JEE ADVANCED

ANSWER KEY

2021

PHYSICS

Paper-1

QUESTION WITH SOLUTION

32700+ SELECTIONS SINCE 2007

MOTION®

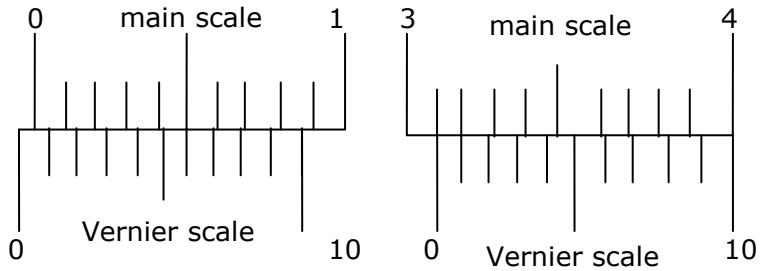
SECTION – 1

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

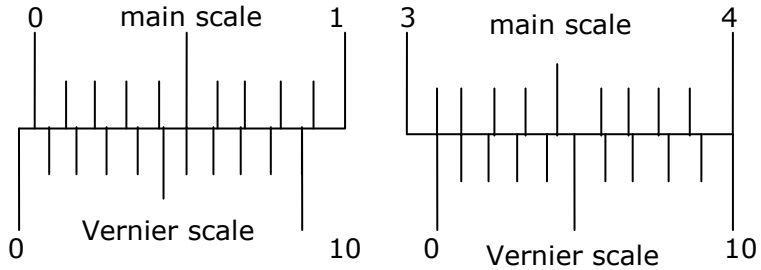
QID: **521096** **ERROR**

1. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is



(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

वर्नियर कैलीपर्स के मुख्य पैमाने पर सबसे छोटा विभाजन 0.1 सेमी है। वर्नियर पैमान के दस भाग मुख्य पैमाने के नो भागो के अनुरूप हैं बायीं ओर नीचे दिया गया चित्र इस कैलीपर्स का पाठ्यांक दिखाता है, जिसके दोनो जबडो के बीच कोई गैप नही है। दाहिनी ओर का चित्र जबड़ो के बीच रखे एक ठोस गोले के साथ पाठ्यांक दर्शाता है। गोले का सही व्यास है –



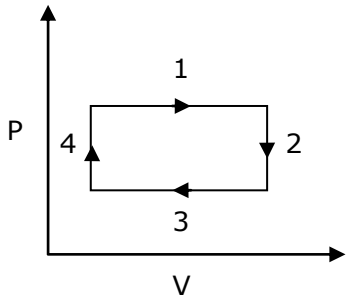
(A) 3.07 cm (B) 3.11 cm (C) 3.15 cm (D) 3.17 cm

Sol. **C**

In the primary figure with setting up the object. The zero of the two scales have a mismatch , where vernier scales start before main scale so it's a negative zero error & 6^{th} division matches.
So zero error (Negative) = $(10 - 6) \times 0.01 = 0.04$ cm
Now in the second figure, the reading from main scale is 3.1 cm will be added to 1^{st} matching division of vernier
So
Reading in 2^{nd} figure
$= 3.1 + 1 \times 0.1 = 3.11$ cm
Actual Reading = 3.11 + (Negative zero error)
= 3.11 + 0.04 = 3.15 cm **Ans.**

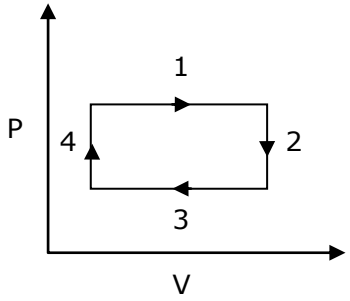
QID: **521099** HEAT-2

2. An ideal gas undergoes a four step cycle as shown in the P–V diagram below. During this cycle, heat is absorbed by the gas in



(A) steps 1 and 2 (B) steps 1 and 3 (C) steps 1 and 4 (D) steps 2 and 4

एक आदर्श गेस नीचे दिए गए P–V आरेख में दर्शाए अनुसार चार स्टेप वाले चक्र से गुजरती है। इस चक्र के दौरान गैस द्वारा उष्मा अवशोषित होती है –



(A) स्टेप 1 व 2 में (B) स्टेप 1 व 3 में (C) स्टेप 1 व 4 में (D) स्टेप 2 व 4 में

Sol. **C**

$1 \rightarrow$ Isobaric expansion $\rightarrow \Delta T > 0$

$2 \rightarrow$ Isochoric expansion $\rightarrow \Delta T < 0$

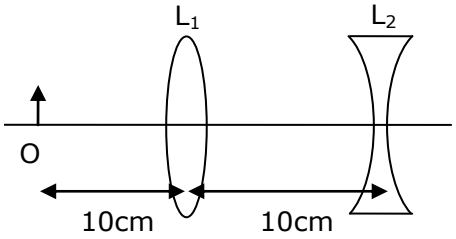
$3 \rightarrow$ Isobaric compression $\rightarrow \Delta T < 0$

$4 \rightarrow$ Isobaric compression $\rightarrow \Delta T > 0$

For 2, 4	for 1,3
$Q = nc_v \Delta T$	$Q = nC_p \Delta T$
$Q>0 \Rightarrow \Delta T > 0$	$Q > 0 \Rightarrow \Delta T > 0$
Process 1	Process 4

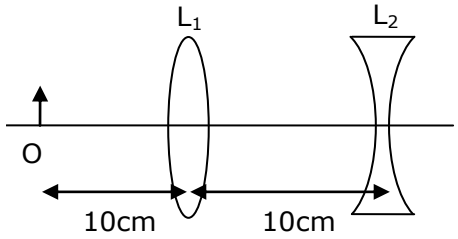
QID: **521101** LENS- IMAGE FORMULA LENS FORMULA & MAGNIFICATION

3. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

एक प्रसारित वस्तु को एक उत्तल लै।स L_1 के सामने 10 cm पर स्थित बिन्दु O पर रखा जाता है तथा अवतल लैंस L_2 को चित्राानुसार इसके पीछे 10 cm पर रखा जाता है। दोनो लैंसो में सभी वक्राकार पृष्ठो की वक्रता त्रिाज्याएं 20 cm हैं। दोनो लैंसो का अपवर्तनांक 1.5 है। इस लैंस निकाय का कुल आवर्धन है –



