JEE ADVANCED

ANSWER KEY

2021

MATHEMATICS

Paper-2

QUESTION WITH ANSWER

32700+ SELECTIONS SINCE 2007

MOTION®

SECTION – A

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks	: +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks	: +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks	: +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	: 0 If unanswered;
Negative Marks	: −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

1. Let
$S_1=\{(i,j,k): i,j,k\in\{1,2,\ldots,10\}\}$,
$S_2=\{(i,j): 1\le i<j+2\le 10,\ i,j\in\{1,2,\ldots,10\}\}$,
$S_3=\{(i,j,k,l): 1\le i<j<k<l,\ i,j,k,l\in\{1,2,\ldots,10\}\}$
and
$S_4=\{(i,j,k,l): i,j,k$ and l are distinct elements in $\{1,2,\ldots,10\}\}$.
If the total number of elements in the set S_r is n_r, $r=1,2,3,4$, then which of the following statements is (are) TRUE ?

(A) $n_1=1000$ (B) $n_2=44$ (C) $n_3=220$ (D) $\frac{n_4}{12}=420$

Ans. **A,B,D**

2. Consider a triangle PQR having sides of lengths p,q and r opposite to the angles P,Q and R, respectively. Then which of the following statements is (are) TRUE ?

(A) $\cos P\ge 1-\frac{p^2}{2qr}$ (B) $\cos R\ge\left(\frac{q-r}{p+q}\right)\cos P+\left(\frac{p-r}{p+q}\right)\cos Q$

(C) $\frac{q+r}{p}<2\frac{\sqrt{\sin Q\sin R}}{\sin P}$ (D) If $p<q$ and $p<r$, then $\cos Q>\frac{p}{r}$ and $\cos R>\frac{p}{q}$

Ans. **A,B**

3. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to\mathbb{R}$ be a continuous function such that

$$f(0)=1 \text{ and } \int_0^{\frac{\pi}{3}} f(t)dt=0$$

Then which of the following statements is (are) TRUE ?

(A) The equation $f(x)-3\cos 3x=0$ has at least one solution in $(0,\frac{\pi}{3})$

(B) The equation $f(x)-3\sin 3x=-\frac{6}{\pi}$ has at least one solution in $(0,\frac{\pi}{3})$

(C) $\lim\limits_{x\to 0}\frac{x\int_0^x f(t)\,dt}{1-e^{x^2}}=-1$

(D) $\lim\limits_{x\to 0}\frac{\sin x\int_0^x f(t)\,dt}{x^2}=-1$

Ans. **A,B,C**

4. For any real numbers α and β, let $y_{\alpha,\beta}(x), x\in\mathbb{R}$, be the solution of the differential equation

$$\frac{dy}{dx}+\alpha y = xe^{\beta x},\ y(1)=1$$

Let $S=\{y_{\alpha,\beta}(x):\alpha,\beta\in\mathbb{R}\}$. Then which of the following functions belong(s) to the set S ?

(A) $f(x)=\frac{x^2}{2}e^{-x}+\left(e-\frac{1}{2}\right)e^{-x}$ (B) $f(x)=-\frac{x^2}{2}e^{-x}+\left(e-\frac{1}{2}\right)e^{-x}$

(C) $f(x)=\frac{e^x}{2}\left(x-\frac{1}{2}\right)+\left(e-\frac{e^2}{4}\right)e^{-x}$ (D) $f(x)=\frac{e^x}{2}\left(\frac{1}{2}-x\right)+\left(e+\frac{e^2}{4}\right)e^{-x}$

Ans. **A,C**

5. Let O be the origin and $\overrightarrow{OA}=2\hat{i}+2\hat{j}+\hat{k}$, $\overrightarrow{OB}=\hat{i}-2\hat{j}+2\hat{k}$ and $\overrightarrow{OC}=\frac{1}{2}(\overrightarrow{OB}-\lambda\overrightarrow{OA})$ for some $\lambda>0$. If $|\overrightarrow{OB}\times\overrightarrow{OC}|=\frac{9}{2}$, then which of the following statements is (are) TRUE ?

(A) Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is $-\frac{3}{2}$

(B) Area of the triangle OAB is $\frac{9}{2}$

(C) Area of the triangle ABC is $\frac{9}{2}$

(D) The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\frac{\pi}{3}$

Ans. **A,B,C**

6. Let E denote the parabola $y^2=8x$. Let P $=(-2,4)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE ?
(A) The triangle PFQ is a right-angled triangle
(B) The triangle QPQ' is a right-angled triangle
(C) The distance between P and F is $5\sqrt{2}$
(D) F lies on the line joining Q and Q'

Ans. A,B

Section – 2

- This section contains THREE (03) question stems.
- There are TWO (02) questions corresponding to each question stem.
- The answer to each question is a NUMERICAL VALUE.
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 7 and 8

Question Stem

Consider the region R $=\{(x,y)\in\mathbb{R}\times\mathbb{R}: x\geq 0$ and $y^2\leq 4-x\}$. Let $\mathcal{F}$ be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in $\mathcal{F}$. Let (α,β) be a point where the circle C meets the curve $y^2=4-x$.

