हमारा विश्वास...
हर एक विद्यार्थी है ख़ास

PAPER WITH SOLUTION

JEE Advanced 2019

MATHEMATICS PAPER - 2

IIT/NIT | NEET / AIIMS | NTSE / IJSO / OLYMPIADS

CRITERIA FOR DIRECT ADMISSION IN STAR BATCHES

V STAR BATCH
XII Pass (JEE M+A)

ELIGIBILITY

- JEE Main'19 %tile > 98%tile
- JEE Advanced'19 Rank (Gen.) < 15,000

J STAR BATCH
XII Pass (NEET/AIIMS)

ELIGIBILITY

- NEET'19 Score > 450 Marks
- AIIMS'19 %tile > 98%tile

P STAR BATCH
XI Moving (JEE M+A)

ELIGIBILITY

- NTSE Stage-1 Qualified or NTSE Score > 160
- 100 marks in Science or Maths in Board Exam

H STAR BATCH
XI Moving (NEET/AIIMS)

ELIGIBILITY

- NTSE Stage-1 Qualified or NTSE Score > 160
- 100 marks in Science or Maths in Board Exam

Scholarship Criteria

JEE Main Percentile	SCHOLARSHIP+ STIPEND
98 - 99	100%
Above 99	100% + ₹ 5000/ month

JEE Advanced Rank	SCHOLARSHIP+ STIPEND
10000-20000	100%
Under 10000	100% + ₹ 5000/ month

NEET 2019 Marks	SCHOLARSHIP+ STIPEND
450	100%
530-550	100% + ₹ 2000/ month
550-560	100% + ₹ 4000/month
560	100% + ₹ 5000/month

NTSE STAGE-1 2019 Marks	SCHOLARSHIP+ STIPEND
160-170	100% + ₹ 2000/ month
171-180	100% + ₹ 4000/month
180+	100% + ₹ 5000/month

FEATURES :

- Batch will be taught by NV Sir & HOD's Only.
- Weekly Quizes apart from regular test.
- Under direct guidance of NV Sir.
- Residential campus facility available.
- 20 CBT (Computer Based Test) for better practice.
- Permanent academic coordinator for personal academic requirement.
- Small batch with only selected student.
- All the top brands material will be discussed.

SECTION–1 (Maximum marks :32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen and both of which are correct
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If two or more options is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases
- For example, in a question, if (A),(B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks
  choosing ONLY (A) and (B) will get +2 marks
  choosing ONLY (A) and (D) will get +2 marks
  choosing ONLY (B) and (D) will get +2 marks
  choosing ONLY (A) will get +1 mark
  choosing ONLY (B) will get +1 mark
  choosing ONLY (D) will get +1 mark
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark

1. Three lines

$L_1 : \vec{r} = \lambda\hat{i} \quad \lambda \in R$

$L_2 : \vec{r} = \hat{k} + \mu\hat{i} , \mu \in R$

$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k} , \ v \in R$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear ?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k} - \frac{1}{2}\hat{j}$ (3) $\hat{k}$ (4) $\hat{k} + \frac{1}{2}\hat{j}$

Ans. 2,4

$L_1 \rightarrow \bar{r} = \lambda\hat{i} \Rightarrow \frac{x-0}{\lambda} = \frac{y-0}{0} = \frac{z-0}{0}$

$L_2 \rightarrow \bar{r} = \hat{k} + \mu\hat{j} \Rightarrow \frac{x-0}{0} = \frac{y-0}{\mu} = \frac{z-1}{0}$

$L_3 \rightarrow \bar{r} = \hat{i} + \hat{j} + v\hat{k} \Rightarrow \frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{v}$

Point P on L_1 $P \equiv (\lambda, 0, 0)$

Point Q on L_2 $Q \equiv (0, \mu, 1)$

Point R on L_3 R $R \equiv (1, 1, v)$

P,Q,R an collinear

$\therefore \overrightarrow{PQ} \,||\, \overline{QR}$

$\overrightarrow{PQ} \,||\, K\overrightarrow{QR}$

$\frac{-\lambda}{1} = \frac{\mu}{1-\mu} = \frac{1}{\nu - 1} = k$

$\lambda = -k$

$\frac{\mu}{1-\mu} = k$

$\mu = k - k\mu$

$\mu(1 + k) = k$

$\mu = \frac{k}{k+1}$

$\frac{1}{\nu - 1} = k$

$\Rightarrow 1 = k\nu - k$

$\frac{1+k}{k} = \nu$

$\therefore\ \mu = \frac{-\lambda}{1-\lambda} = \frac{1}{\nu}$

μ cannot take value 0 & 1

2. Let $f : R \to R$ be given by $f(x) = (x-1)(x-2)(x-5)$. Define

$$F(x) = \int_0^x f(t)dt, \quad x>0$$

Then which of the following options is/are correct ?