(A) 0.4 (B) 0.8 (C) 1.3 (D) 1.6

Sol. **B**

ROC = 20 cm each

and $\mu = 1.5$

$$\frac{1}{f_{concave}} = (1.5 - 1)\left(\frac{-1}{20} - \frac{1}{20}\right)$$

$$= 0.5 \times \left(\frac{-1}{10}\right) = \frac{-1}{20}$$

$f_{concave} = -20cm$

$f_{convex} = +20$ cm

So for 1st Refraction

$\frac{1}{V} + \frac{1}{10} = \frac{1}{20}$ so $m_1 = \frac{v}{u} = 2$

v = –20 cm

for second Refraction

u = –30

so, $\frac{1}{V} + \frac{1}{30} = \frac{-1}{20}$ $\frac{1}{V} = \frac{-1}{20} - \frac{1}{30} = \frac{-3-2}{60} = \frac{-1}{12}$

v = -12 cm

so $m_2 = \frac{-12}{-30} = \left(\frac{2}{5}\right)$

$m_{final} = m_1 m_2 = 0.8$

so option B

QID: **521100** MODERN-II

4. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is

(A) 50 (B) 75 (C) 350 (D) 525

अर्धायु 20 मिनट वाला एक भारी नाभिक Q, 60% की प्रायिकता के साथ एल्फा–क्षय से तथा 40% की प्रायिकता के साथ बीटा–क्षय से गुजरता है। प्रारंभ में, Q नाभिको की संख्या 1000 पहले एक घण्टे में एल्फा–क्षय की संख्या है –

(A) 50 (B) 75 (C) 350 (D) 525

Sol. **D**

Given that half life = 20 minutes

Number of nucleus of Q at t=0

N_o = 1000

No. of nucleus remaining after t =1hour = N = $No\left(\frac{1}{2}\right)^n$

$n = \frac{t}{t_{1/2}} = \frac{60 \text{ minutes}}{20 \text{ minutes}}$

[n = 3]

So $N_{remaining} = 1000 \cdot \left(\frac{1}{2}\right)^3$

$= \left(\frac{1000}{8}\right)$

So number of total decay in 1 hour

= No - $N_{remaining}$

$= \left(\frac{7000}{8}\right)$

α decay has probability of 60%

So number of α decay

$\frac{7000}{8} \times \frac{60}{100} = 525$ α decays

IInd Method

$1000 \xrightarrow{20\,min} 500 \xrightarrow{20\,min} 250 \xrightarrow{20\,min} 125$

Decay = 1000 – 125 = 875

$\Rightarrow$ 875 × 60% = 525

Section – 2

- This section contains **THREE (03)** question stems.
- There are **TWO (02)** questions corresponding to each question stem.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated <u>according to the following marking scheme:</u>

  Full Marks : +2 If ONLY the correct numerical value is entered at the designated place;

  Zero Marks : 0 In all other cases.

Question Stem for Question Nos. 5 and 6

Question Stem

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity g=10 m/s^2.

QID: **521103** KINEMATICS

5. The value of t is ___ .

Sol. **0.50**

QID: **521104** KINEMATICS

6. The value of x is ___ .

एक प्रक्षेप्य को धरातल पर स्थित एक बिन्दु O से उर्ध्वाधर के साथ 45° के कोण पर m/s की चाल से प्रक्षेपित किया जाता है। अपने प्रक्षेप–पथ के उच्चिष्ठ बिन्दु पर प्रक्षेप्य दो समान भागो में विभक्त हो जाता है। विभक्त होने के 0.5 s पश्चातृ, एक भाग धरातल पर उर्ध्वाधर नीचे की ओर गिरता है। विभक्त होने के t सैकण्ड पश्चात्, दूसरा भाग बिन्दु O से x मीटर की दूरी पर धरातल पर गिरता है। गुरूत्वीय त्वरण g =10 m/s^2

QID: **521103** KINEMATICS

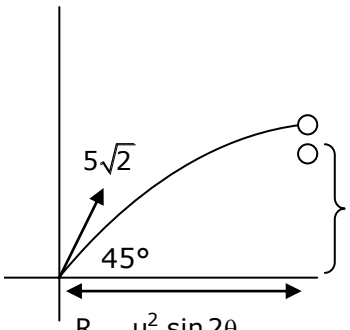
5. t का मान _____ है।

Sol. **0.50**

QID: **521104** KINEMATICS

6. x का मान _____ है।

Sol. **7.50**



$$H = H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(5)^2}{2\times 10} = 1.25\,m$$

$$\frac{R}{2} = \frac{u^2 \sin 2\theta}{2g} = 2.5m$$

Given that particle (2m) splits to two equal halves (m,m)

Let velocities after split be

$u_1 = u_{x_1}\hat{i} + u_{y_1}\hat{j}$

$u_2 = u_{x_2}\hat{i} + u_{y_2}\hat{j}$

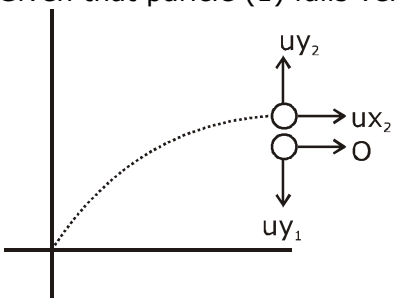
from com

$mu_1 + mu_2 = 2mu \cos 45° \left(\hat{i}\right)$

$\Rightarrow u_{x_1} + u_{x_2} = 10$ ....(1)

$\Rightarrow u_{y_1} + u_{y_2} = 0$ ....(2)

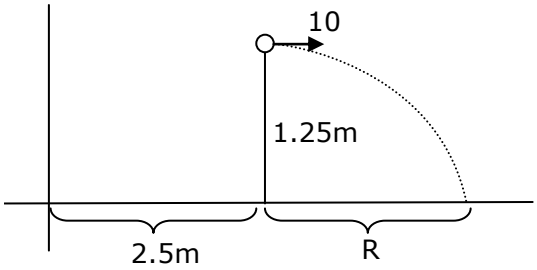
Given that paricle (1) falls vertically down in



0.5 sec $\Rightarrow u_{x_1} = 0$

Now $\Rightarrow H_{max} = u_{x_1} t + \frac{1}{2} gt^2$

$\Rightarrow \frac{1.25}{0.5} = u_{y_1} + 5(0.5) \Rightarrow u_{y_1} = 0$



$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(1.25)}{10}} = \sqrt{0.25} = 0.5$ sec

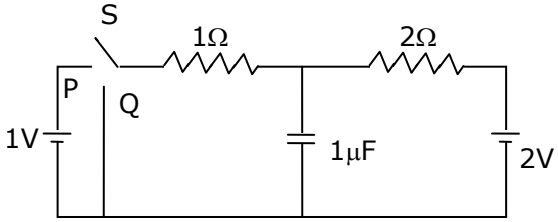
$x = 2.5 + R = 2.5 + u\sqrt{\frac{2H}{g}}$

