हमारा विश्वास...
हर एक विद्यार्थी है ख़ास

PAPER WITH SOLUTION

JEE Advanced 2019

PHYSICS PAPER - 2

हमारा विश्वास... हर एक विद्यार्थी है ख़ास

MOTION™
Nurturing potential through education

CRITERIA FOR DIRECT ADMISSION IN STAR BATCHES

V STAR BATCH
XII Pass (JEE M+A)

ELIGIBILITY

JEE Main'19 %tile > 98%tile

JEE Advanced'19 Rank (Gen.) < 15,000

J STAR BATCH
XII Pass (NEET/AIIMS)

ELIGIBILITY

NEET'19 Score > 450 Marks

AIIMS'19 %tile > 98%tile

P STAR BATCH
XI Moving (JEE M+A)

ELIGIBILITY

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

H STAR BATCH
XI Moving (NEET/AIIMS)

ELIGIBILITY

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

Scholarship Criteria

JEE Main Percentile	SCHOLARSHIP+ STIPEND
98 - 99	100%
Above 99	100% + ₹ 5000/ month

JEE Advanced Rank	SCHOLARSHIP+ STIPEND
10000-20000	100%
Under 10000	100% + ₹ 5000/ month

NEET 2019 Marks	SCHOLARSHIP+ STIPEND
450	100%
530-550	100% + ₹ 2000/ month
550-560	100% + ₹ 4000/month
560	100% + ₹ 5000/month

NTSE STAGE-1 2019 Marks	SCHOLARSHIP+ STIPEND
160-170	100% + ₹ 2000/ month
171-180	100% + ₹ 4000/month
180+	100% + ₹ 5000/month

FEATURES :

- Batch will be taught by NV Sir & HOD's Only.
- Weekly Quizes apart from regular test.
- Under direct guidance of NV Sir.
- Residential campus facility available.
- 20 CBT (Computer Based Test) for better practice.
- Permanent academic coordinator for personal academic requirement.
- Small batch with only selected student.
- All the top brands material will be discussed.

SECTION-I

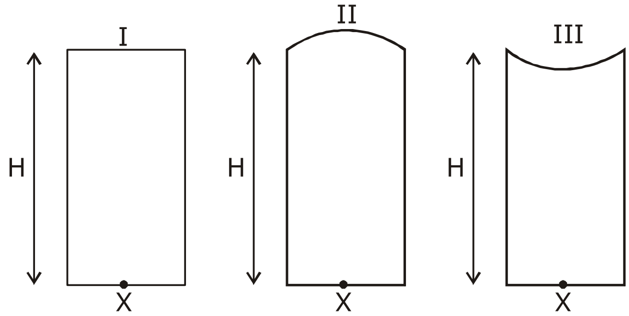
- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** option. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s)
- For each question, choose the option(s) corresponding to (all) the correct answer(s)
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : + 4 If only (all) the correct option(s) is(are)chosen;
Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.
Partial Marks : + 2 If three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks : + 1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : – 1 In all other cases

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get + 4 marks;
choosing ONLY (A) and (B) will get + 2 marks;
choosing ONLY (A) and (D) will get + 2 marks :
choosing ONLY (B) and (D) will get + 2 marks;
choosing ONLY (A) will get + 1 Marks ;
Choosing ONLY (B) will get + 1 mark;
Choosing ONLY (D) will get + 1 mark ;
choosing no option (i.e. the question is unanswered) will get 0 marks ; and choosing any other combination of options will get – 1 mark

1. Three glass cylinders of equal height H = 30 cm and same refractive index n = 1.5 are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3m). If H_1, H_2 and H_3 are the apparent depths of a point X on the bottom of the three cylinder, respectively, the correct statement(s) is/are :



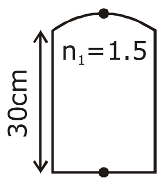
(1) $H_3 > H_1$
(2) $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$
(3) $H_2 > H_1$
(4) $H_2 > H_3$

