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PAPER WITH ANSWER

# JEE Advanced 2019

## MATHEMATICS PAPER - 2

IIT/NIT | NEET/AIIMS | NTSE/IJSO/OLYMPIADS

## कोटा का रिपिटर्स (12th पास) का सर्वश्रेष्ठ रिजल्ट देने वाला संस्थान

### JEE ADVANCED 2018 RESULT

AIR 82
Sarthak Behera

AIR 120
Pankaj

AIR 146
Varun Goyal

AIR 148
Mukul Kumar

Total Selection
709/2084 = 34.02%

### JEE MAIN 2019 RESULT

AIR 79
Shiv Kumar Modi

AIR 85
Anuj Chaudhary

AIR 96
Shubham Kumar

AIR 120
Eeshaan Jain

Students Qualified for JEE ADVANCED
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# CRITERIA FOR DIRECT ADMISSION IN STAR BATCHES

## V STAR BATCH
### XII Pass (JEE M+A)

**ELIGIBILITY**

JEE Main'19 %tile > 98%tile

JEE Advanced'19 Rank (Gen.) < 15,000

## J STAR BATCH
### XII Pass (NEET/AIIMS)

**ELIGIBILITY**

NEET'19 Score > 450 Marks

AIIMS'19 %tile > 98%tile

## P STAR BATCH
### XI Moving (JEE M+A)

**ELIGIBILITY**

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

## H STAR BATCH
### XI Moving (NEET/AIIMS)

**ELIGIBILITY**

NTSE Stage-1 Qualified or NTSE Score > 160

100 marks in Science or Maths in Board Exam

## Scholarship Criteria

| JEE Main Percentile | SCHOLARSHIP+ STIPEND |
|---|---|
| 98 - 99 | 100% |
| Above 99 | 100% + ₹ 5000/ month |

| JEE Advanced Rank | SCHOLARSHIP+ STIPEND |
|---|---|
| 10000-20000 | 100% |
| Under 10000 | 100% + ₹ 5000/ month |

| NEET 2019 Marks | SCHOLARSHIP+ STIPEND |
|---|---|
| 450 | 100% |
| 530-550 | 100% + ₹ 2000/ month |
| 550-560 | 100% + ₹ 4000/month |
| 560 | 100% + ₹ 5000/month |

| NTSE STAGE-1 2019 Marks | SCHOLARSHIP+ STIPEND |
|---|---|
| 160-170 | 100% + ₹ 2000/ month |
| 171-180 | 100% + ₹ 4000/month |
| 180+ | 100% + ₹ 5000/month |

## FEATURES :

- Batch will be taught by NV Sir & HOD's Only.
- Weekly Quizes apart from regular test.
- Under direct guidance of NV Sir.
- Residential campus facility available.
- 20 CBT (Computer Based Test) for better practice.
- Permanent academic coordinator for personal academic requirement.
- Small batch with only selected student.
- All the top brands material will be discussed.

# MATHS [ JEE ADVANCED - 2019 ] PAPER - 2

## SECTION–1 (Maximum marks :32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen and both of which are correct
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.
  Zero Marks : 0 If two or more options is chosen (i.e. the question is unanswered)
  Negative Marks : –1 in all other cases
- For example, in a question, if (A),(B) and (D) are the ONLY three options corresponding to correct answer, then
  choosing ONLY (A), (B) and (D) will get +4 marks
  choosing ONLY (A) and (B) will get +2 marks
  choosing ONLY (A) and (D) will get +2 marks
  choosing ONLY (B) and (D) will get +2 marks
  choosing ONLY (A) will get +1 mark
  choosing ONLY (B) will get +1 mark
  choosing ONLY (D) will get +1 mark
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark

**1.** Three lines

$$L_1 : \vec{r} = \lambda\hat{i} \quad \lambda \in R$$

$$L_2 : \vec{r} = \hat{k} + \mu\hat{i}, \mu \in R$$

$$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu\hat{k}, \ \nu \in R$$

are given. For which point(s) Q on $L_2$ can we find a point P on $L_1$ and a point R on $L_3$ so that P,Q and R are collinear ?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k} - \frac{1}{2}\hat{j}$ (3) $\hat{k}$ (4) $\hat{k} + \frac{1}{2}\hat{j}$

**Sol.** **2,4**

**2.** Let $f : R \to R$ be given by f(x) = (x–1)(x–2)(x–5). Define

$$F(x) = \int_0^x f(t)dt, \quad x>0$$

Then which of the following options is/are correct ?