$= 2.5 + 10\ (0.5) = 7.5$ m

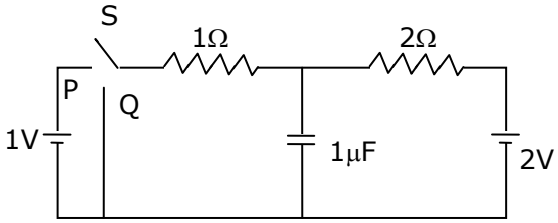
Question Stem for Question Nos. 7 and 8

Question Stem

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



नीचे दर्शाये गये परिपथ में, स्विच S को लम्बे समय के लिए स्थिति P से संयोजित किया जाता है, ताकि संधारित्रा पर आवेश q_1 μC हो जाए। फिर S को स्थिति Q पर स्थानान्तरित यिका जाता है। लम्बे समय पश्चात्, संधारित्रा पर आवेश q_2 μC है।



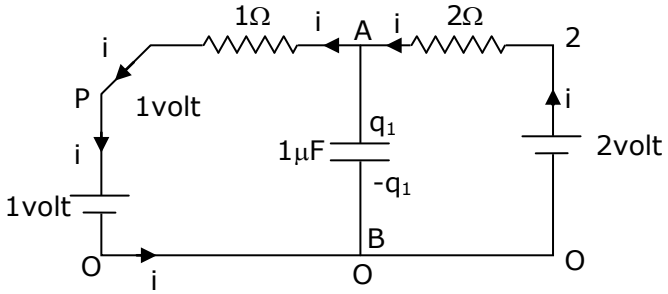
QID: **521212** CAPACITANCE

7. The magnitude of q_1 is ___ .

q_1 का परिमाण ___ है।

Sol. **1.33**

When switch is connected to P for long enough interval.



There will be no current through capacitor

$\Delta V_{capacitor} = V_A - V_B$

$i = \frac{2-1}{3} = \frac{1}{3}$ Amp.

so, $V_A = 2 - i \times 2 = 2 - \frac{2}{3} = \frac{4}{3}$ volt

So, $V_A - V_B = \Delta V_{cap} = \frac{4}{3}$ volt

Hence, $q_1 = C.\Delta V_{cap} = 1 \times \frac{4}{3}$ μC

$= \frac{4}{3}$ $\mu C = 1.33$ μC

$|q_1| = 1.33$

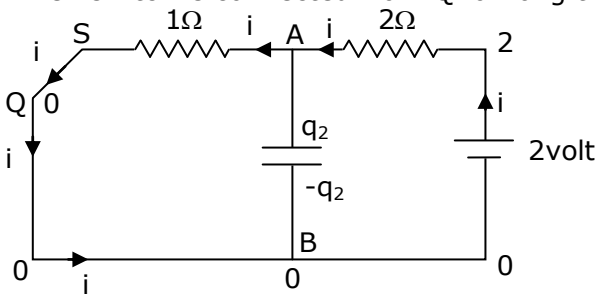
QID: **521215** CAPACITANCE

8. The magnitude of q_2 is ___ .

q_2 का परिमाण ___ है।

Sol. **0.67**

When switch is connected from Q for long time



Again no current from capacitor and $\Delta V_{cap} = V_A - V_B$

$i = \frac{2}{3}$ amp.

& $V_A = 2 - i \times 2 = 2 - \frac{2}{3} \times 2 = \frac{2}{3}$ volt

So, $V_A - V_B = \frac{2}{3} - 0 = \frac{2}{3}$ volt

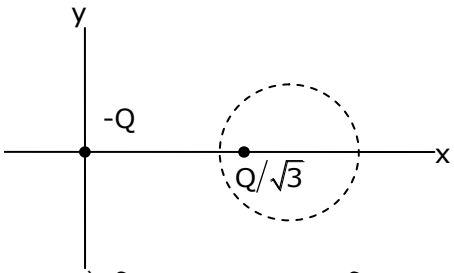
& $q_2 = C\Delta V = \frac{2}{3}$ μC

$|q_2| = 0.67$

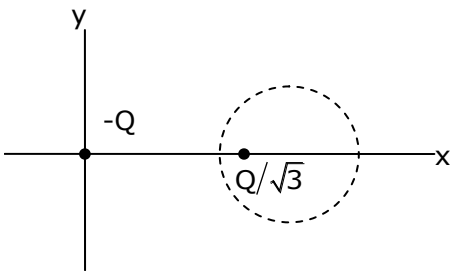
Question Stem for Question Nos. 9 and 10

Question Stem

Two point charges –Q and + $Q/\sqrt{3}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential V=0 in the xy-plane with its center at (b, 0). All lengths are measured in meters.

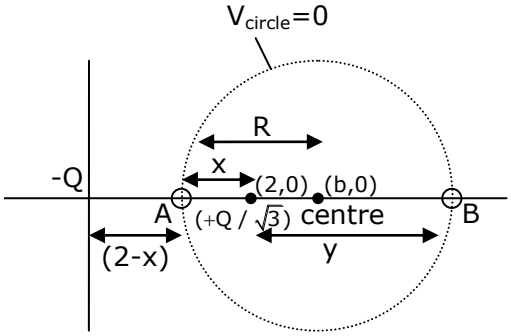


दो बिन्दु आवेशो –Q तथा + को xy-तल में चित्राानुसार क्रमशः मूलबिन्दु (0, 0) तथा बिन्दु (2, 0) पर रखा गया है। इसके परिणामस्वरूप xy-तल में R त्रिाज्या तथा विभव V = 0 का एक समविभव वृत्त निर्मित होता है,जिसका केन्द्र (b, 0) पर है। सभी लम्बाईयो को मीटर में मापा गया है।



9. The value of R is ___ meter.
R का मान ___ मीटर है।

Sol. **1.73**



The question says the circle is equipotential and $V_{circle} = 0$
So, $V_A = V_B = 0$

So, $V_A = \frac{-kQ}{(2-x)} + \frac{kQ}{\sqrt{3}(x)} = 0$

$\frac{1}{2-x} = \frac{1}{\sqrt{3}x} \Rightarrow \sqrt{3}x = 2 - x$

$x = \frac{2}{(\sqrt{3}+1)} = (\sqrt{3}-1)$ ....(1)

$V_B = \frac{-KQ}{(2+y)} + \frac{KQ}{\sqrt{3}(y)} = 0$

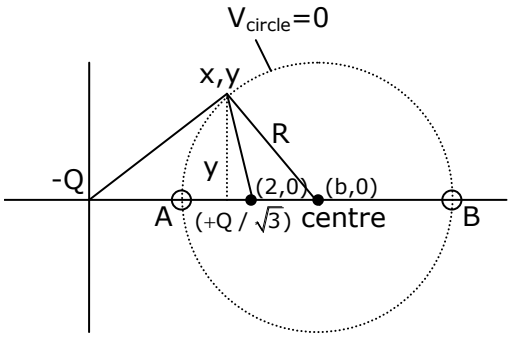
$\Rightarrow \frac{1}{\sqrt{3}y} = \frac{1}{2+y} \Rightarrow 2 + y = \sqrt{3}y$