Sol. **3, 4**

$$\frac{d'}{d} = \frac{n_2}{n_1}$$

$$\Rightarrow \frac{d'}{30} = \frac{1}{3/2}$$

$$\Rightarrow d' = 20 \text{ cm}$$

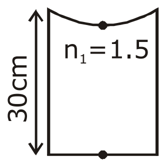


$$\frac{1}{v} + \frac{1.5}{30} = \frac{(1-1.5)}{-300}$$

$$\Rightarrow \frac{1}{v} = +\frac{1}{600} - \frac{3}{60} = \frac{1-30}{600}$$

$$\Rightarrow \frac{1}{v} = -\frac{29}{600}$$

$$\Rightarrow v = \frac{600}{29} = -20.68 \text{ cm}$$

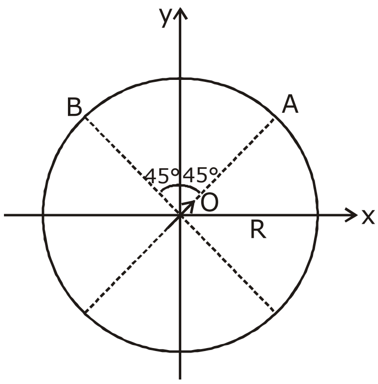


$$\frac{1}{v} - \frac{1.5}{-30} = \frac{1-1.5}{300}$$

$$\Rightarrow \frac{1}{v} = \frac{-1}{600} - \frac{3}{60} = \frac{-1-30}{600} = \frac{-31}{600}$$

$$\Rightarrow v = -\frac{600}{31} = -19.35$$

2. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are :
(ε_0 is permittivity of free space. R >> dipole size)



(1) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

(2) Total electric field at point B is $\vec{E}_B = 0$

(3) $R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

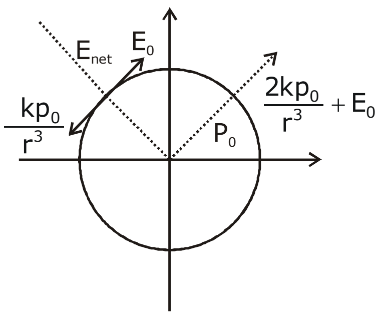
(4) The magnitude of total electric field on any two points of the circle will be same

Sol. **2,3**

E_{net} should be $\perp$ to surface so $\frac{kp_0}{r^3} = E_0 \Rightarrow r = \left(\frac{kp_0}{E_0}\right)^{1/3}$

$(E_A)_{net} = \frac{2kp_0}{r^3} + E_0 = 3E_0$

$(E_B)_{net} = 0$



3. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state n =1 to the state n =4. Immediately after that the electron jumps to n = m state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and ΔP_e, respectively. If $\lambda_a/\lambda_e = \frac{1}{5}$, which of the option(s) is/are correct ?

[Use hc = 1242 eV nm ; 1 nm =10^{-9} m, h and c are Plank's constant and speed of light, respectively]

(1) $\Delta p_a/\Delta p_e = \frac{1}{2}$

(2) The ratio of kinetic energy of the electron in the state n = m to the state n = 1 is $\frac{1}{4}$

(3) λ_e = 418 nm

(4) m = 2

Sol. **2,4**

$$E_a = \frac{hc}{\lambda}$$

$$\frac{\lambda_a}{\lambda_e} = \frac{E_4 - E_1}{E_4 - E_m} = \frac{1-\frac{1}{16}}{\frac{1}{n^2}-\frac{1}{16}} = \frac{1}{5}$$

$\Rightarrow$ m = 2

$(4 \to 2)\ E_e = \frac{hc}{\lambda}$ $\qquad \lambda = \frac{h}{\sqrt{2mKE}}$

$$\Rightarrow \lambda_e = \frac{12400}{E_e} = \frac{12400}{2.75}$$

= 4509 Å

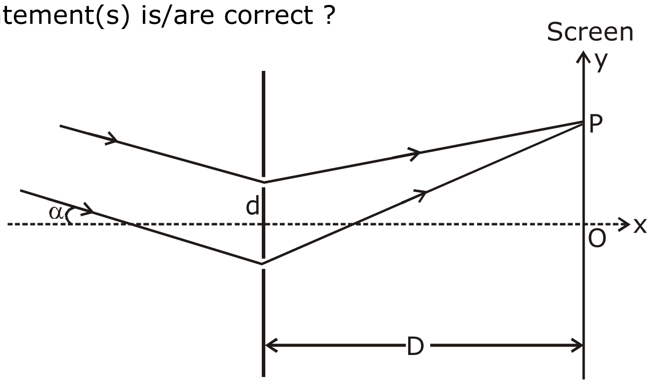
= 450 nm

(2) n = 2 $\to$ – 3.4

n = 1 $\to$ – 13.6

$$\therefore \frac{E_2}{E_1} = \frac{3.4}{13.6} = \frac{1}{4}$$

4. In a Young's double slit experiment, the slit separation d is 0.3 mm and the screen distance D is 1 m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle α as shown in figure. On the screen, the point O is equidistant from the slits and distance PO is 11.0 mm. Which of the following statement(s) is/are correct ?



