + t^6)^{2/3}}$$

$$= \int \frac{dt}{t^7 \left(\frac{1}{t^6} + 1\right)^{2/3}}$$

$$\frac{1}{t^6} + 1 = z^3$$

$$= \int \frac{3z^2 dz}{-6z^2} = -\frac{1}{2} \int dz = -\frac{1}{2} \left(1 + \frac{1}{\sin^6 x}\right)^{1/3} + C$$

$$= \frac{-1}{2 \sin^2 x} (1 + \sin^6 x)^{1/3} + C$$

$$f(x) = -\frac{1}{2} \operatorname{cosec}^2 x \quad \& \quad \lambda = 3$$

$$\lambda f\left(\frac{\pi}{3}\right) = 3 \cdot \left(-\frac{1}{2} \cdot \frac{4}{3}\right) = -2$$

16. Let the volume of a parallelepiped whose coterminous edges are given by $\vec{u} = \hat{i} + \hat{j} + \lambda \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ be 1 cu. unit. If θ be the angle between the edges \vec{u} and \vec{w} , then $\cos\theta$ can be :

माना एक समानतर षट्फलक, जिसके एक ही शीर्ष से होकर जाने वाले किनारे $\vec{u} = \hat{i} + \hat{j} + \lambda \hat{k}$, $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$ तथा $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$ द्वारा प्रदत्त हैं, का आयतन 1 घन इकाई है। यदि किनारों \vec{u} तथा \vec{w} के बीच का कोण θ है तो $\cos\theta$ हो सकता है :

- (1) $\frac{5}{7}$ (2) $\frac{7}{6\sqrt{6}}$ (3) $\frac{5}{3\sqrt{3}}$ (4) $\frac{7}{6\sqrt{3}}$

Sol. 4

$$\vec{u} = \langle 1, 1, \lambda \rangle$$

$$\vec{v} = \langle 1, 1, 3 \rangle$$

$$\vec{w} = \langle 2, 1, 1 \rangle$$

$$\text{volume} = [\vec{u} \vec{v} \vec{w}] = 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\lambda = 2 \text{ or } \lambda = 4$$

For $\lambda = 2$

$$\cos\theta = \frac{2+1+2}{\sqrt{6}\sqrt{6}} = \frac{5}{6}$$

for $\lambda = 4$

$$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$$

17. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, $x \in (-1, 1)$, is

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1, 1) \text{ का व्युत्क्रम फलन है } \underline{\hspace{2cm}}$$

$$(1) \frac{1}{4}(\log_8 e) \log_e \left(\frac{1+x}{1-x} \right) \qquad (2) \frac{1}{4} \log_e \left(\frac{1+x}{1-x} \right)$$

$$(3) \frac{1}{4}(\log_8 e) \log_e \left(\frac{1-x}{1+x} \right) \qquad (4) \frac{1}{4} \log_e \left(\frac{1-x}{1+x} \right)$$

Sol. 1

$$f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}, x \in (-1, 1) \Rightarrow y = \frac{8^{4x} - 1}{8^{4x} + 1}$$

$$8^{4x}y + y = 8^{4x} - 1 \Rightarrow 8^{4x}(y - 1) = -1 - y$$

$$8^{4x} = \frac{y+1}{1-y} \Rightarrow 4x = \log_8 \left(\frac{y+1}{1-y} \right)$$

$$x = \frac{1}{4} \log_8 \left(\frac{1+y}{1-y} \right) \Rightarrow f^{-1}(x) = \frac{1}{4} \log_8 \left(\frac{1+x}{1-x} \right)$$

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score above 240

18. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{1/x^2}$ is equal to :

$\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{1/x^2}$ बराबर है :

- (1) $\frac{1}{e^2}$ (2) e^2 (3) e (4) $\frac{1}{e}$

Sol. 1

$$\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{1/x^2} \quad (1^\infty)$$

e^L

$$L = \lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} - 1 \right) \frac{1}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-4}{7x^2 + 2} = -2 \Rightarrow e^{-2}$$

19. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and $(0, \beta)$, then β is equal to
माना रेखा $y = mx$ तथा दीर्घवत्त $2x^2 + y^2 = 1$ प्रथम चतुर्थांश में स्थित एक बिन्दु P पर काटते है। यदि इस दीर्घवत्त का P पर अभिलंब, निर्देशांक अक्षों को क्रमशः $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ तथा $(0, \beta)$ पर मिलता है, तो β का मान है

- (1) $\frac{2}{3}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2\sqrt{2}}{3}$

Sol. 2

Let P be (x_1, y_1)

Equation of normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$

It passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right) \Rightarrow \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

So $y_1 = \frac{2\sqrt{2}}{3}$ (as P lies in Ist quadrant)

so $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

20. Let $f(x) = x \cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true ?

(1) $f'(0) = -\frac{\pi}{2}$

(2) f is not differentiable at $x = 0$

(3) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(4) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

माना $f(x) = x \cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ है, तब निम्न में से कौनसा सत्य है ?