$y = \frac{2}{(\sqrt{3}-1)} = (\sqrt{3}+1)$

$R = \frac{x+y}{2} = \frac{\sqrt{3}-1+\sqrt{3}+1}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$R = \sqrt{3}$

IInd Method



$V = \frac{KQ}{r}$

$\frac{-KQ}{\sqrt{r^2 + y^2}} + \frac{KQ}{\sqrt{3}\sqrt{(2-x)^2 + y^2}} = 0$

$\frac{KQ}{\sqrt{x^2 + y^2}} = \frac{KQ}{\sqrt{3}\sqrt{4 + x^2 - 4x + y^2}}$

$x^2 + y^2 = 3\,(x^2 + y^2 - 4x + 4)$

$x^2 + y^2 = 3x^2 + 3y^2 - 12x + 12$

$2x^2 + 2y^2 - 12x + 12 = 0$

$x^2 + y^2 - 6x + 6 = 0$

$x^2 + y^2 + 2gx + 2fy + c = 0$

centre $(-g, -f) \equiv (3,0)$, $R = \sqrt{g^2 + f^2 - c}$

$(b,0) \equiv (3,0)$ $= \sqrt{(3)^2 + 0^2 - 6}$

$b = 3$ $R = \sqrt{3}$ $R = \sqrt{3}$

QID: **521113** ELECTROSTATICS-1

10. The value of b is ___ meter.
b का मान ___ मीटर है।

Sol. **3**

So, from A to centre displacement = R

$A \rightarrow$ centre

is also equal to $b - (2 + x)$

So, $b - (2 + x) = R$

$b - 2 + \sqrt{3} - 1 = \sqrt{3}$

$b = 3$

$b = 3$

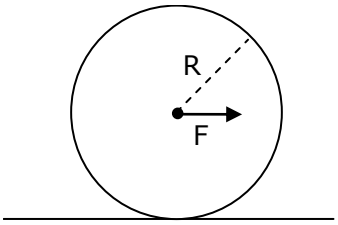
Section – 3

- This section contains SIX (06) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If only (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;

choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

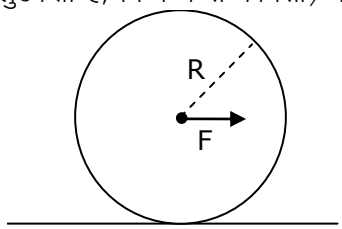
QID: **521117** ROTATIONAL MOTION

11. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The center of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



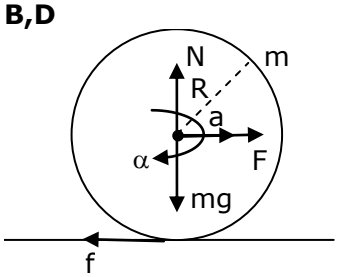
(A) For the same F, the value of a does not depend on whether the cylinder is solid or hollow
(B) For a solid cylinder,the maximum possible value of a is $2\mu g$
(C) The magnitude of the frictional force on the object due to the ground is always μmg
(D) For a thin-walled hollow cylinder, $a = \frac{F}{2m}$

एक क्षैतिज बल F को m द्रव्यमान व R त्रिज्या की एक बेलनाकार वस्तु के द्रव्यमान केन्द्र पर चित्राानुसार इसके अक्ष के लम्बवत् आरोपित किया जाता है। वस्तु तथा धरातल के मध्य घर्षण गुणांक μ है। वस्तु के द्रव्यमान केन्द्र का त्वरण a है। गुरूत्वीय त्वरण g है। दिया गया है कि वस्तु बिना फिसले लुढकती है, निम्न में से कौनसा/कौनसे कथन सही है/है ?



(A) समान F के लिए, a का मान इस पर निर्भर नही करता है कि बेलन ठोस है या खोखला
(B) एक ठोस बेलन के लिए, a का अधिकतम संभावित मान $2\mu g$ है
(C) धरातल के कारण वस्तु पर घर्षण बल का परिमाण सदैव μmg है
(D) पतली दीवार वाले खोखले बेलन के लिए, $a = \frac{F}{2m}$

Sol. **B,D**



Friction has to act backwards to provide α in acw sense

such that a= Rα for pure rolling

By NLM (Translational)

F – f = ma ... (1)

N = mg

NLM (Rotation)

$\Sigma\tau_o = I_0\alpha$

$fR = I_0\alpha$

$$fR = \frac{I_0 a}{R}\left(\because a = R\alpha\right)$$

$$f = \frac{I_0 a}{R^2}...(2)$$

eq.2 in eq.1 gives

$$F - \frac{I_0 a}{R^2} = ma$$

$$\Rightarrow a = \frac{F}{\left(m + \frac{I_0}{R^2}\right)}$$

(A) for solid cylinder $I_0 = \frac{mR^2}{2}$ and for hollow cylinder $I_0 = mR^2$

$\Rightarrow$ $a_{\text{solid cylinder}} = \frac{2F}{3m}$; $a_{\text{hollow cylinder}} = \frac{F}{2m}$. Thus a depends on I_0 also. (A) is wrong.

(B) $f = \frac{I_0 a}{R^2} \Rightarrow$ as a increases f increases

$$\Rightarrow a_{max} = \frac{f_r R^2}{I_0} = \frac{\mu N R^2}{I_0} = \frac{\mu mg R^2}{I_0}$$

for solid cylinder

$a_{max} = \frac{\mu mgR^2}{\frac{mR^2}{2}} = 2\mu g$. Thus (B) is correct.

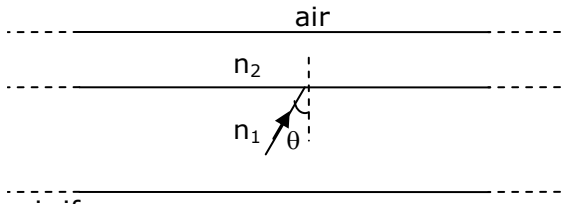
(C) As long as there is pure rolling the friction is static and self adjusting, $f = \frac{I_0 a}{R^2}$. f adjusts its value according to 'a' as long as limiting friction is reached. (C) is wrong.

(D) We have already seen that $a_{\text{hollow cylinder}} = \frac{F}{2m}$. (D) is right.

The correct options are (B) and (D).

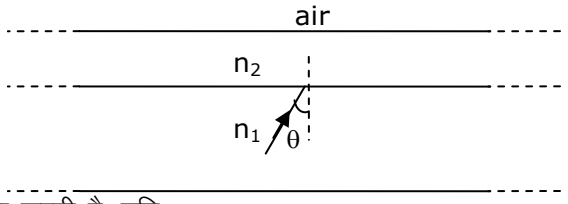
QID: **521128** GEOMETRICAL OPTICS

12. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where $\sin\theta$ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct?