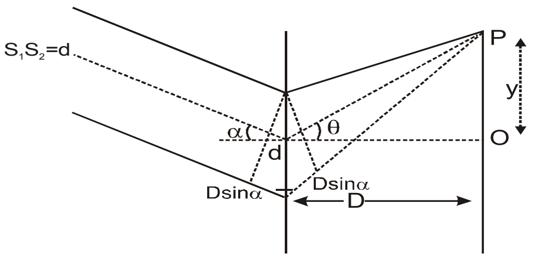
(1) Fringe spacing depends on α

(2) For $\alpha = 0$, there will be constructive interference at point P

(3) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point P.

(4) For $\alpha = \frac{0.36}{\pi}$ degree, there will be destructive interference at point O.

Sol. **3**



$\Delta x = d\sin\alpha + d\sin\theta$

$\theta\alpha$:small angle $\qquad \sin\theta \simeq \tan\theta = \frac{y}{D}$

$\Delta x = d\alpha + \frac{dy}{D}$

(1) Fringe width does not depend on α.

(2) $\Delta x = 0$

So constructive intereference

(3) $\Delta x = 3mm \times \frac{0.36}{\pi} \times \frac{\pi}{180} + \frac{3mm \times 11mm}{1} = 3900$ nm

$3900nm = (2n-1)\frac{\lambda}{2} = (2n-1) \times \frac{600nm}{2}$

$n = 7$

destructive interference happend

(4) $\Delta x = 3mm \times \frac{0.36}{\pi} \times \frac{\pi}{180} + 0 = 600nm$

600 nm $= n\lambda$

$n = 1$

constructive interference

5. A thin and uniform rod of mass M and length L is held vertical on floor with large friction. The rod is released from rest so that is falls by rotating about is contact-point with the floor without slipping. Which of the following statement(s) si/are correct. When the rod makes an angle 60° with vertical? [g is the acceleration due to gravity]

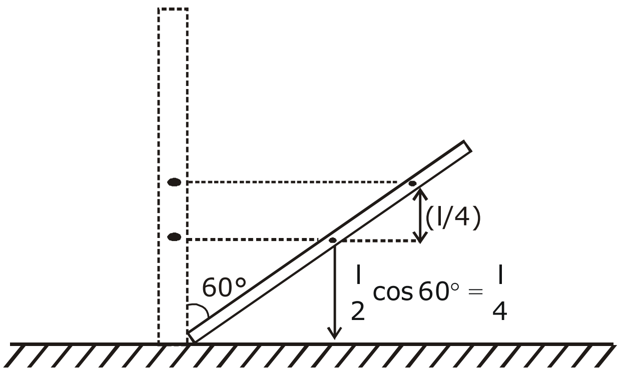
(1) The Normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(4) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

Sol. **1,3,4**



By E.C.

$\frac{1}{2}Iw^2 = +mg\frac{\ell}{4}$

$\Rightarrow \frac{1}{2}\frac{m\ell}{3}\omega^2 = mg\frac{\ell}{4}$

$\Rightarrow \omega = \frac{m\ell^2}{3}\omega^2 = mg\frac{\ell}{4^2}$

$\Rightarrow \omega = \sqrt{\frac{3g}{2l}}$

$a_c = \omega^2 R = \frac{3g}{2l}\times\frac{l}{2} = \frac{3g}{4}$

$\alpha = \frac{\tau}{I}$

$= \frac{mg\frac{l}{2}\sin 60°}{\frac{ml^2}{3}} = \frac{3\sqrt{3}g}{4l}$

$a = \alpha\frac{l}{2}\sin 60° + \omega^2\frac{l}{2}\cos 60°$

$(R\alpha)$

$= \frac{9g}{16} + \frac{6g}{16} = \frac{15g}{16}$

$\therefore mg - N = ma$

$\Rightarrow N = mg - \frac{15mg}{16}$

$N = \frac{Mg}{16}$

6. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0. Volume V_0. and temperature T_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,

(Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(1) Adiabatic constant of the gas mixture is 1.6

(2) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$.

(3) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$.

(4) The work |W| done during the process is $13RT_0$.

Sol. **1,3,4**

$$\left.\begin{matrix}\text{monoatomic} & (5\text{ moles}) \\ \text{diatomic} & (1\text{ mole})\end{matrix}\right\} P_0, V_0, T_0$$

adiabatically comperession to $\frac{V_0}{4}$

$$\gamma_{mix} = \frac{n_1 C_{p1} + n_2 C_{P_2}}{n_1 C_{Y_1} + n_2 C_{V_2}} = \frac{8}{5}$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