(1) F has two local maxima and one local minimum in $(0, \infty)$

(2) F has a local maximum at x = 2

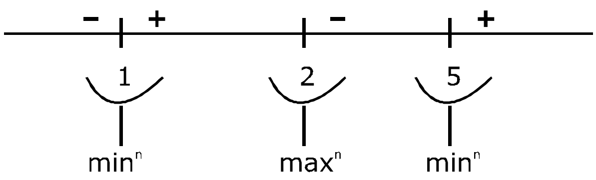
(3) $F(x) \neq 0$ for all $x \in (0,5)$

(4) F has a local minimum at x = 1

Sol. **2, 4, 3**

$F'(x) = f(x)$

$F'(x) = (x-1)(x-2)(x-5)$



at x = 1, 5 $\rightarrow$ minima

x = 2 $\rightarrow$ maxima

Now

$F'(x) = x^3 - x^2 + 17x - 10$

Integrate

$$F(x) = \frac{x^4}{4} - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x + C$$

$F(0) = 0 \Rightarrow C = 0$

$$F(x) = \frac{x^4}{4} - \frac{8}{3}x^3 + \frac{17}{2}x^2 - 10x$$

For $x \in (0, 5) \Rightarrow F(x) \neq 0$

3. Let $f : R \to R$ be a function We say that f has

PROPERTY 1 if $\lim_{h\to 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct ?
(1) f(x) = |x| has PROPERTY 1
(2) f(x) = x|x| has PROPERTY 2
(3) $f(x) = x^{2/3}$ has PROPERTY 1
(4) f(x) = sin x has PROPERTY 2

Sol. **1,3**

(a) f(x) = |x|

Property I $\quad \text{limit}_{h\to 0} \frac{|h|-0}{\sqrt{|h|}} \Rightarrow \text{limit}_{n\to 0} \sqrt{|h|} = 0$

Property II $\quad \text{limit}_{h\to 0} \frac{|h|-0}{h^2} \Rightarrow \text{limit}_{h\to 0} 1/h \to \infty$ (Not Satisfies)

(b) f(x) = x |x|

property I $\quad \text{limit}_{h\to 0} \frac{h|h|-0}{\sqrt{|h|}} = 0$

Property II $\quad \text{limit}_{h\to 0} \frac{h|h|-0}{h^2} \to$ does not exist

(c) $f(x) = x^{2/3}$

Property I $\quad \text{limit}_{h\to 0} \frac{h^{2/3}-0}{\sqrt{|h|}}$

$h \to 0^+ \quad \text{limit}_{h\to 0} \frac{h^{2/3}}{h^{1/2}} = 0$

$h \to O^- \quad \text{limit}_{h\to 0} \frac{h^{2/3}}{-h^{1/2}} = 0$

Property II $\quad \text{limit}_{h\to 0} \frac{h^{2/3}}{h^{1/2}} \to \infty$

(d) f(x) = sin x

property 2 $\quad \text{limit}_{h\to 0} \frac{\sin h - 0}{h^2} \to \infty$

4. For non-negative integers n, let

$$f(n) = \frac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$, which of the following options is/are correct ?

(1) $\sin(7\cos^{-1} f(5))=0$

(2) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2+2\alpha-1=0$

(3) $\lim_{n\to 0} f(n) = \frac{1}{2}$

(4) $f(4)=\frac{\sqrt{3}}{2}$

Sol. **1,2,4**

$$f(n) = \frac{\sum_{K=0}^{n} \cos\left[\left(\frac{K+1}{n+2}\right)-\left(\frac{K+2}{n+2}\right)\pi\right] - \cos\left[\left(\frac{K+1}{n+2}+\frac{K+2}{n+2}\right)\pi\right]}{\sum_{K=0}^{n} 2\sin^2\left(\frac{K+1}{n+2}\right)\pi}$$

$$= \frac{\sum_{K=0}^{n}\left[\left(\cos\frac{\pi}{n+2}\right) - \cos\left(\frac{2K+3}{n+2}\right)\pi\right]}{\sum_{K=0}^{n}\left[1-\cos 2\left(\frac{K+1}{n+2}\right)\pi\right]}$$