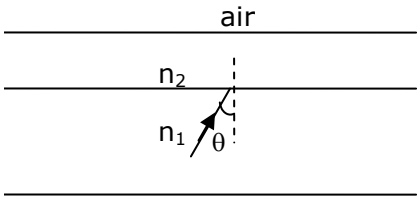
(A) The light ray enters air if $n_2 = n_1$
(B) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$
(C) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$
(D) The light ray is reflected back into the medium of refractive index n_1 if $n_2=1$

n_1 तथा n_2 अवर्तनांक के दो माध्यमो से बनी एक चोड़ी पट्टिका को चित्रानुसार वायु में रखा गया है। प्रकाश की एक किरण माध्यम n_1 से n_2 पर कोण θ पर आपतित होती है, जहाँ $\sin\theta$ का मान $1/n_1$ से थोड़ा सा अधिक है। वायु का अपवर्तनांक 1 लीजिए। निम्न में से कौनसा/कौनसे कथन सही है/है ?

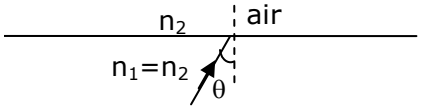


(A) प्रकाश किरण वायु में प्रवेश करती है, यदि $n_2 = n_1$
(B) प्रकाश किरण अन्ततः अपवर्तनांक n_1 के माध्यम में पुनः परावर्तित हो जाती है, यदि $n_2 < n_1$
(C) प्रकाश किरण अन्ततः अपवर्तनांक n_1 के माध्यम में पुनः परावर्तित हो जाती है, यदि $n_2 > n_1$
(D) प्रकाश किरण अपवर्तनांक n_1 के माध्यम में पुनः परावर्तित हो जाती है, यदि $n_2=1$

Sol. **B,C,D**



(1) If $n_2 = n_1$
As there is one medium, wave will travel in straight line

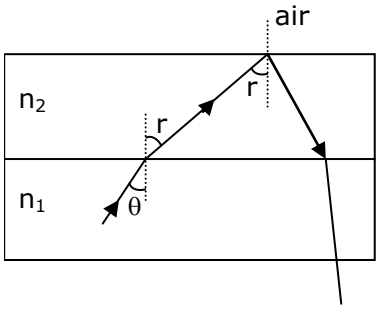


$n_1 \sin\theta = 1 \times \sin r$

as given $\sin\theta > \frac{1}{n_1}$ so sin r will become more than 1 which is not possible.

So, ray will never pass to air.

A is wrong.



as given $\sin\theta \frac{1}{n_1}$

$n_1 \sin\theta = n_2 \sin r$

$\Rightarrow \frac{n_1}{n_2} \sin\theta = \sin r$

as $\sin\theta > \frac{1}{n_1}$ so $\sin r > \frac{1}{n_2}$

$\Rightarrow$ T.I.R. (as $i > \theta c$)

so it gets refracted back to n_1 – B & C

& if $n_2 = 1$ then it is line air so given

$\sin\theta > \frac{1}{n_1}$ so T.I.R. again reflected back to n_1

QID: **521133** ROTATIONAL MOTION

13. A particle of mass M=0.2 kg is initially at rest in the xy-plane at a point (x=–l,y=–h), where l = 10 m and h=1 m. The particle is accelerated at time t=0 with a constant acceleration a=10 m/s^2 along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct?

(A) The particle arrives at the point (x=l, y=–h) at time t = 2s

(B) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point (x=l, y=–h)

(C) $\vec{L} = 4\hat{k}$ when the particle passes through the point (x=l, y=–h)

(D) $\vec{\tau} = \hat{k}$ when the particle passes through the point (x=0, y=–h)

द्रव्यमान M=0.2 kg का एक कण प्रारंभ में xy-तल में बिन्दु (x=–l,y=–h) पर विराम में है जहाँ l = 10 m तथा h=1 m है। समय t = 0 पर कण धनात्मक x-दिशा के अनुदिश नियत त्वरण a = 10 m/s² से त्वरित होता है। मूलबिन्दु के सापेक्ष इसके कोणीय संवेग तथा बल–आघूर्ण को SI मात्राको में क्रमशः $\vec{L}$ तथा $\vec{\tau}$, द्वारा प्रदर्शित किया जाता है। $\hat{i},\hat{j}$ तथा क्रमशः धनात्मक x, y तथा z-दिशाओ के अनुदिश एकांक सदिश है। यदि $\hat{k}=\hat{i}\times\hat{j}$ तब निम्न में से कौनसा/कौनसे कथन सही है/है ?

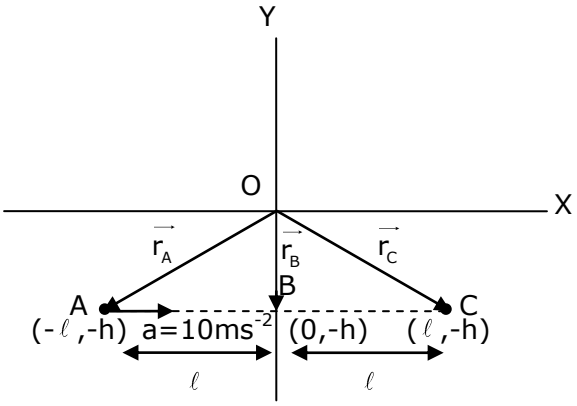
(A) कण, समय t = 2 s पर बिन्दु (x=l, y=–h) पर पहुंचता है

(B) $\vec{\tau}=2\hat{k}$ जब कण बिन्दु (x = l, y = –h) से गुजरता है

(C) $\vec{L}=4\hat{k}$ जब कण बिन्दु (x = l, y = –h) से गुजरता है

(D) $\vec{\tau}=\hat{k}$ जब कण बिन्दु (x = 0, y = –h) से गुजरता है

Sol. A,B,C



$\vec{r}_A=-\ell\hat{i}-h\hat{j}$

$\vec{r}_B=-h\hat{j}$

$\vec{r}_C=\ell\hat{i}-h\hat{j}$

$\vec{f}=m\vec{a}=0.2(10)\hat{i}=2\hat{i}$

(A) From A to C

S = 2ℓ

u = 0

a= 10

$\Rightarrow 2\ell=\frac{1}{2}(10)t^2$

$\Rightarrow t^2=\frac{4\ell}{10}=4\ (\ell=10)$

t = 2s. (A) is correct.

(B) $\vec{\tau}_C=\vec{r}_C\times\vec{F}$

$=(\ell\hat{i}-h\hat{j})\times 2\hat{i}$

$=-2h\left(-\hat{k}\right)$

$=2h\hat{k}$

$= 2\hat{k} \qquad (h = 1)$

(B) is correct.