7. The radius of the circle C is ___ .

Ans. 1.5

8. The value of α is ___ .

Ans. 2

Question Stem for Question Nos. 9 and 10

Question Stem

Let $f_1:(0,\infty)\rightarrow\mathbb{R}$ and $f_2:(0,\infty)\rightarrow\mathbb{R}$ be defined by

$f_1(x)=\int_0^x \prod_{j=1}^{21}(t-j)^j\,dt$, $x>0$

and $f_2(x)=98(x-1)^{50}-600(x-1)^{49}+2450$, $x>0$,

where, for any positive integer n and real numbers $a_1,a_2,\ldots,an$, $\prod_{i=1}^{n} a_i$ denotes the product of $a_1,a_2,\ldots,an$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, $i=1,2$, in the interval $(0,\infty)$.

9. The value of $2m_1+3n_1+m_1n_1$ is ___ .

Ans. 57

10. The value of $6m_2+4n_2+8m_2n_2$ is ___.

Ans. 6

Question Stem for Question Nos. 11 and 12

Question Stem

Let $g_i:\left[\frac{\pi}{8},\frac{3\pi}{8}\right]\to\mathbb{R}, i=1,2$, and $f:\left[\frac{\pi}{8},\frac{3\pi}{8}\right]\to\mathbb{R}$ be functions such that

$g_1(x)=1$, $g_2(x)=|4x-\pi|$ and $f(x)=\sin^2 x$, for all $x\in\left[\frac{\pi}{8},\frac{3\pi}{8}\right]$

Define

$$S_i=\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).g_i(x)\,dx, i=1,2$$

11. The value of $\frac{16S_1}{\pi}$ is _____.

Ans. 2

12. The value of $\frac{48S_2}{\pi^2}$ is _____.

Ans. 1.5

Section – 3

- This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Paragraph

Let

$$M=\{(x,y)\in\mathbb{R}\times\mathbb{R}: x^2+y^2\le r^2\},$$

where $r>0$. Consider the geometric progression $a_n=\frac{1}{2^{n-1}}, n=1,2,3,\ldots$. Let $S_0=0$ and, for $n\ge1$, let S_n denote the sum of the first n terms of this progression. For $n\ge1$, let C_n denote the circle with center $(S_{n-1},0)$ and radius a_n, and D_n denote the circle with center (S_{n-1},S_{n-1}) and radius a_n.

13. Consider M with $r=\frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k+2l=22$ (B) $2k+l=26$ (C) $2k+3l=34$ (D) $3k+2l=40$

Ans. D

14. Consider M with $r=\frac{\left(2^{199}-1\right)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

Ans. B

Paragraph

Let $\psi_1:[0,\infty)\to\mathbb{R}$, $\psi_2:[0,\infty)\to\mathbb{R}$, $f:[0,\infty)\to\mathbb{R}$ and $g:[0,\infty)\to\mathbb{R}$ be functions such that $f(0)=g(0)=0$,

$$\psi_1(x)=e^{-x}+x,\ x\geq 0,$$

$$\psi_2(x)=x^2-2x-2e^{-x}+2,\ x\geq 0,$$

$$f(x)=\int_{-x}^{x}\left(|t|-t^2\right)e^{-t^2}\,dt,\ x>0$$

and

$$g(x)=\int_0^{x^2}\sqrt{t}\,e^{-t}\,dt,\ x>0.$$

15. Which of the following statements is TRUE ?

(A) $f\left(\sqrt{\ln 3}\right)+g\left(\sqrt{\ln 3}\right)=\frac{1}{3}$

(B)For every $x>1$, there exists an $\alpha\in(1,x)$ such that $\psi_1(x)=1+\alpha x$

(C)For every $x>0$, there exists a $\beta\in(0,x)$ such that $\psi_2(x)=2x(\psi_1(\beta)-1)$

(D)f is an increasing function on the interval $\left[0,\frac{3}{2}\right]$

Ans. C

16. Which of the following statements is TRUE ?

(A)$\psi_1(x)\leq 1$, for all $x>0$

(B)$\psi_2(x)\leq 0$, for all $x>0$

(C)$f(x)\geq 1-e^{-x^2}-\frac{2}{3}x^3+\frac{2}{5}x^5$, for all $x\in\left(0,\frac{1}{2}\right)$

(D) $g(x)\leq\frac{2}{3}x^3-\frac{2}{5}x^5+\frac{1}{7}x^7$, for all $x\in\left(0,\frac{1}{2}\right)$

Ans. D

SECTION 4

- This section contains THREE (03) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

17. A number is chosen at random from the set {1,2,3,...,2000}. Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7. Then the value of 500p is ___ .

Ans. 214

18. Let E be the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. For any three distinct points P,Q and Q' on E, let M(P, Q) be the mid-point of the line segment joining P and Q, and M(P, Q') be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between M(P, Q) and M(P, Q'), as P, Q and Q' vary on E, is ___.

Ans. **4**

19. For any real number x, let $[x]$ denote the largest integer less than or equal to x. If

$$I=\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx,$$

then the value of 9I is _____.

Ans. **182**