$$= \frac{\left(\cos\left(\frac{\pi}{n+2}\right)\right)(n+1) - \left[\cos\left(\frac{3\pi}{n+2}\right)+\cos\left(\frac{5\pi}{n+2}\right)+\ldots+\cos\left(\frac{2n+3}{n+2}\right)\pi\right]}{(n+1)-\sum_{K=0}^{n}\cos 2\left(\frac{K+1}{n+2}\right)\pi}$$

$$= \frac{\left(\cos\left(\frac{\pi}{n+2}\right)\right)(n+1) - \frac{\sin(n+1)\frac{\pi}{n+2}}{\sin\left(\frac{\pi}{n+2}\right)}\cos\left[\left(\frac{n+3}{n+2}\right)\pi\right]}{(n+1)-\frac{\sin\left((n+1)\frac{\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.(\cos\pi)}$$

$$= \frac{\left(\cos\left(\frac{\pi}{n+2}\right)\right)(n+1)+\cos\left(\frac{\pi}{n+2}\right)}{n+2}$$

f(n) = $\dfrac{\cos\left(\dfrac{\pi}{n+2}\right)(n+2)}{(n+2)}$

F(n) = $\cos\left(\dfrac{\pi}{n+2}\right)$

(a) f(5) = $\cos\left(\dfrac{\pi}{7}\right)$

$\sin\left(7\dfrac{\pi}{7}\right) = 0$

(b) $\alpha = \tan[\cos^{-1}(\cos\pi/8)]$

$= \tan\dfrac{\pi}{8}$

$\alpha = \sqrt{2} - 1$

Then $\alpha^2 + 2\alpha - 1 = 0$

(c) $\lim\limits_{n\to\infty}\cos\left(\dfrac{\pi}{n+2}\right) = 1$

(d) $f(4) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

5. Let $x \in R$ and let

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$ and $R = PQP^{-1}$.

Then which of the following options is/are correct ?

(1) For x = 0, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then a+b = 5

(2) There exists a real number x such that PQ = QP

(3) For x = 1, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(4) det R = det $\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix}$ +8, for all $x \in R$

Sol. **1,4**

RP = PQ

det(R) det(P) = (det P) (dep Q)
(det R) (6) = (6) $(12 - x^2)$ (4)
det R = $48 - 4x^2$ $\longrightarrow$ optioin D correct

$$\text{Now } P{-}1 = \frac{1}{6}\begin{bmatrix} 6 & -3 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix}$$

R = PQP^{-1}

$$R = \frac{1}{6}\begin{bmatrix} 6x+12 & 3x+6 & 4-10x \\ 12x & 24 & 8-4x \\ 18x & 0 & 36-6x \end{bmatrix}$$

Option I $\rightarrow$ x = 0

$$R = \frac{1}{6}\begin{bmatrix} 12 & 6 & 4 \\ 0 & 24 & 8 \\ 0 & 0 & 36 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\Rightarrow R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} 2 + a + \frac{2}{3}b \\ 4a + \frac{4}{3}b \\ 6b \end{bmatrix} = \begin{bmatrix} 6 \\ 6a \\ 6b \end{bmatrix}$$

$a + \frac{2}{3}b = 4$, $\frac{4}{3} - b = 2a \Rightarrow a = 2, b = 3$
a + b = 5
Option (b) PQ = QP Not possible
Option (c) x = 1

$$R = \frac{1}{6}\begin{bmatrix} 18 & 9 & -6 \\ 12 & 24 & 4 \\ 18 & 0 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3/2 & -1 \\ 2 & 4 & 2/3 \\ 3 & 0 & 5 \end{bmatrix}$$

$$R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} 3\alpha + \frac{3}{2}\beta - \gamma &= 0 \\ 2\alpha + 4\beta + \frac{2}{3}\gamma &= 0 \\ 3\alpha + 5\gamma &= 0 \end{aligned}\right]$$

$\gamma = \frac{-3\alpha}{5}$, $\beta = \frac{-2\alpha}{5}$

$\alpha + \beta + \gamma = 0$

$\alpha, \frac{-2\alpha}{5}, \frac{-3\alpha}{5}$

5, − 2, − 3 [Not unit vector]

6. Let $f(x) = \frac{\sin \pi x}{x^2}$, $x > 0$.

Let $x_1 < x_2 < x_3 < \ldots < x_n < \ldots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \ldots. < y_n < \ldots.$ be all the points of local minimum of f.
Then which of the following options is/are correct ?

