JEE ADVANCED

ANSWER KEY

2021

MATHEMETICS

Paper-1

QUESTION WITH ANSWER

32700+ SELECTIONS SINCE 2007

MOTION®

SECTION – 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

1. Consider a triangle Δ whose two sides lie on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1,1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2 + y^2 - 3x + y = 0$ (B) $x^2 + y^2 + x + 3y = 0$

(C) $x^2 + y^2 + 2y - 1 = 0$ (D) $x^2 + y^2 + x + y = 0$

Ans. B

2. The area of the region $\left\{(x,y): 0 \le x \le \frac{9}{4},\quad 0 \le y \le 1,\quad x \ge 3y,\quad x+y \ge 2\right\}$ is

(A) $\frac{11}{32}$ (B) $\frac{35}{96}$ (C) $\frac{37}{96}$ (D) $\frac{13}{32}$

Ans. A

3. Consider three sets $E_1 = \{1,2,3\}$, $F_1 = \{1,3,4\}$ and $G_1 = \{2,3,4,5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1,2\}$. Then the value of p is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$

Ans. A

4. Let $\theta_1,\theta_2,\ldots,\theta_{10}$ be positive valued angles (in radian) such that $\theta_1+\theta_2+\cdots+\theta_{10}=2\pi$. Define the complex numbers $z_1=e^{i\theta_1}, z_k=z_{k-1}e^{i\theta_k}$ for $k=2,3,\ldots,10$, where $i=\sqrt{-1}$. Consider the statements P and Q given below:

$$P: |z_2-z_1|+|z_3-z_2|+\ldots.+|z_{10}-z_9|+|z_1-z_{10}|\le 2\pi$$

$$Q: |z_2^2-z_1^2|+|z_3^2-z_2^2|+\ldots.+|z_{10}^2-z_9^2|+|z_1^2-z_{10}^2|\le 4\pi$$

Then,

(A) P is **TRUE** and Q is **FALSE**

(B) Q is **TRUE** and P IS **FALSE**

(C) Both P and Q are **TRUE**

(D) Both P and Q are **FALSE**

Ans. **C**

Section – 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;
  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

Three numbers are chosen at random, one after another with replacement, from the set $S=\{1,2,3,\ldots,100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

5. The value of $\frac{625}{4}p_1$ is _____.

Ans. **76.25**

6. The value of $\frac{125}{4}p_2$ is _____.

Ans. **24.5**

Question Stem for Question Nos. 7 and 8

Question Stem

Let α, β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$
$$4x + 5y + 6z = \beta$$
$$7x + 8y + 9z = \gamma - 1$$

is consistent. Let |M| represent the determinant of the matrix

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those $(\alpha,,\gamma)$ for which the above system of linear equations is consistent, and D be the square of the distance of the point (0,1,0) from the plane P.

7. The value of |M| is _____.

Ans. **1**

8. The value of D is _____.

Ans. **1.5**

Question Stem for Question Nos. 9 and 10

Question Stem

Consider the lines L_1 and L_2 defined by

$$L_1 : x\sqrt{2} + y - 1 = 0 \text{ and } L_2 : x\sqrt{2} - y + 1 = 0$$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y=2x+1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S'. Let D be the square of the distance between R' and S'.

9. The value of λ^2 is _____.

Ans. **9**

10. The value of D is _____.

Ans. **77.14**

Section – 3

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If unanswered;
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

11. For any 3 × 3 matrix M, let |M| denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3 × 3, then which of the following statements is (are) **TRUE**?

(A) $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

(C) $|(EF)^3| > |EF|^2$

(D) Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E = P^{-1}FP$

Ans. **A,B,D**

12. Let $f : R \to R$ be defined by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is (are) **TRUE**?

(A) f is decreasing in the interval (–2, –1)

(B) f is increasing in the interval (1, 2)

(C) f is onto

(D) Range of f is $\left[-\frac{3}{2}, 2\right]$

Ans. **A,B**

13. Let E, F and G be three events having probabilities

$P(E) = \frac{1}{8}$, $P(F) = \frac{1}{6}$ and $P(G) = \frac{1}{4}$, and let $P(E \cap F \cap G) = \frac{1}{10}$.

For any event H, if H^c denotes its complement, then which of the following statements is (are) **TRUE**?

(A) $P(E \cap F \cap G^c) \le \frac{1}{40}$

(B) $P(E^c \cap F \cap G) \le \frac{1}{15}$

(B) $P(E \cap F \cap G) \le \frac{13}{24}$

(D) $P(E^c \cap F^c \cap G^c) \le \frac{5}{12}$

Ans. **A,B,C**

14. For any 3 × 3 matrix M, let |M| denote the determinant of M. Let I be the 3 × 3 identity matrix. Let E and F be two 3 × 3 matrices such that (I – EF) is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) **TRUE**?

(A) |FE| = |I – FE||FGE|

(B) (I – FE) (I + FGE) = I

(C) EFG = GEF

(D) (I – FE) (I – FGE) = I

Ans. **A,B,C**

15. For any positive integer n, let S_n: $(0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1 + k(k+1)x^2}{x}\right),$$

where for any $x \in R$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is (are) **TRUE**?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{n\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

Ans. A,B

16. For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x, y) lies on the circle

$$x^2 + y^2 + 5x - 3y + 4 = 0$$

Then which of the following statements is (are) **TRUE** ?

(A) $\alpha = -1$ (B) $\alpha\beta = 4$ (C) $\alpha\beta = -4$ (D) $\beta = 4$

Ans. B,D

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>
  Full Marks : +4 If ONLY the correct integer is entered;
  Zero Marks : 0 In all other cases.

17. For $x \in R$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is _____.

Ans. 4

18. In a triangle ABC, let $AB = \sqrt{23}$, $BC = 3$ and $CA = 4$. Then the value of $\frac{\cot A + \cot C}{\cot B}$ is _____.

Ans. 2

19. Let $\vec{u}, \vec{v}$ and $\vec{w}$, be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u}.\vec{w} = 1$, $\vec{v}.\vec{w} = 1$, $\vec{w}.\vec{w} = 4$

If the volume of the parallelopiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w}$, is $\sqrt{2}$ then the value of $|3\vec{u} + 5\vec{v}|$ is _____.

Ans. 7