(1) F has two local maxima and one local minimum in $(0,\infty)$

(2) F has a local maximum at x = 2

(3) $F(x) \neq 0$ for all $x \in (0,5)$

(4) F has a local minimum at x = 1

**Sol.** **2,3,4**

**3.** Let f : $R \to R$ be a function We say that f has

PROPERTY 1 if $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct ?
(1) f(x) = |x| has PROPERTY 1
(2) f(x) = x|x| has PROPERTY 2
(3) $f(x) = x^{2/3}$ has PROPERTY 1
(4) f(x) = sin x has PROPERTY 2

**Sol.** **1,3**

**4.** For non-negative integers n, let

$$f(n) = \frac{\sum\limits_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum\limits_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$, which of the following options is/are correct ?
(1) $\sin(7\cos^{-1} f(5))=0$
(2) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2+2\alpha-1=0$

(3) $\lim\limits_{n\to 0} f(n) = \dfrac{1}{2}$ (4) $f(4)=\dfrac{\sqrt{3}}{2}$

**Sol.** **1,2,4**

**5.** Let $x \in R$ and let

$$P = \begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 2\\ 0 & 0 & 3\end{bmatrix}, \qquad Q = \begin{bmatrix}2 & x & x\\ 0 & 4 & 0\\ x & x & 6\end{bmatrix} \text{ and } R = PQP^{-1}.$$

Then which of the following options is/are correct ?

(1) For x = 0, if $R\begin{bmatrix}1\\ a\\ b\end{bmatrix}=6\begin{bmatrix}1\\ a\\ b\end{bmatrix}$, then a+b = 5

(2) There exists a real number x such that PQ = QP

(3) For x = 1, there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix}\alpha\\ \beta\\ \gamma\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

(4) det R = $\det\begin{bmatrix}2 & x & x\\ 0 & 4 & 0\\ x & x & 5\end{bmatrix}$ +8, for all $x \in R$

**Sol.** **1,4**

**6.** Let $f(x) = \frac{\sin \pi x}{x^2}$, $x > 0$.

Let $x_1 < x_2 < x_3 < \ldots < x_n < \ldots$ be all the points of local maximum of f and $y_1 < y_2 < y_3 < \ldots < y_n < \ldots$ be all the points of local minimum of f.
Then which of the following options is/are correct ?

(1) $x_1 < y_1$

(2) $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ for every n

(3) $|x_n - y_n| > 1$ for every n

(4) $x_{n+1} - x_n > 2$ for every n

**Sol.** **1,3,4**

**7.** Let $p_1 = 1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $p_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $p_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

$p_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $p_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $p_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_K^T$

Where $P_K^T$ denotes the transpose of the matrix $P_K$. Then which of the following options is/are correct ?

(1) The sum of diagonal entries of X is 18

(2) If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

(3) X is a symmetric matrix

(4) X – 30I is an invertible matrix

**Sol.** **1,2,3**

**8.** For

For $a \in R |a| > 1$, let

$$\lim_{n\to\infty} \left( \frac{1 + \sqrt[3]{2} + \ldots + \sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \ldots + \frac{1}{(an+n)^2}\right)} \right) = 54.$$

Then the possible value(s) of a is/are

(1) 7 (2) –6 (3) 8 (4) –9

**Sol.** **3,4**

**Section 2**

- This section contains **SIX (06)** qeustions. The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme;
  Full Marks : +3 If ONLY the correct numerical value is entered
  Zero Marks : 0 in all other cases.