(C) $\vec{L}_C = \vec{r}_C \times \vec{P}_C$

$\vec{P}_C = m\vec{V}_C$

$V_C = u + at$

$= 0 + 10(2)$

$= 20 ms^{-1}$

$\vec{P}_C = 0.2(20)\hat{i}$

$= 4\hat{i}$

$\vec{L}_C = \left(\ell\hat{i} - h\hat{j}\right) \times 4\hat{i}$

$= -4h\left(-\hat{k}\right)$

$= 4\hat{k}$

(C) is correct.

(D) $\vec{\tau}_B = \vec{r}_B \times \vec{F}$

$= -h\hat{j} \times 2\hat{i}$

$= -2h\left(-\hat{k}\right)$

$= 2\hat{k} \quad (h = 1)$

(D) is wrong. Infact $\vec{\tau}$ remains the same at all points along the line of motion w.r.t. any point on the x –axis.

The correct options are (A), (B) and (C)

QID: **521144** MODERN-I

14. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom?

(A) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5

(B) There is an overlap between the wavelength ranges of Balmer and Paschen series

(C) The wavelengths of Lyman series are given by $\left(1 + \frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(D) The wavelength ranges of Lyman and Balmer series do not overlap

हाइड्रोजन परमाणु के स्पैक्ट्रम के बारे में, निम्न में से कौनसा/कौनसे कथन सही है/है ?

(A) बॉमर श्रेणी में दीर्घत्तम तरंगदैर्ध्य व लघुत्तम तरंगदैर्ध्य का अनुपात 9/5 है।

(B) बॉमर तथा पाश्चन श्रेणी की तरंगदैर्ध्य परासो के मध्य अतिव्यापन होता है।

(C) लाइमन श्रेणी की तरंगदैर्ध्य को $\left(1 + \frac{1}{m^2}\right)\lambda_0$ द्वारा व्यक्त किया जाता है, जहाँ λ_0 लाइमन श्रेणी की लघुत्तम तरंगदैर्ध्य है तथा m एक पूर्णांक है।

(D) लाइमन व बॉमर श्रेणी की तरंगदैर्ध्य परास अतिव्यापित नही होती है।

Sol. **A,D**

For hydrogen atom

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

(A) for Balmer series

$n_1 = 2$; $n_2 = 3,4,5,...$

$$\frac{1}{\lambda_{longest}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\frac{1}{\lambda_{shortest}} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4}$$

$$\Rightarrow \frac{\lambda_{longest}}{\lambda_{shortest}} = \frac{36}{5R} \times \frac{R}{4}$$

$$= \frac{9}{5}$$

(A) is correct

(B) for Paschan series

$n_1 = 3$, $n_2 = 4,5, ...$

$$\frac{1}{\lambda_{longest}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$= \frac{7R}{144}$$

$$\frac{1}{\lambda_{shortest}} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right]$$

$$= \frac{R}{9}$$

For Balmer Series

λ ranges from $\frac{4}{R}$ to $\frac{36}{5R} = \frac{7.2}{R}$

for Paschen Series

λ ranges from $\frac{9}{R}$ to $\frac{144}{7R} = \frac{20.57}{R}$

Thus there is no overlap. (B) is wrong. In fact Lyman falls is UV region and Balmer falls in Visible region.

(C) In general for Lyman Series,

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{n_2^2}\right); n_2 = 2,3,4,...$$

and $\lambda_{shortest} = \lambda_0 = \frac{1}{R}$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_0}\left(1 - \frac{1}{n_2^2}\right) (\text{let } n_2 \text{ be } m)$$

$$\lambda = \lambda_0 \left(1 - \frac{1}{n_2^2}\right)^{-1}$$

$\lambda = \lambda_0\left(1+\frac{1}{n_2^2}\right)$

$\lambda = \lambda_0\left(1+\frac{1}{m^2}\right)$

Thus (C) is wrong.

(A) For balmer, λ ranges from $\frac{4}{R}$ to $\frac{36}{5R}$

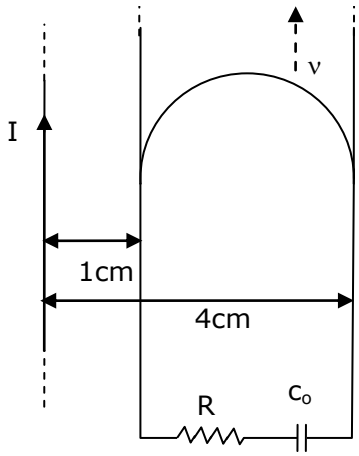
For Lyman, λ ranges from $\frac{1}{R}$ to $\frac{4}{3R}=\frac{1.33}{R}$

It's clear that they do not overlap.

QID: **521149** MAGNETISM

15. A long straight wire carries a current, I=2 ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time t=0, the rod starts moving on the rails with a speed v=3.0 m/s (see the figure).

A resistor R =1.4 Ω and a capacitor C_0=5.0 μF are connected in series between the rails. At time t=0, C_0 is uncharged. Which of the following statement(s) is(are) correct? [$\mu_0=4\pi\times10^{-7}$ SI units. Take ln2=0.7]



(A) Maximum current through R is 1.2 $\times10^{-6}$ ampere

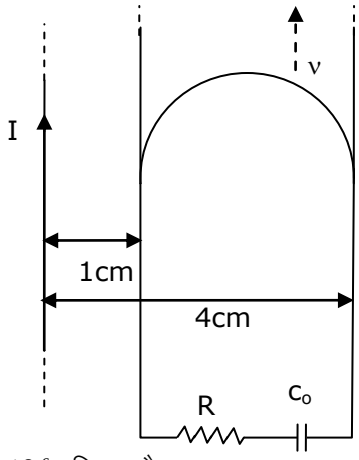
(B) Maximum current through R is 3.8 $\times10^{-6}$ ampere

(C) Maximum charge on capacitor C_0 is 8.4 $\times10^{-12}$ coulomb

(D) Maximum charge on capacitor C_0 is 2.4 $\times10^{-12}$ coulomb

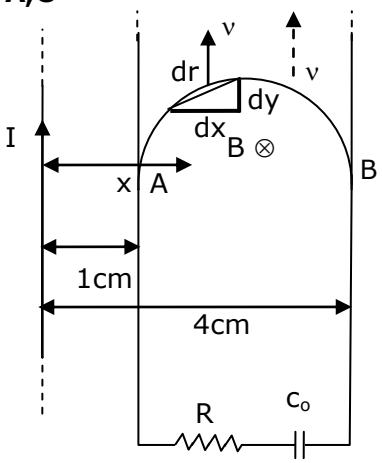
एक लम्बा सीधा तार धारा I = 2 एम्पियर ग्रहण किए हुए है। एक अर्द्धवृत्तीय चालक छड़ को इसके पीछे, नगण्य प्रतिरोध की दो चालक समान्तर पटरियो पर रखा जाता है। दोनो पटरियाँ तार के समान्तर है। तार, छड़ तथा पटरियाँ चित्रानुसार समान क्षैतिज तल में स्थित है। अर्द्धवृत्तीय छड़ के दोनो सिरे तार से 1 cm तथा 4 cm की दूरी पर है। समय t = 0 पर, छड़ पटरियो पर चाल v = 3.0 m/s से गति करना प्रारंभ करती है (चित्रा देखे)।