(1) $x_1 < y_1$

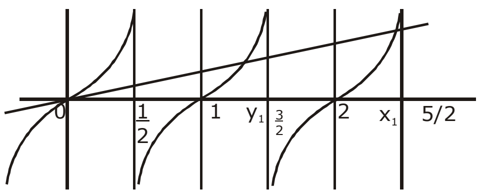
(2) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(3) $|x_n - y_n| > 1$ for every n

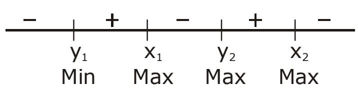
(4) $x_{n+1} - x_n > 2$ for every n

Sol. **1,3,4**

$$f'(x) = \frac{2x \cos \pi x \left(\frac{\pi x}{2} - \tan \pi x\right)}{x^4}$$



For f'(x)



7. Let $P_1 = I = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$, $P_2 = \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$, $P_3 = \begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$,

$P_4 = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}$, $P_5 = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$, $P_6 = \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$,

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix} P_K^T$

Where P_K^T denotes the transpose of the matrix P_K. Then which of the following options is/are correct ?

(1) The sum of diagonal entries of X is 18

(2) If $X\begin{bmatrix}1\\1\\1\end{bmatrix} = \alpha\begin{bmatrix}1\\1\\1\end{bmatrix}$, then $\alpha = 30$

(3) X is a symmetric matrix

(4) X – 30I is an invertible matrix

Sol. **1,2, 3**

Clearly $P_1 = P^T{}_1 = P_1{}^{-1}$

$P_2 = P^T{}_2 = P_2{}^{-1}$

$P_6 = P_6^T = P_6^{-1}$

and $A^1 = A$, where $A = \begin{bmatrix}2&1&3\\1&0&2\\3&2&1\end{bmatrix}$

Using formula $(A+B)^1 = A^1+B^1$

$X^1 = \left(P_1AP_1^T + ...P_6AP_2^T\right)^T + ...P_6A^TP_6^T = x$ $\Rightarrow$ x is symmetric

Let $B = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$XB = P_1AP_1TB+P_2AP_2{}^TB+....+ P_6APT_6B = P_1AB+PAB+....+P_6AB$

$XB = (P_1+P_2+.......P_6)\begin{bmatrix}6\\3\\6\end{bmatrix}$

$$= \begin{bmatrix} 6\times2+3\times2+6\times2 \\ 6\times2+3\times2+6\times2 \\ 6\times2+3\times2+6\times2 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 300 \Rightarrow \alpha = 30$$

Since $x \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow$ (x–301)B = 0 has a non trivial solution $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow$ $|x - 30| = 0$

$x = P_1AP^T_1 + \ldots\ldots + P_6AP^T_6$

traco (x) = $t_1 (P_1AP^T_1) + \ldots (P_6P^T_6) = (2+0+1)+\ldots.+(2+0+1)=3\times6=18$

8. For

For $a \in R|a| > 1$, let

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+\ldots+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+\ldots+\frac{1}{(an+n)^2}\right)}\right)=54.$$

Then the possible value(s) of a is/are

(1) 7 (2) –6 (3) 8 (4) –9

Sol. **3,4**

$$\lim_{n\to\infty}\frac{1^{\frac{1}{3}}+2^{\frac{1}{3}}+\ldots.+n^{\frac{1}{3}}}{\frac{n^{\frac{1}{3}}n\,n}{n^2}\left[\frac{1}{\left(a+\frac{1}{n}\right)^2}+\frac{1}{\left(a+\frac{2}{n}\right)^2}+\ldots\ldots+\frac{1}{\left(a+\frac{n}{n}\right)^2}\right]}$$

$$\lim_{n\to\infty}\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{\frac{1}{3}}}{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{1}{a+\frac{r}{n}}\right)^2} \Rightarrow \frac{\int_0^1 x^{\frac{1}{3}}dx}{\int_0^1\frac{dx}{(a+x)^2}} \Rightarrow \frac{\frac{3}{4}\left(x^{\frac{4}{3}}\right)_0^1}{-\left(\frac{1}{a+x}\right)_0^1} = 54$$

$$\Rightarrow \frac{\frac{3}{4}}{\left(\frac{1}{a+1}-\frac{1}{a}\right)} = 54 \Rightarrow \frac{3}{4}a(a+1) = 54$$

$a^2 + a - 72 = 0 \Rightarrow a = -9, 8$

Section 2

- This section contains **SIX (06)** qeustions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme;
  Full Marks : +3 If ONLY the correct numerical value is entered
  Zero Marks : 0 in all other cases.

1. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. $\alpha, \beta, \in$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$ equals_____

Sol. **18**

$$\frac{\bar{C}.[\bar{a} + \bar{b}]}{|\bar{a} + \bar{b}|} = 3\sqrt{2}$$

$\bar{C} = \alpha(2,1,-1)+\beta(1,2,1)$

$= (2\alpha+\beta,\ \alpha+2\beta,\ -\alpha+\beta)$

$$\frac{\bar{C} - \bar{a} + \bar{c} - \bar{b}}{|\bar{a} + \bar{b}|} = 3\sqrt{2}$$

$\bar{C} . \bar{a} = 2(2\alpha+\beta) + \alpha+2\beta+ \alpha-\beta = 6\alpha + 3\beta$

$$\frac{(6\alpha + 3\beta) + (6\beta + 3\alpha)}{3\sqrt{2}} = 3\sqrt{2}$$

$\bar{c} . \bar{b} = 2\alpha+\beta + 2\alpha+4\beta+\alpha+\beta = 6\beta + 3\alpha$

$\boxed{(\alpha + \beta) = 2}$

$|\bar{a} + \bar{b}|^2 = 6+6+23$

$|\bar{a} + \bar{b}| = \sqrt{18} = 3\sqrt{2}$

Now $(\bar{c} - (\bar{a} \times \bar{b})).\bar{c} = |\bar{c}|^2 - [\bar{a}\ \bar{b}\ \bar{c}]$ $\quad \because [\bar{a}\ \bar{b}\ \bar{c}] = 0$

$= (2\alpha+\beta)^2 + (\alpha+2\beta)^2+(-\alpha+\beta)^2$

$= 6\alpha^2 + 6\beta^2 + 6\alpha\beta = 6(\alpha^2+\beta^2+\alpha\beta)$

For minimum value $= \alpha = \beta = 1$

we get minimum value = 18

2. Suppose

$$\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_k k^2 \\ \sum_{k=0}^{n} {}^{n}C_k k & \sum_{k=0}^{n} {}^{n}C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^{n}C_k}{k+1}$ equals_____

Sol. **6.2**

$$\sum_{k=0}^{n}\left(k^2 - k + k\right){}^{n}G_c = \sum_{k=0}^{n}(k-1)k\,\frac{n}{k}\cdot\frac{n-1}{k-1}\,{}^{n-2}G_{c-2} + \sum_{r=0}^{n} k\,\frac{n}{k}\,{}^{n-1}C_{k-1}$$

$= n(n-1).\ 2^{n-2} + n\ 2^{n-1}$

$= n.2^{n-2}\ [(n-1)+2]$

$= n(n+1)\ 2^{n-2}$

$$\sum_{k=0}^{n} k\,\frac{n}{k}\,{}^{n-1}C_{n-1} = n2^{n-1}$$

$$\sum_{k=0}^{n} 3k\,{}^{n-1}C_{n-1} = {}^{n}C_o + 3^1({}^{n}C_1) + (3^2)\,n_e + \ldots + 3^n\,({}^{n}C_n)$$

Now $\begin{vmatrix} \frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n2^{n-1} & 4^n \end{vmatrix} = 0$

$\begin{vmatrix} 2 & 2^n \\ n2^{n-1} & 4^n \end{vmatrix} = 0\quad 2^{2n+1} = n.\ 2^{2n-1}$

$= \boxed{n = 4}$

$$\sum_{k=0}^{4}\frac{{}^{4}C_{1c}}{k+1} = \frac{{}^{4}C_0}{1} + \frac{{}^{4}C_1}{2} + \frac{{}^{4}C_2}{3} + \frac{{}^{4}C_3}{4} + \frac{{}^{4}C_4}{5}$$

$= 1+2+2+1+1/5$

$= \frac{31}{5}$

$= 6.20$

3. Let |X| denote the number of elements in a set X, Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \le |B| < |A|$, equals_____