**1.** Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. $\alpha,\beta, \in$ . If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $\left(\vec{c} - (\vec{a} \times \vec{b})\right) \cdot \vec{c}$ equals_____

**Sol. 18**

**2.** Suppose

$$\det \begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_k k^2 \\ \sum_{k=0}^{n} {}^{n}C_k k & \sum_{k=0}^{n} {}^{n}C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^{n}C_k}{k+1}$ equals_____

**Sol. 6.2**

**3.** Let |X| denote the number of elements in a set X, Let S = {1,2,3,4,5,6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \leq |B| < |A|$, equals_____

**Sol. 1523**

**4.** The value of the integral

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{(\sqrt{\cos\theta} + \sqrt{\sin\theta})^5} d\theta$$

equals _

**Sol. 0.5**

**5.** Five persons A,B,C,D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distrubuting the hats such that the persons seated in adjacent seats get different coloured hats is_____

**Sol. 30.00**

**6.** The value of

$$\text{Sec}^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\text{Sec}\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\text{Sec}\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals _____

**Sol.** **0**

**Section 3**

- This section contains **TWO (02)** List -Match sets
- Each List Match set has **TWO (02)** Multiple Choice Questions.
- Each List Match set has two lists. List I and List II
- **List I** has Four entries (I), (II),(III) and (IV) and List II has Six entries (P),(Q)(R),(S),(T)and (U)
- Four options are given in each multiple choice question based on List I and List II and only one of these four options satisfies the condition asked in the multiple choice question.
- Answer to each question will be evaluated according to the following marking scheme.
  Full marks +3 If ONLY the option corresponding to the correct combination is chosen
  Zero Marks 0 If none of the options is chosen (i.e., the question is unanswered)
  Negative marks –1 in all other cases.
  Anser the following by appropriately matching the lists based on the information given in the paragraph

**1.** Let f(x) = sin(πcosx) and g(x) = cos(2π sinx) be two functions defined for x > 0. Define the following sets whose elements are written in the increasing order :

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$,
$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$,

List -I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

| **List-I** | **List-II** |
|---|---|
| (I) X | (P) $\supseteq \left\{\frac{\pi}{2},\frac{3\pi}{2},4\pi,7\pi\right\}$ |
| (II) Y | (Q) an arithmetic progression |
| (III) Z | (R) NOT an arithmetic progression |
| (IV) W | (S) $\supseteq \left\{\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\right\}$ |
| | (T) $\supseteq \left\{\frac{\pi}{3},\frac{2\pi}{3},\pi\right\}$ |
| | (U) $\supseteq \left\{\frac{\pi}{6},\frac{3\pi}{4}\right\}$ |

Which of the following is the only CORRECT combination ?
(1) (III), (P), (Q), (U)
(2) (IV), (P), (R), (S)
(3) (III), (R), (U)
(4) (IV), (Q), (T)

**Sol.** **2**

**2.** Answer the following appropriately matching the list based on the information given in the paragraph.

Let $f(x) = \sin(\pi\cos x)$ and $g(x)=\cos(2\pi \sin x)$ be two functions for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x)=0\}$, $Y = \{x, f(x) =0\}$

$Z = \{x : g(x)=0\}$, $W = \{x : g'(x)=0\}$

List I contains the sets X,Y,Z and W. List II contains some information regarding these sets.

| | **List I** | | **List II** |
|---|---|---|---|
| (I) | X | (P) | $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$ |
| (II) | Y | (Q) | an arithemetic progression |
| (III) | Z | (R) | NOT an arithemetic progression |
| (IV) | W | (S) | $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$ |
| | | (T) | $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$ |
| | | (U) | $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$ |

Which of the following is the only CORRECT combination ?

(1) (I ), (Q), (U) (2) (II),(Q), (T) (3) (I), (P), (R) (4) (II), (R), (S)

**Sol. 2**

**3.** Answer the following by appropriately matching the list based on the information given in the paragraph.

Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$ , intersect at the points X and Y.

Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions

(i) centre of $C_3$ is collinear with the centres of $C_1$ and $C_2$.