एक प्रतिरोधक R = 1.4 Ω तथा संधारित्रा C_0=5.0 μF को पटरियो के मध्य श्रेणी में संयोजित किया जाता है। समय t = 0 पर C_0 अनावेशित है। निम्न में से कौनसा/कौनसे कथन सही है/है ? [$\mu_0=4\pi\times10^{-7}$ SI इकाई, ln 2 = 0.7 लीजिए]



(A) R से प्रवाहित अधिकतम धारा 1.2×10^{-6} एम्पियर है
(B) R से प्रवाहित अधिकतम धारा 3.8×10^{-6} एम्पियर है
(C) संधारित्रा C_0 पर अधिकतम आवेश 8.4×10^{-12} कूलॉम है
(D) संधारित्रा C_0 पर अधिकतम आवेश 2.4×10^{-12} कूलॉम है

Sol. **A,C**



Consider a small element 'dr' of the semi-ring. The emf is induced due to the component ⊥ to V i.e., dx and no emf is induced due to dy. Thus,

$d\varepsilon = BdxV$

Here, b is the field due to the infinitely long current carrying wire.

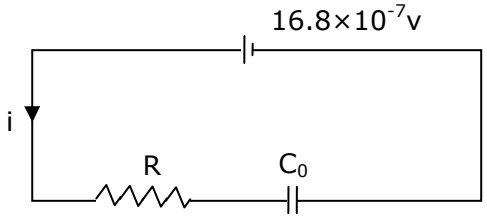
$$B = \frac{\mu_0 i}{2\pi x} \Rightarrow d\varepsilon = \frac{\mu_0 i}{2\pi x} V\ dx$$

$$\varepsilon = \frac{\mu_0 iv}{2\pi}\int_{1cm}^{4cm}\frac{dx}{x}$$

$= \frac{\mu_0 iv}{2\pi} \times \left[\ln(4) - \ln(1)\right] = \frac{2\mu_0 iv \ln(2)}{2\pi}$

$= \frac{4\pi \times 10^{-7} \times 2 \times 3 \times 0.7}{\pi} = 16.8 \times 10^{-7}$ v.

Now the diagram can be re-written as a circuit as shown.



Since this is a RC charging circuit,
Current is maximum at t= 0, when capacitor can be shorted.

$i_{max} = \frac{E_0}{R} = \frac{16.8 \times 10^{-7}}{1.4} = 1.2 \times 10^{-6}$ A.

During steady state i = 0 and charge (q_0) on the capacitor is maximum.
Thus,

$\frac{q_0}{C_0} = 16.8 \times 10^{-7}$

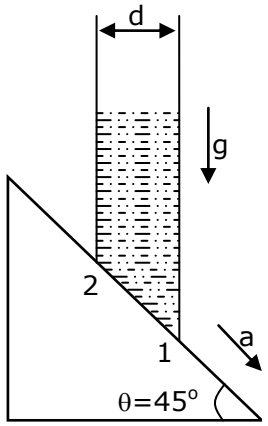
$q_0 = 16.8 \times 10^{-7} \times 5 \times 10^{-6}$

$= 8.4 \times 10^{-12}$C

(A) and (C) are correct options

QID: **521153** FLUIDS

16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle θ=45°. P_1 and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let β=(P_1− P_2)/(ρgd), where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct?



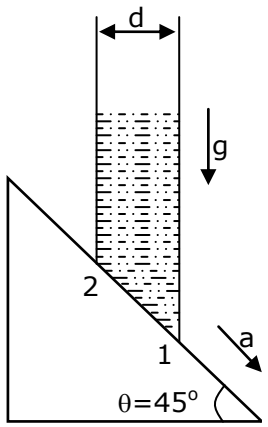
(A) $\beta = 0$ when $a = g/\sqrt{2}$

(B) $\beta > 0$ when $a = g/\sqrt{2}$

(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when a = g/2

(D) $\beta = \frac{1}{\sqrt{2}}$ when a = g/2

एक बेलनाकार नली जिसका आधार चित्रा में दर्शाए अनुसार है, को जल से भरा जाता है। यह $\theta = 45^\circ$ कोण वाले दृढ़ित आनत तल के अनुदिश नियत त्वरण a से नीचे की ओर गतिशील है। P_1 तथा P_2 नली के आधार पर स्थित क्रमशः बिन्दुओ 1 तथा 2 पर दाब हैं। माना $\beta=(P_1 - P_2)/(\rho gd)$, जहाँ ρ जल का घनत्व है, d नली का आन्तरिक व्यास है तथा g गुरूत्वीय त्वरण है। निम्न में से कौनसा/कौनसे कथन सही है/है ?



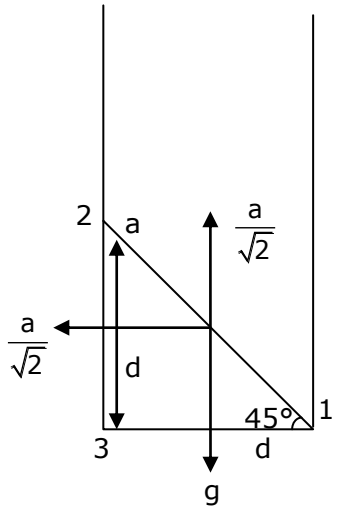
(A) $\beta = 0$ when $a = g/\sqrt{2}$

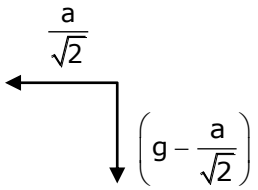
(B) $\beta > 0$ when $a = g/\sqrt{2}$

(C) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when a = g/2

(D) $\beta = \frac{1}{\sqrt{2}}$ when a = g/2

Sol. **A,C**





$\beta = (P_1 - P_2)/\delta gd$

$P_3 = P_2 + \sigma\left(g - \frac{a}{\sqrt{2}}\right) d$

$P_1 = P_3 - \sigma \frac{a}{\sqrt{2}} d$

$P_1 = P_2 + \sigma\left(g - \frac{a}{\sqrt{2}}\right) d - \frac{\sigma d}{\sqrt{2}}$

$P_1 = P_2 - \sigma gd - \sqrt{2}\ \sigma ad$

$\frac{P_1 - P_2}{\sigma gd} = \left(1 - \sqrt{2}\frac{a}{g}\right) = \beta$

if $a = \frac{g}{\sqrt{2}}$, $\beta = 0$ ....(A)

if $a = \frac{g}{2}$, $\beta = \frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$ ....(C)

SECTION 4

- This section contains **THREE (03)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

QID: **521159** **MODERN - I**

17. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulfur ion move in circular orbits of radii r_α and r_S, respectively. The ratio (r_S/r_α) is ___.