Sol. **422**

Let No. of element in A = α $\quad\alpha > \beta \geq 1$

No. of element in B = β

& No. of elemens in $A \cap B$ = Z

$\because$ A & B are independent events

then $P(A \cap B) = P(A).\ P(B)$

$$\frac{z}{6} = \frac{\alpha}{6}.\frac{\beta}{6} \Rightarrow 6z = \alpha.\beta$$

Now Case I : if z = 1

(i) $\alpha = 6, \beta = 1 \Rightarrow {}^6C_6.\ {}^6C_1 = 6$

(ii) $\alpha = 3, \beta = 2 \Rightarrow {}^6C_3.{}^3C_1.{}^3C_1 = 180$

Case II : if z = 2

(i) $\alpha = 6, \beta = 2 \Rightarrow {}^6C_5.\ {}^6C_2 = 1.15 = 15$

(ii) $\alpha = 4, \beta = 3 \Rightarrow {}^6C_4.{}^3C_2.{}^2C_1 = 180$

Case III : if z = 3

(i) $\alpha = 6, \beta = 3 \Rightarrow {}^6C_6.\ {}^6C_3 = 1.20 = 20$

Case IV : if z = 4

(i) $\alpha = 6, \beta = 4 \Rightarrow {}^6C_6.\ {}^6C_4 = 1.15 = 15$

Case V : if z = 5

(i) $\alpha = 6, \beta = 5 \Rightarrow {}^6C_6.\ {}^6C_5 = 1.6 = 6$

= 422

4. The value of the integral

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} d\theta$$

equals _

Sol. **0.5**

$$I = \int_0^{\pi 2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta + \sqrt{\sin\theta}}\right)^5}$$

King prop and add.

$$2I = \int_0^{\pi 2} \frac{3}{\left(\sqrt{\cos\theta + \sqrt{\sin\theta}}\right)^4}$$

$$I = \frac{3}{2}\int_0^{\pi 2} \frac{3\sec^2\theta}{\left(\sqrt{1\sqrt{\sin\theta}}\right)^4}$$

$= \tan\theta = t^2$

$= \sec^2 q d\theta = 2\tan$

I $= \frac{3}{2}\int_0^{\infty}\frac{2+dt}{(1+1)^4}$

$= 3\int_0^{\infty}\frac{t+1-1}{(t+1)^4}dt$

$= 3\left[\int_0^{\infty}\left[\frac{1}{(t+1)^3}-\frac{1}{(t+1)^4}\right]\right]dt$

$= \left[\frac{-1}{2(t+1)^2}+\frac{1}{3}\frac{1}{(t+1)^3}\right]_0^{\infty}$

$= 3\left[0-\left(-\frac{1}{2}+\frac{1}{3}\right)\right]$

$=$ 3/6 $=$ 0.5

5. Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is_____

Sol. **(30.00)**

Maximum number of hats used the same colour are 2. They cannot be 3 otherwise atleast 2 hats of same colour are consecutive.

Now, Let hats used are R, R, G, G, B

(Which can be selected in 3 ways. It can be RGGBB or RRGBB also)

Now, numbers of ways of disturbing blue hat (single one) in 5 person equal to 5

Let blue hat goes to person A.

Now, either position B & D are filled by green hats and C & E are filled by Rads hats or B & D are filled by Red hats and C & E are filled by Green hats

$\Rightarrow$ 2 ways are possible

Hence total number of ways=3×5×2=30ways

6. The value of

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\text{Sec}\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\text{Sec}\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals ______

Sol. **0**

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{1}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cos\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)}\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\frac{\sin\left(\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)-\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\right)}{\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cos\left(\frac{7\pi}{12}+(k+1)\frac{\pi}{2}\right)}\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\left(\tan\left(\frac{7\pi}{12}+(k+1)\frac{\pi}{2}\right)-\tan\left(\frac{7\pi}{2}+k\frac{\pi}{2}\right)\right)\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\left(\tan\left(\frac{7\pi}{12}+\frac{\pi}{2}\right)\right)+\left(\tan\left(\frac{7\pi}{12}+\frac{2\pi}{2}\right)\right)\right)\text{-tan}\left(\frac{7\pi}{12}+\frac{\pi}{2}\right)$$

$$+\ldots+\left(\tan\left(\frac{7\pi}{12}+\frac{11\pi}{2}\right)\right)-\tan\left(\frac{7\pi}{2}+\frac{10\pi}{2}\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\left(\tan\frac{13\pi}{12}-\tan\frac{7\pi}{12}\right)\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\left(\tan\left(\frac{\pi}{12}\right)-\tan\left(\frac{7\pi}{12}\right)\right)\right)$$

$$\text{Sec}^{-1}\left(\frac{1}{4}\left(\left(2-\sqrt{3}\right)+12+\sqrt{3}\right)\right)$$

$\text{Sec}^{-1}(1)$

= 0.00

Section 3

- This section contains **TWO (02)** List -Match sets
- Each List Match set has **TWO (02)** Multiple Choice Questions.
- Each List Match set has two lists. List I and List II
- **List I** has Four entries (I), (II),(III) and (IV) and List II has Six entries (P),(Q)(R),(S),(T)and (U)
- Four options are given in each multiple choice question based on List I and List II and only one of these four options satisfies the condition asked in the multiple choice question.
- Answer to each question will be evaluated according to the following marking scheme.