(1) $f'(0) = -\frac{\pi}{2}$

(2) $x=0$ पर f अवकलनीय नहीं है

(3) f' , $\left(-\frac{\pi}{2}, 0\right)$ में ह्रासमान तथा $\left(0, \frac{\pi}{2}\right)$ में वर्धमान है

(4) f' , $\left(-\frac{\pi}{2}, 0\right)$ में वर्धमान तथा $\left(0, \frac{\pi}{2}\right)$ ह्रासमान है।

Sol. 3

$$f'(x) = x(\pi - \cos^{-1}(\sin|x|)) = x\left(\pi - \left(\frac{\pi}{2} - \sin^{-1}(\sin|x|)\right)\right) = x\left(\frac{\pi}{2} + |x|\right)$$

$$f(x) = \begin{cases} x\left(\frac{\pi}{2} + x\right) & x \geq 0 \\ x\left(\frac{\pi}{2} - x\right) & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{\pi}{2} + 2x & x \geq 0 \\ \frac{\pi}{2} - 2x & x < 0 \end{cases}$$

$f'(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$ and decreasing in $\left(-\frac{\pi}{2}, 0\right)$

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score above 240

21. An urn contains 5 red marbles, 4 black marbles and 3 white marbles, then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is :

एक कलश में 5 लाल मार्बल, 4 काले मार्बल तथा 3 सफेद मार्बल हैं, तो इसमें से 4 मार्बल इस प्रकार निकालते ताकि उनमें से अधिक से अधिक तीन लाल रंग के हों, के तरीकों की संख्या है

Sol. 490

$$5R + 4B + 3W$$

$$\binom{0}{4} \binom{R}{U} + \binom{1}{3} \binom{R}{U} + \binom{2}{2} \binom{R}{U} + \binom{3}{1} \binom{R}{U}$$

$$= {}^7C_4 + {}^5C_1 \cdot {}^7C_3 + {}^5C_2 \cdot {}^7C_2 + {}^5C_3 \cdot {}^7C_1 = 490$$

22. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is

'a' का वह न्यूनतम धनात्मक मान, जिसके लिए समीकरण $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ के वास्तविक मूल हैं, है

.....

Sol. 8

$$2x^2 + (a - 10)x + \frac{33}{2} = 2a$$

$$D \geq 0$$

$$\Rightarrow (a - 10)^2 - 4 \cdot 2 \cdot \left(\frac{33}{2} - 2a\right) \geq 0 \Rightarrow a^2 + 100 - 20a - 132 + 16a \geq 0$$

$$a^2 - 4a - 32 \geq 0 \Rightarrow (a - 8)(a + 4) \geq 0$$

$$a \leq -4 \cup a \geq 8$$

23. The sum $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$ is

योगफल $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$ है

Sol. 1540

$$\sum_{k=1}^{20} \frac{k(k+1)}{2} = \frac{1}{2} \left(\sum_{k=1}^{20} k^2 + \sum_{k=1}^{20} k \right) \text{ use formula} = 1540$$

24. The number of all 3×3 matrices A, with entries from the set $\{-1, 0, 1\}$ such that the sum of the diagonal elements of AA^T is 3, is

ऐसे सभी 3×3 आव्यूहों A की संख्या, जिनके अवयव समुच्चय $\{-1, 0, 1\}$ से हैं तथा AA^T के विकर्ण के अवयवों का योगफल 3 है, है.....

Sol. 672

Let

$$A = [a_{ij}]_{3 \times 3}$$

$$\text{tr}(AA^T) = 3$$

$$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \dots + a_{33}^2 = 3$$

possible cases

$$\left. \begin{array}{l} 0, 0, 0, 0, 0, 0, 1, 1, 1 \rightarrow 1 \\ 0, 0, 0, 0, 0, 0, 1, 1, -1 \rightarrow 1 \\ 0, 0, 0, 0, 0, 0, 1, 1, -1 \rightarrow 3 \\ 0, 0, 0, 0, 0, 0, -1, 1, -1 \rightarrow 3 \end{array} \right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$

25. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y - axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to

माना वक्र $y^2 - 3x^2 + y + 10 = 0$ के बिन्दु P पर खींचा गया अभिलंब, y - अक्ष को $\left(0, \frac{3}{2}\right)$ पर काटता है। यदि P पर वक्र की स्पर्श रेखा का ढाल m है, तो $|m|$ बराबर है.....

Sol.

4

$$P \equiv (x_1, y_1)$$

$$2yy' - 6x + y' = 0 \Rightarrow y' = \left(\frac{6x_1}{1 + 2y_1}\right)$$

$$\left(\frac{\frac{3}{2} - y_1}{-x_1}\right) = -\left(\frac{1 + 2y_1}{6x_1}\right)$$

$$9 - 6y_1 = 1 + 2y_1 \Rightarrow y_1 = 1$$

$$\therefore x_1 = \pm 2$$

$$\therefore \text{slope of tangent} = \left(\frac{\pm 12}{3}\right)$$

$$= \pm 4$$

$$\therefore |m| = 4$$

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