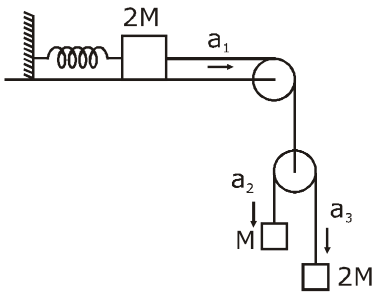
and $P_0 V_0^{8/5} = P_2 \left(\frac{V_0}{4}\right)^{8/5}$

$\therefore P_2 = 9.2P_0$

$$\therefore \quad W = \frac{P_0 V_0 - (9.2P_0)(V_0/4)}{3/5} = -13RT_0$$

$|W| = 13RT_0$

7. A block of mass 2M is attached to a massless spring with spring-constant K. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in the figure. The system is released from rest with The spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ? [g is the acceleration due to gravity. Neglect friction]



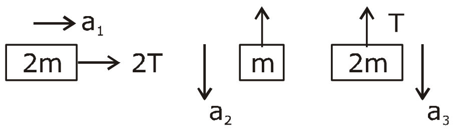
(1) $x_0 = \frac{4Mg}{k}$

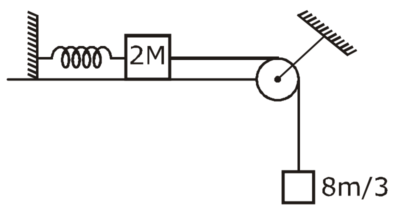
(2) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$.

(3) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(4) $a_2 - a_1 = a_1 - a_3$

Sol. **4**





We know

$2a_1 = a_2 + a_3$

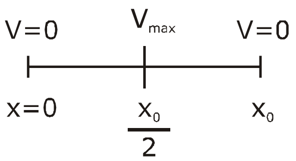
or $a_2 - a_1 = a_1 - a_3$

and $\frac{T}{g_{app}} = \frac{8m}{3}$

$$\frac{1}{2}Kx_0^2 = \left(\frac{8m}{3}\right)gx_0$$

$$x_0 = \frac{16mg}{3k}$$

amplitude of SHM in $\frac{x_0}{2}$



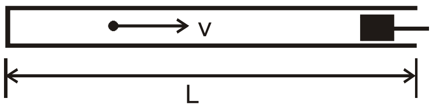
$$U_{max} = \omega A = \left(\sqrt{\frac{K}{2m + 8m/3}}\right).\left(\frac{x}{2}\right) = \frac{x_0}{2}\sqrt{\frac{3K}{14m}}$$

$$= \frac{8mg}{3K}.\sqrt{\frac{3K}{14m}} = g\sqrt{\frac{64 \times 3m}{9 \times 14k}}$$

ACC at $\frac{x_0}{4}$

$$a = \omega^2 x = \frac{3K}{14m}.\frac{x_0}{4} = \frac{3K}{14m} \times \frac{4ms}{3k} = \frac{4g}{14} = \frac{2}{7}g$$

8. A small particle of mass m moving inside a heavy hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V << \frac{dL}{L}V_0$. Where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



(1) After each collision with the piston, the particle speed increases by 2V

(2) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(3) The rate at which the particle strikes the piston is v/L.

(4) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$.

Sol. **1, 4**

Initial : $v = v_0$

Distance $x = L_0$

$$dt = \frac{dx}{v}$$

$$dt' = \frac{2x}{v}$$

no. of collision = $n = \frac{v}{2x}$

Total in dt = N = $\frac{v}{2x}.\frac{dx}{v}$

$\Delta V = 2v$

Speed change in dx shifting = dv = $\frac{vdx}{2xv}.2v$

$$\therefore dv = \frac{vdx}{x}$$

$$\Rightarrow dv = v\frac{dL}{L}$$

$$\Rightarrow \int_{V_0}^{V'}\frac{dv}{v} = \int_{\ell}^{\ell/2}\frac{dL}{L}$$

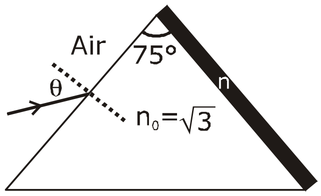
$$\Rightarrow V' = 2V_0$$

$\therefore$ KE is 4 times

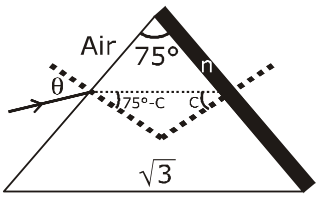
Secction-2 (Maximum Marks : 18)

- This section contains SIX (06) questions The answer to each questions is a NUMERICAL VALUE.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places. truncate/roung-off the value of TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : +3 If **ONLY the** correct numerical value is entered.
  Zero Marks : 0 In all other cases.

1. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $\eta_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of $\theta \le 60°$. The value of n^2 is ___________.