Full marks +3 If ONLY the option corresponding to the correct combination is chosen
Zero Marks 0 If none of the options is chosen (i.e., the question is unanswered)
Negative marks –1 in all other cases.

Anser the following by appropriately matching the lists based on the information given in the paragraph

1. Let f(x) = sin(πcosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose elements are written in the increasing order :

X = {x : f(x) = 0}, Y = {x : f'(x) = 0},
Z = {x : g(x) = 0}, W = {x : g'(x) = 0},

List -I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I | **List-II**

List-I	List-II
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?
(1) (III), (P), (Q), (U)
(2) (IV), (P), (R), (S)
(3) (III), (R), (U)
(4) (IV), (Q), (T)

Sol. **2**

2. Answer the following appropriately matching the list based on the information given in the paragraph.
Let f(x) = sin (πcosx) and g(x)=cos(2π sin x) be two functions defined for x > 0. Define the following sets whose elements are written in the increasing order.

X = {x : f(x)=0}, Y = {x : f'(x) =0}
Z = {x : g(x)=0}, W = {x : g'(x)=0}

List I contains the sets X,Y,Z and W. List II contains some information regarding these sets.

List I		List II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithemetic progression
(III)	Z	(R)	NOT an arithemetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination ?
(1) (I), (Q), (U) (2) (II),(Q), (T) (3) (I), (P), (R) (4) (II), (R), (S)

Sol. **2**
f(x) = 0 $\Rightarrow$ sin(πcosx) = 0
$\Rightarrow \pi$ cosx = n$\pi \Rightarrow$ cosx = n

$\Rightarrow$ cosx =−1, 0, 1 $\Rightarrow$ X = $(n\pi,(2n+1)\ \frac{\pi}{2})$

= $(n\frac{\pi}{2},n)$

f'(x) = 0 $\Rightarrow$ cos(πcosx) (−π sinx) = 0

$\Rightarrow$ πcosx = (2n + 1) $\frac{\pi}{2}$ or x = nπ

$\Rightarrow$ cosx =−1, 0, 1 $\Rightarrow$ x =$(n\pi(2n+1)\ \frac{\pi}{2})$ = $(n\frac{\pi}{2},n \in))$

f'(x) = 0 $\Rightarrow$ cos(πcosx) (−πsinx) = 0

$\Rightarrow$ cosx = n + $\frac{1}{2}$ or X nπ

$\Rightarrow$ cos x = $\pm\frac{1}{2}$ of x = nπ

$\Rightarrow$ y = $\left\{an\pi = \frac{\pi}{3}2\pi r = \frac{2\pi}{3}, n\pi n\right\}$

g(x) = 0 $\Rightarrow$ cos(2xsinx) = 0 $\Rightarrow$ 2πsinx = (2n+1) $\frac{\pi}{2}$

$\Rightarrow$ sinx = $\frac{2n+1}{2}$ − $\frac{1}{2}$ + $\frac{3}{2}$

$\Rightarrow$ cosx = n + $\frac{1}{2}$ or x nπ $\Rightarrow$ cosx = $\pm\frac{1}{2}$ or x = nπ

$\Rightarrow$ y = $\left\{2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n\pi, n \in 1\right\}$

g(x) = 0 $\Rightarrow$ cos(2πsinx) = 0

$\Rightarrow$ 2π sinx = (2n + 1) $\frac{\pi}{2}$ $\Rightarrow$ sinx = $\frac{2n+1}{4} = \pm\frac{1}{4} \pm \frac{3}{4}$

$\Rightarrow$ Z = $\left\{n\pi \pm \sin 1\frac{1}{4}, n\pi \pm \sin 1\frac{3}{4}, n \in 1\right\}$

g'(x)= 0 $\Rightarrow$ −sin(2πsinx) (2πcosx) = 0

$\Rightarrow$ 2π sinx = nπ or x = (2n + 1) $\frac{\pi}{2}$ $\Rightarrow$ sinx = $\frac{n}{2}$ = 0, $\pm\frac{1}{2}$, ±1 or x = (2n + 1) $\frac{\pi}{2}$

$\Rightarrow$ W = $\left\{n\pi, (2n+1)\frac{\pi}{2}, np\frac{\pi}{6}, n \in 1\right\}$

3. Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i) centre of C_3 is collinear with the centres of C_1 and C_2.