(ii) $C_1$ and $C_2$ both lie inside $C_3$, and

(iii) $C_3$ touches $C_1$ at M and $C_2$ at N

Let the line through X and Y intersect $C_3$ at Z and W, and let a common tangent of $C_1$ and $C_3$ be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List I whose values are given in List II below :

| | **List I** | | **List II** |
|---|---|---|---|
| (I) | $2h + k$ | (P) | 6 |
| (II) | $\frac{\text{Length of ZW}}{\text{Length of XY}}$ | (Q) | $\sqrt{6}$ |
| (III) | $\frac{\text{Area of triangle MZN}}{\text{Area of trangle ZMW}}$ | (R) | $\frac{5}{4}$ |
| (IV) | $\alpha$ | (S) | $\frac{21}{5}$ |
| | | (T) | $2\sqrt{6}$ |
| | | (U) | $\frac{10}{3}$ |

Which of the following is the only CORRECT combination ?
(1) (II), (T) (2) (I), (U) (3) (I), (S) (4) (II), (Q)

**Sol.** **4**

**4.** Answer the following by appropriately matching the list based on the information given in the paragraph.
Let the circles $C_1 : x^2+y^2=9$ and $C_2 : (x-3)^2+(y-4)^2=16$ , intersect at the points X and Y.
Suppose that another circle $C_3 : (x-h)^2+(y-k)^2=r^2$ satisfies the following conditions
(i) centre of $C_3$ is collinear with the centres of $C_1$ and $C_2$.
(ii) $C_1$ and $C_2$ both lie inside $C_3$, and
(iii) $C_3$ touches $C_1$ at M and $C_2$ at N
Let the line through X and Y intersect $C_3$ at Z and W, and let a common tangent of $C_1$ and $C_3$ be a tangent to the parabola $x^2 = 8\alpha y$.
There are some expressions given in the List I whose values are given in List II below :

| | **List I** | | **List II** |
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| (I) | $2h + k$ | (P) | 6 |
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| (III) | $\frac{\text{Area of triangle MZN}}{\text{Area of trangle ZMW}}$ | (R) | $\frac{5}{4}$ |
| (IV) | $\alpha$ | (S) | $\frac{21}{5}$ |
| | | (T) | $2\sqrt{6}$ |
| | | (U) | $\frac{10}{3}$ |

Which of the following is the only CORRECT combination ?
(1) (IV), (S) (2) (IV), (U) (3) (I), (P) (4) (III), (R)

**Sol.** **1**

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## Based on JEE Advanced'19

| MARKS | FEE (After Scholarship) |
|---|---|
| 140 above | Drona Residential Program Free |
| 120 to 139 | ₹ 0 |
| 100 to 120 | ₹ 14,500 |
| 90 to 99 | ₹ 29,000 |
| 80 to 89 | ₹ 43,500 |
| 69 to 79 | ₹ 58,000 |
| 40 to 69 | ₹ 87,000 |

*Scholarship Applicable at Kota Center Only

## Based on JEE Main'19

| JEE Main Percentile | English | Hindi |
|---|---|---|
| | Fees (After Scholarship) | |
| 99 & Above | Drona Residential Program Free | |
| 97.5 To 99 | ₹ 0 | ₹ 0 |
| 97 To 97.5 | ₹ 14,500 | ₹ 14,500 |
| 96.5 To 97 | ₹ 29,000 | ₹ 29,000 |
| 96 To 96.5 | ₹ 58,000 | ₹ 58,000 |
| 95.5 To 96 | ₹ 65,250 | ₹ 65,250 |
| 95 To 95.5 | ₹ 72,500 | ₹ 72,500 |
| 93 To 95 | ₹ 87,000 | ₹ 87,000 |
| 90 To 93 | ₹ 1,01,500 | ₹ 94,250 |
| 85 To 90 | ₹ 1,08,750 | ₹ 1,01,500 |
| 80 To 85 | ₹ 1,16,000 | ₹ 1,08,750 |
| 75 To 80 | ₹ 1,30,500 | ₹ 1,23,250 |

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