Sol. 1.5



For TIR at coating $\sin c = \frac{n}{\sqrt{3}}$

applying snell's law

$\sin\theta = \sqrt{3}\sin(75° - C)$

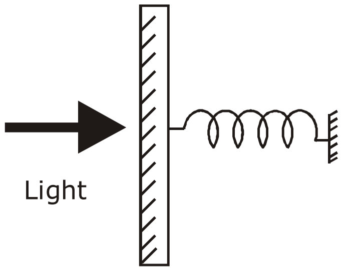
For limiting condition θ = 60°

$\sin 60° = \sqrt{3}\sin(75° - C)$

$\therefore$ C = 45°

and $n^2 = \frac{3}{2} = 1.5$

2. A perfectly reflecting mirror of mass *M* mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\frac{4\pi M\Omega}{h} = 10^{24} m^{-2}$ with *h* as Planck's constant. *N* photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by $1\,\mu m$. If the value of N is x x 10^{12}, then the value of x is ___
[Consider the spring as massless]



Sol. **1**

$mV_{max} = \left(\frac{2h}{\lambda}\right)N$

$m.\omega A = \frac{2h}{\lambda}N$

$N = \frac{m\omega\lambda A}{2h}$

$N = \frac{10^{24}}{4\pi} \times \frac{8\pi \times 10^{-6}}{2}\left(10^{-6}\right)$

$N = 10^{12}$

$\therefore X = 1$

3. Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α – decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ – decay. The energy of the emitted γ photon is ___keV.

[Given: atomic mass of $^{226}_{88}Ra = 226.005u$ atomic mass of $\frac{222}{86}Rn = 222.000u$, atomic mass of α particle = 4.000 u, 1 u =931 MeV/c^2, c is speed of the light]

Sol. **135 KeV**

$^{226}_{88}Ra \longrightarrow {}^{222}_{86}R_n^* + {}_2He^4 + Q_1$

Given: $(K.E.)_\alpha = 4.44$ MeV

$Q_1 = \Delta m \times 931$ MeV

$\Delta m = m_{Ra} - (m_{Rn} + m_{He})$

$\Rightarrow 226.005 - (222 + 4)$

$\Rightarrow 226.005 - 226$

$\Rightarrow 0.005$

So, $Q = 0.005 \times 931$

$\Rightarrow 4.655$ MeV

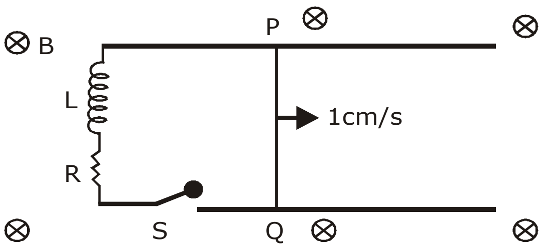
Now, the energy of γ-photon

$k_\alpha = (Q - E)\left(\frac{A-4}{A}\right)$

$\Rightarrow 4.44 = (4.655 - E)\left(\frac{222}{226}\right)$

$\Rightarrow E = 0.135$ MeV
$\Rightarrow E = 135$ KeV

4. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L=1 mH and a resistance R=1 Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B=1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is x x 10^{-3} A, where the value of x is ___
[Assume the velocity of wire PQ remain constant (1cm/s) after key S is closed. Given: e^{-1}=0.37, where e is base of the natural logarithm]



Sol. 0.63
As we know relation of e.m.f. for rail road problems
$e = Bvl \sin\theta$
$(\theta = 90°)$
$e = Bvl$
$l = 10$ cm
$\Rightarrow 10^{-1}$ m
Given B = 1 Tesla
v = 1 cm/sec
$\Rightarrow 10^{-2}$ m/sec
$e = 1 \times 10^{-2} \times 10^{-1} \Rightarrow 10^{-3}$ volt
and also we know

$$i = i_0\left(1 - e^{\frac{-L}{R}}\right)$$

$$i = \frac{e}{R}\left(1 - e^{\frac{-L}{R}}\right)$$

$$i = \frac{10^{-3}}{1}\left(1 - e^{\frac{-L}{R}}\right) \Rightarrow \frac{10^{-3}}{1}(1 - e^{-1})$$

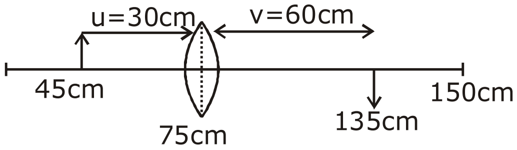
$\Rightarrow 10^{-3}(1 - 0.37)$
$\Rightarrow 10^{-3}(0.63) \Rightarrow 0.63$ mA