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

	List I		**List II**
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only CORRECT combination ?

(1) (II), (T) (2) (I), (U) (3) (I), (S) (4) (II), (Q)

Sol. 4

4. Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$, intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i) centre of C_3 is collinear with the centres of C_1 and C_2.

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

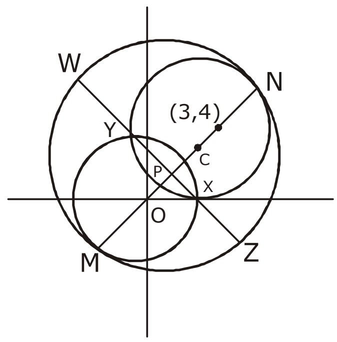
There are some expressions given in the List I whose values are given in List II below :

	List I		**List II**
(I)	$2h + k$	(P)	6
(II)	$\frac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}}$	(R)	$\frac{5}{4}$
(IV)	α	(S)	$\frac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\frac{10}{3}$

Which of the following is the only CORRECT combination ?

(1) (IV), (S) (2) (IV), (U) (3) (I), (P) (4) (III), (R)

Sol. **1**



(i) $2r = MN = 3 + \sqrt{3^2 + 4^2} + 4 = 12 \Rightarrow r = 6$

Contre C of circle C_3 lies on $y = \frac{4}{3}x$

Let $C\left(h, \frac{4}{3}h\right)$

$OC = MC = OM = \frac{12}{2} - 3 = 3$

$\therefore \sqrt{h^2 + \frac{16}{9}h^2} = 3 \Rightarrow \frac{5h}{3} = 3 \Rightarrow h = \frac{9}{5}$

$K = \frac{4}{3}h = \frac{12`}{3}$

$\therefore 2h + K = \frac{18}{5} + \frac{12}{5} = 6$

(ii) Equation of line ZW

$C_1 - C_2 = 0 \Rightarrow 3x + 4y = 9$

Distance of ZW from (0,0)

$\frac{|-9|}{\sqrt{3^2 + 4^2}} = \frac{9}{5}$

Length of XY $= 2\sqrt{3^2 - \left(\sqrt{\frac{9}{5}}\right)^2} = \frac{24}{5}$

Distance of ZW from C $\Rightarrow \frac{\left|\frac{3 \times 9}{5} + 4\frac{12}{5} - 9\right|}{\sqrt{3^2 + 4^2}} = \frac{24\sqrt{6}}{5}$

$\therefore \frac{\text{Length of ZW}}{\text{length of XY}} = \sqrt{6}$

(iii) Area of $\Delta MZN = \frac{1}{2} MN \left(\frac{1}{2} ZW\right) = \frac{72\sqrt{6}}{5}$

Area of $\Delta ZMW = \frac{1}{2} ZW\,(OM + OP) = \frac{1}{2}\ \frac{24\sqrt{6}}{5}$

$\left(3 + \frac{9}{5}\right) = \frac{288\sqrt{6}}{25}$ $\quad\therefore\quad$ $\frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{5}{4}$

(iv) Slop of tangent to C_1 at $M = \frac{-1}{4/3} = -\frac{3}{4}$

$\therefore$ Equation of tangent $y = mx\ -3\sqrt{1+m^2}$

$y = -\frac{3}{4} \times -3\sqrt{1+\frac{9}{16}}$

$y = \frac{-3x}{4} - \frac{15}{4} \Rightarrow \quad x = -\frac{4y}{3} - 3$.....(i)

tangent to $x^2 = 4(2\alpha)y$ is

$x = my + \frac{2\alpha}{m}$ ....(ii)

Compare (i) and (ii)

$m = -\frac{4}{3}$ and $\frac{2\alpha}{m} = -5 \Rightarrow \quad \alpha = \frac{10}{3}$

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95.5 To 96	₹ 65,250	₹ 65,250
95 To 95.5	₹ 72,500	₹ 72,500
93 To 95	₹ 87,000	₹ 87,000
90 To 93	₹ 1,01,500	₹ 94,250
85 To 90	₹ 1,08,750	₹ 1,01,500
80 To 85	₹ 1,16,000	₹ 1,08,750
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