JEE MAIN April'19

PAPER WITH SOLUTION

9 April 2019 _ Evening _ Maths

1. A water tank has the shape of an inverted right circular cone, whose semi- vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic meter per minute. The the rate (in m/min). at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is :

(1) $1/10\pi$ (2) $1/15\pi$ (3) $1/5\pi$ (4) $2/\pi$

Sol. **3**

$\frac{dv}{dt} = 5$ cm^3/min

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\tan\theta = \frac{1}{2} = \frac{r}{h} \Rightarrow 2r = h$

volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2(h)$

$v = \frac{\pi}{12}.h^3$

$\frac{dv}{dt} = \frac{\pi}{12}.3h^2.\frac{dh}{dt}$

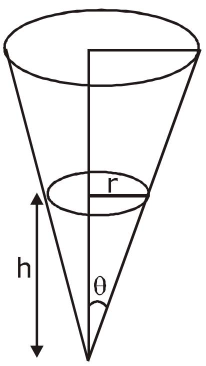
$\frac{dv}{dt} = \frac{\pi}{4}.h^2.\frac{dh}{dt}$

Put h = 10

$5 = \frac{\pi}{4}(10)^2\frac{dh}{dt}$

$\frac{20}{\pi(100)} = \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = \frac{1}{5\pi}$



2. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} terms is :

(1) –25 (2) 25 (3) –36 (4) –35

Sol. **1**

a, a + d, a + 2d are in A.P

a + a + d + a + 2d = 33

3(a+d) = 33

a + d = 11 ....(1)

(a) (a + d) (a + 2d) = 1155

(a)(11)(a + 2d) = 1155

(a)(a + 2d) = 105

$(a)(a + 2(11 - a)) = 105$ {$\because$ $d = 11 - a$}
$a^2 - 22a + 105 = 0$
$(a - 7)(a - 15) = 0$
$a = 7$ or $a = 15$
$\therefore$ $d = 4$ or $d = -4$
$\therefore$ $T_{11} = a + 10d$ or $T_{11} = a + 10d$
$T_{11} = 7 + 10(4)$ or $T_{11} = 15 + 10(-4)$
$T_{11} = 47$ or $T_{11} = -25$

3. The mean and the median of the folowing ten numbers in increasing order 10,22,26,29,34,x,42,67,70,y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to :

(1) 7/3 (2) 9/4 (3) 7/2 (4) 8/3

Sol. **1**

Mean = $\frac{10+22+26+29+34+4+42+67+70+y}{10}$

$42 = \frac{300+x+y}{10} \Rightarrow 420 = 300 + x + y$

$x + y = 120$

median = $\frac{x+34}{2}$

$35 = \frac{x+34}{2} \Rightarrow x = 70 - 34 \Rightarrow x = 36$

as $x + y = 120$
$\therefore$ $y = 120 - 36$
$y = 84$

$\therefore \frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

4. If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$, where [x] denotes the greatest integer function, then :

(1) $\lim_{x\to 4^-} f(x)$ exists but $\lim_{x\to 4^+} f(x)$ does not exists

(2) $\lim_{x\to 4^+} f(x)$ exists but $\lim_{x\to 4^-} f(x)$ does not exists.

(3) f is continuous at $x = 4$

(4) Both $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$ exists but are not equal

Sol. **3**

$f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$

$\lim_{x\to 4^+} f(x) = \lim_{h\to 0} [4+h] - \left[\frac{4+h}{4}\right]$

$= 4 - 1 = 3$

$\lim_{x\to 4^-} f(x) = \lim_{h\to 0}[4-h] - \left[\frac{4-h}{4}\right]$

= 3 – 0 = 3

f(4) = [4] – $\left[\frac{4}{4}\right]$

= 4 – 1 = 3

∴ f(x) is continuous at x = 4

5. If $\cos x\frac{dy}{dx}$ – y sinx = 6x, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right)$ = 0 , then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{\pi^2}{2\sqrt{3}}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$ (3) $-\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2}$

Sol. 3

$\cos x\frac{dy}{dx}$ – y sinx = 6x $\left(0 < x < \frac{\pi}{2}\right)$

$\frac{dy}{dx} - y\tan x$ = 6x. secx

Linear differential equation

∴ I.F. = $e^{\int -\tan x dx}$

= $e^{\int -\tan x dx}$ = $e^{-(-\ln\cos x)}$ = |cosx|

But x ∈ (0, π/2) ∴ Cosx

∴ I.F. = cosx

∴ y(IF) = $6\int x\sec x.\cos x\, dx$

ycosx = $6.\frac{x^2}{2} + C$

ycosx = $3x^2$ + C

given $y\left(\frac{\pi}{3}\right)$ = 0 ⇒ $(0).\cos\frac{\pi}{3}$ = $3\left(\frac{\pi^2}{9}\right) + C$