5. An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is___

Sol. **1.38**

Least count = $\frac{1}{4}$ cm

= 0.25 cm



lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f}$$

$$\frac{3}{60} = \frac{1}{f}$$

f = 20 cm

By error formula

$$\frac{dv}{u^2} + \frac{du}{u^2} = \frac{df}{f^2}$$

(double error because we need to measure u also)

$$\frac{2\times0.25}{60^2} + \frac{2\times0.25}{30^2} = \frac{df}{(20)^2}$$

$$2 \times 0.25 \times 20 \times 20 \left(\frac{1}{3600} + \frac{1}{900}\right) = df$$

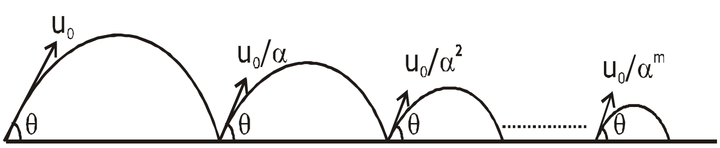
$$2 \times 100 \times \left(\frac{1+4}{3600}\right) = df$$

$$\% \text{ error} = \frac{df}{f} \times 100\% = \frac{5}{36} \times \frac{1}{20} \times 100 \times 2$$

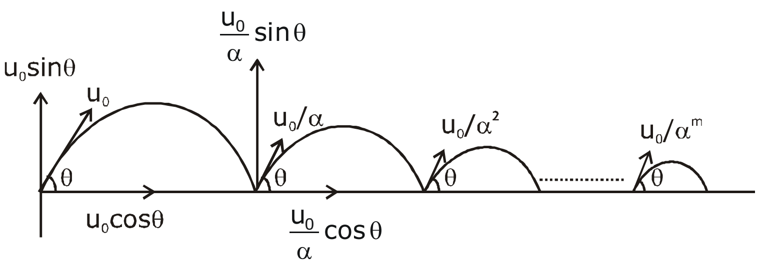
= 0.69 % × 2

= 1.38%

6. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1. After hitting the ground, the ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is 0.8 V_1,. the value of α is ________.



Sol. **4**



$$V_{avg} = \frac{R_1 + R_2 + R_3 + ...}{T_1 + T_2 + T_3 + ...}$$

$$V_{avg} = \frac{\frac{2u_{x_1}u_{y_1}}{g} + \frac{2u_{x_2}u_{y_2}}{g} + ...}{\frac{2u_{y_1}}{g} + \frac{2u_{y_2}}{g} + ...}$$

$$= \frac{u_0\cos\theta . u_0\sin\theta + \frac{u_0}{\alpha}\cos\theta . \frac{u_0}{\alpha}\sin\theta + ...}{u_0\sin\theta + \frac{u_0}{\alpha}\sin\theta + \frac{u_0}{\alpha^2}\sin\theta + ...}$$

$$= \frac{u_0\cos\theta\left[1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ... \frac{1}{\alpha^{2n}}\right]}{\left[1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ... + \frac{1}{\alpha^n}\right]}$$

Given, $u_0 \cos\theta = v_1$

& $v_{avg} = 0.8\ v_1$

$$0.8\ v_1 = \frac{v_1\left[1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + ... + \frac{1}{\alpha^{2n}}\right]}{1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + ... + \frac{1}{\alpha^n}}$$

$$0.8 = \frac{\left[\frac{1}{1 - \frac{1}{\alpha^2}}\right]}{\left[\frac{1}{1 - \frac{1}{\alpha}}\right]}$$

$$= \frac{\frac{1}{\left(1 - \frac{1}{\alpha}\right)\left(1 + \frac{1}{\alpha}\right)}}{\frac{1}{\left(1 - \frac{1}{\alpha}\right)}}$$

$$0.8 = \frac{1}{\left(1+\frac{1}{\alpha}\right)}$$

$$1 + \frac{1}{\alpha} = \frac{1}{0.8}$$

$$\frac{1}{\alpha} = \frac{10}{8} - 1$$

$$\frac{1}{\alpha} = \frac{2}{8}$$

$$\alpha = \frac{8}{2} = 4$$

SECTION - 3 [MAXIMUM MARKS : 12]

- This section contains TWO (02) List-Match Sets.
- Each List-Match set has TWO (02) Multiple Choice Questions.
- Each List-Match Set has Two lists : List-I and List-II
- List-I has Four entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- FOUR Options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen ;
  Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
  Negative Marks : −1 In all other cases

1. A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

List-I gives the above four strings while list-II lists the magnitude of some quantity.