एक α-कण (द्रव्यमान 4 amu) और एक अकेला आवेशित सल्फर आयन (द्रव्यमान 32 amu) प्रारंभ में विरामावस्था में है। उन्हे एक विभवान्तर V के माध्यम से त्वरित किया जाता है और फिर एकसमान चुंबकीय क्षेत्र के एक क्षेत्र में से गुजारा जाता है, जो कि कणो के वेगो के लंबत्व है। इस क्षेत्र के अंदर, α-कण तथा सल्फर आयन क्रमशः त्रिज्या r_α तथा r_S की वृत्ताकार कक्षाओ में गति करते है। अनुपात (r_S/r_α) है ___.

Sol. **4**

$r = \frac{P}{qB} = \frac{\sqrt{2mE}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$\Rightarrow r\ \alpha \sqrt{\frac{m}{q}}$

$\Rightarrow \frac{r_s}{r_\alpha} = \sqrt{\frac{m_s}{m_\alpha} \times \frac{q_\alpha}{q_s}}$

$= \sqrt{\frac{32}{4} \times \frac{2e}{e}}$

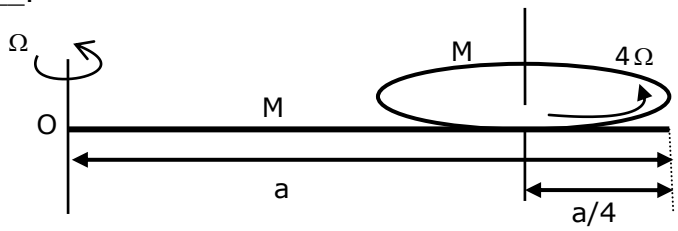
$= 4$

QID: **521183** **ROTATIONAL MOTION**

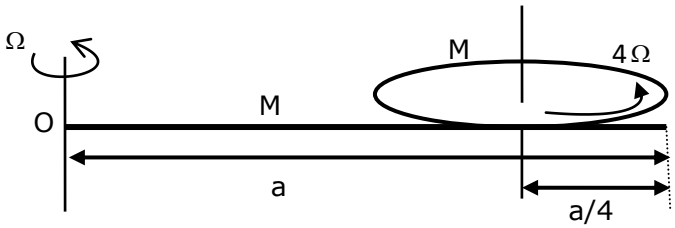
18. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius a/4 is pivoted on this rod with its center at a distance a/4 from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω.

The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)$ n.

The value of n is_____.



M द्रव्यमान व a लम्बाई की एक पतली छड़ बिन्दु O से गुजरते दृढ़ित उर्ध्वाधर अक्ष के सापेक्ष क्षैतिज तल में घूर्णन करने के लिए स्वतंत्रा है। M द्रव्यमान व a/4 त्रिाज्या की एक पतली वृत्तीय डिस्क इस छड़ पर कीलकीत है तथा इसका केन्द्र मुक्त सिरे से a/4 दूरी पर है, ताकि यह चित्रा में दर्शाए अनुसार इसके उर्ध्वाधर अक्ष के सापेक्ष स्वतंत्रा रूप से घूर्णन कर सके। माना छड़ तथा डिस्क दोनो का घनत्व एकसमान है तथा ये गति के दौरान क्षैतिज रहते है। एक बाह्य स्थिर प्रेक्षक को ज्ञात होता है कि छड़ कोणीय वेग Ω से घूर्णन कर रही है तथा डिस्क कोणीय वेग 4Ω से इसके उर्ध्वाधर अक्ष के सापेक्ष घूर्णन कर रही है। बिन्दु O के सापेक्ष निकाय का कुल कोणीय संवेग $\left(\frac{Ma^2\Omega}{48}\right)$ n n है। n का मान _____ है।



Sol. **49**

For rod →

$L_{rod} = I_{rod}\,\omega$

$\Rightarrow \frac{1}{3}Ma^2\Omega$

For disc

$L_{disc} = M\left(\vec{r_{cm}} \times \vec{v_{cm}}\right) + I_{disc}(4\Omega)$

$V_{cm} = \frac{3a}{4}$

$V_{cm} = \frac{3a}{4}\Omega$

$\Rightarrow L_{disc} = M\left(\frac{9a^2}{16}\Omega\right) + \frac{Ma^2}{2(16)} \times 4\Omega$

$\Rightarrow Ma^2\,\Omega\left(\frac{9}{16}+\frac{2}{16}\right) \Rightarrow \frac{11Ma^2\Omega}{16}$

$L_{net} = L_{rod} + L_{disc} \Rightarrow \frac{Ma^2\Omega}{3}+\frac{11Ma^2\Omega}{16}$

$\Rightarrow \frac{Ma^2\Omega}{48}(16+33) \Rightarrow \frac{Ma^2\Omega}{48}(49)$

n = 49

QID: **521190** **HEAT-1**

19. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time t=0, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t=t_1$ and 50 K at $t=t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_2/t_1) is ____.

एक छोटी वस्तु को बड़े निर्वातित खोखले गोलीय पात्रा के केन्द्र पर रखा गया है। माना कि पात्रा 0 K पर व्यवस्थित है। समय t = 0 पर, वस्तु का ताप 200 K है। वस्तु का ताप $t=t_1$ पर 100 K तथा $t=t_2$ पर 50 K हो जाता है। माना वस्तु तथा पात्रा आदर्श कृष्णिका हैं। वस्तु की ऊष्मा धारिता ताप पर निर्भर नहीं करती है। अनुपात (t_2/t_1) का मान _____ है।

Sol. **9**

Using Stephan Boltzmann Law

$P = \sigma A\,(T^4 - O^4)$

$P = \sigma AT^4$

$\frac{dQ}{dt} = \sigma AT^4$

$\Rightarrow -mc\frac{dT}{dt} = \sigma AT^4$

$\Rightarrow \int_{200}^{T}\frac{dT}{T^4} = \int_0^t -k\,dt$

$\Rightarrow \left[\frac{1}{3T^3}\right]_{200}^{T} = k(t-o)$

$\Rightarrow \frac{1}{3}\left[\frac{1}{T^3}-\frac{1}{(200)^3}\right] = k\,t$

$\Rightarrow$ If T = 100 $\quad t = t_1$

T = 50 $\quad t = t_2$

$\frac{t_2}{t_1}\frac{\frac{1}{50^3}-\frac{1}{200^3}}{\frac{1}{100^3}-\frac{1}{200^3}} = \frac{1-\frac{1}{64}}{\frac{1}{8}-\frac{1}{64}}$

$\frac{t_2}{t_1} = \frac{\frac{63}{64}}{\frac{7}{64}} = 9$