List-I	List-II
(I) String - 1 (μ)	(P) 1
(II) String - 2 (2μ)	(Q) 1/2
(III) String - 3 (3μ)	(R) $1/\sqrt{2}$
(IV) String - 4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be -

(1) I - Q, II - S, III - R, IV - P
(2) I - P, II - R, III - S, IV - Q
(3) I - P, II - Q, III - T, IV - S
(4) I - Q, II - P, III - R, IV - T

Sol. 2

Fundamental frequency $f_0 = f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{u}} \rightarrow (1)$

Ist string $f_1 = \frac{1}{2\ell_0}\sqrt{\frac{T_0}{\mu}}$

$f_1 = f_0$

IInd string $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}}$

$f_2 = \frac{1}{\sqrt{2}}f_0$

IIIrd string

$f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}}$

$f_3 = \frac{1}{\sqrt{3}}f_0$

IVth string

$f_4 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}}$

$f_4 = \frac{1}{2}f_0$

2. A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.
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(III) String - 3 (3μ)	(R) $1/\sqrt{2}$
(IV) String - 4 (4μ)	(S) $1/\sqrt{3}$
	(T) 3/16
	(U) 1/16

The length of the strings 1,2,3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$, respectively. S trings 1,2,3 and 4 are vibrated at their 1st, 3rd, 5th and 14th harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be -

(1) I - P, II - R, III - T, IV - U
(2) I - P, II - Q, III - T, IV - U
(3) I - T, II - Q, III - R, IV - U
(4) I - P, II - Q, III - R, IV - T

Sol. **2**

I→P; II→Q;III→T; IV→U

fundamental frequency

$$f_0 = \frac{1}{2\ell}\sqrt{\frac{T_0}{\mu}}$$

I^{st} string → 1 st harmonic

$$f_1 = \frac{1`}{2\ell}\sqrt{\frac{T_0}{\mu}}$$

$f_1 = f_0$

so $\Rightarrow T_1 = T_0$

II^{nd} string → IIIrd Harmonic

$$\ell = \frac{3\ell_0}{2}$$

$$f_2 = 3\times\frac{1}{2\times\frac{3\ell_0}{2}}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{\ell_0}\sqrt{\frac{T_2}{2\mu}}$$

$\because f_2 = f_0$

So $T_2 = \frac{T_0}{2}$

IIIrd string

$\ell = \frac{5\ell_0}{4}$ → Fifth harmonic

$$f_3 = 5\times\frac{1}{2\times\frac{5\ell_0}{4}}\times\sqrt{\frac{T_3}{3\mu}}$$

$$f_3 = \frac{2}{\ell_0}\sqrt{\frac{T_3}{3\mu}}$$

$\because f_3 = f_0$

$$\frac{2}{\ell_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3T_0}{16}$$

IVth streing → IV harmonic

$$f_4 = 14 \times \frac{1}{\frac{2 \times 7\ell_0}{4}} \sqrt{\frac{T_4}{4\mu}}$$

$$f_4 = \frac{4}{\mu_0} \sqrt{\frac{T}{4\mu}}$$

$f_4 = f_0$

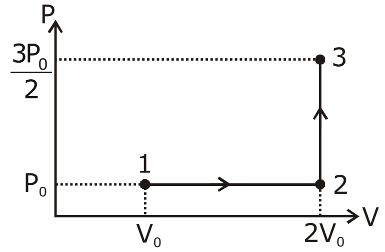
$$T_4 = \frac{T_0}{16}$$

3. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List-I	List-II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3} RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3} RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3} RT_0$
	(T) $\frac{1}{3} RT_0 (3 + \ln 2)$
	(U) $\frac{5}{6} RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV - diagram with $P_0V_0 = \frac{1}{3} RT_0$. The correct match is -



(A) I - Q, II - S, III - R, IV - U
(B) I - S, II - R, III - Q, IV - T
(C) I - Q, II - R, III - P, IV - U
(D) I - Q, II - R, III - S, IV - U

Sol. **4**

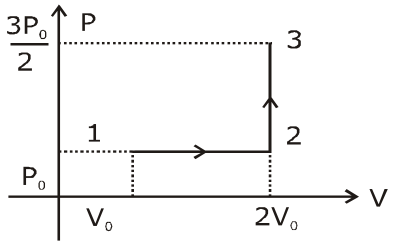
I→Q; II→R; III→S; IV→U

$$X = \frac{3}{2}R \quad \ell n\frac{T}{T_A} + R\ell n\frac{V}{V_A}$$

$$dx = \frac{3}{2}R\frac{1}{T}dT + \frac{R}{V}dv$$

$d\theta = Tdx$

$$d\theta = \frac{3}{2}RdT + \frac{RT}{V}dv$$



$$\int dQ = \int \frac{3}{2}RdT + \int_{V_0}^{2V_0}\int \frac{RT}{V}dv$$

(iv) $$Q_{1\to2} = \int \frac{3}{2}RdT + \int_{V_0}^{2V_0}\frac{RT}{V}dv$$

$$Q = \frac{3}{2}R\int_{\frac{2T_0}{3}}^{T_0} dT + \int pdv$$

$$Q = \frac{3}{2}R\frac{T_0}{3} + P_0V_0$$

$$Q_{1\to2} = \frac{RT_0}{2} + \frac{RT_0}{3} = \frac{5}{6}RT_0 \quad \{\because P_0V_0 = \frac{RT_0}{3}$$

(iii) $$Q_{2\to3} = \int_{\frac{2T_0}{3}}^{T_0}\frac{3}{2}RdT + 0$$

$$Q_{2\to3} = \frac{RT_0}{2}$$

$$Q_{1\to3} = \frac{5}{6}RT_0 + \frac{RT_0}{2} = \frac{4}{3}RT_0$$

(ii) $$\Delta u = \frac{f}{2}nR\Delta T = \frac{3}{2}R\frac{2T_0}{3} = RT_0$$

(1) $dq = dw + du$

$dw = dq - du$

$$w = \frac{4}{3}RT_0 - RT_0$$

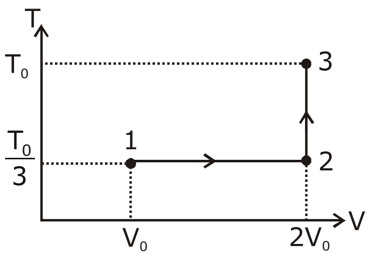
$$W = \frac{RT_0}{3}$$

4. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2} R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$. Here, R is gas constant, V is volume of gas, T_A and V_A are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

List - I	List - II
(I) Work done by the system in process $1 \to 2 \to 3$	(P) $\frac{1}{3} RT_0 \ln 2$
(II) Change in internal energy in process $1 \to 2 \to 3$	(Q) $\frac{1}{3} RT_0$
(III) Heat absorbed by the system in process $1 \to 2 \to 3$	(R) RT_0
(IV) Heat absorbed by the system in process $1 \to 2$	(S) $\frac{4}{3} RT_0$
	(T) $\frac{1}{3} RT_0 (3 + \ln 2)$
	(U) $\frac{5}{6} RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in the TV - diagram with $P_0V_0 = \frac{1}{3} RT_0$. The correct match is -



(A) I - P, II - R, III - T, IV - P
(B) I - P, II - R, III - T, IV - S
(C) I - S, II - T, III - Q, IV - U
(D) I - P, II - T, III - Q, IV - T

Sol. **1**

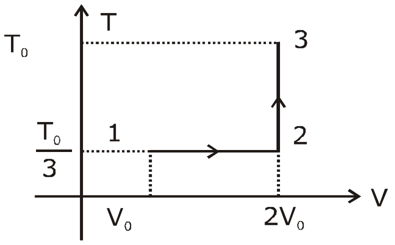
I→P; II→R; III→T; IV→P

$$X = \frac{3}{2}R \quad \ell n\frac{T}{T_A} + R\ell n\frac{V}{V_A}$$

$$dx = \frac{3}{2}R\frac{1}{T}dt + \frac{R}{V}dv$$

dQ = Tdx

$dQ = \frac{3}{2}RdT + \frac{RT}{V}dv$



(iv) $\int dQ = \int \frac{3}{2}Rdt + \int \frac{RT}{V}dv$

$Q = 0 + \frac{RT_0}{3}(\ell nv)_{v_0}^{2v_0}$

$Q_{1\to2} = \frac{RT_0}{3}\ell n2$

(iii) $Q_{2\to3} = \int_{\frac{T_0}{3}}^{T_0} \frac{3}{2}Rdt + 0 = \frac{3}{2}Rto\frac{2}{3} = RT_0$

$Q_{1\to3} = Q_{1\to2} + Q_{2\to3}$

$= \frac{RT_0}{3}(3 + \ell n2)$

(ii) Change in internal energy

$\Delta u = \frac{f}{2}nR\Delta t$

$\Delta u = \frac{3}{2}R\left(T_0 - \frac{T_0}{3}\right)$

$\Delta u = RT_0$

(i) $dq = dw + \Delta u$

$dw = dq - \Delta u$

$= \frac{1}{3}RT_0\ell n2$

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95.5 To 96	₹ 65,250	₹ 65,250
95 To 95.5	₹ 72,500	₹ 72,500
93 To 95	₹ 87,000	₹ 87,000
90 To 93	₹ 1,01,500	₹ 94,250
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