C = $\frac{-\pi^2}{3}$

Put x = $\frac{\pi}{6}$

$y.\cos\frac{\pi}{6}$ = $3\left(\frac{\pi^2}{6}\right)$ – $\frac{\pi^2}{3}$

$y.\left(\frac{\sqrt{3}}{2}\right)$ = $\frac{\pi^2}{12} - \frac{\pi^2}{3}$

$\therefore\ y = \frac{-\pi^2}{2\sqrt{3}}$

6. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1,2) and the x -axis is :

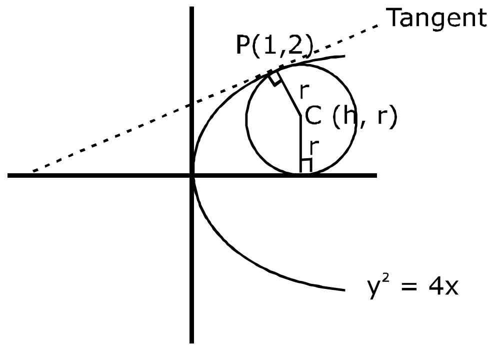
(1) $8\pi(2-\sqrt{2})$ (2) $4\pi(3+\sqrt{2})$ (3) $4\pi(2-\sqrt{2})$ (4) $8\pi(3-2\sqrt{2})$

Sol. **4**

Centre (h,r)

r = r

Tangent for parabola
at P(1,2) is
T = 0



i.e. $y(2) - 4\left(\frac{x+1}{2}\right) = 0$

$x - y - 1 = 0$

normal at P is $x + y - 3 = 0$

centre is on $x + y - 3 = 0$

$\therefore\ h + r - 3 = 0$

$h = 3 - r\ \therefore\ c \equiv (3 - r, r)$

as PC = r

$(PC)^2 = r^2$

$(3 - r - 1)^2 + (r - 2)^2 = r^2$

$4 + r^2 - 4r + r^2 + 4 - 4r = r^2$

$r^2 - 8r + 8 = 0$

$r = 4+2\sqrt{2}$ or $r = 4-2\sqrt{2}$

($4 + 2\sqrt{2}$ will give x-coordinate negative which is note possible)

$\therefore$ area = πr^2

$= \pi\left(4-2\sqrt{2}\right)^2$

area = $\pi\left(16+8-16\sqrt{2}\right) = \pi\left(24-16\sqrt{2}\right)$

$= 8\pi\left(3-2\sqrt{2}\right)$

7. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is :

(1) 964 (2) 227 (3) 625 (4) 232

Sol. **4**

${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} :: 2 : 15 : 70$

$$\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{2}{15} \Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{2}{15}$$

$$\frac{(r+1)!(n-r-1)!}{r!(n-r)!} = \frac{2}{15} \Rightarrow \frac{(r+1)!(n-r-1)!}{r!(n-r)(n-r-1)!} = \frac{2}{15}$$

$\therefore \frac{r+1}{n-r} = \frac{2}{15}$ ....(1)

$\frac{{}^nC_{r+1}}{{}^nC_{r+2}} = \frac{15}{70} \Rightarrow \frac{r+2}{n-r-1} = \frac{15}{70} = \frac{3}{14}$ ....(2)

From (1)
15 (r+1) = 2(n–r)
15r + 15 = 2n – 2r
17r + 15 = 2n
17r = 2n – 15
From (2)
14(r + 2) = 3(n – r – 1)
14r + 28 = 3n – 3r - 3
17r + 31 = 3n
17r = 3n – 31
$\therefore$ 2n – 15 = 3n – 31
n = 16 & r = 1

$\therefore$ Average $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$

$= \frac{696}{3} = 232$

8. The value of integral $\int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$ is :

(1) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (2) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$ (3) $\frac{\pi}{4} - \log_e 2$ (4) $\frac{\pi}{2} - \log_e 2$

Sol. 2

$I = \int_0^1 x\cot^{-1}(1-x^2+x^4)\,dx$

$I = \int_0^1 x\tan^{-1}\left(\frac{1}{1-x^2+x^4}\right)dx$

Put $x^2 = t$ as $x \to 0, t \to 0$
$2xdx = dt$ $x \to 1, t \to 1$

$I = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{1}{1-t+t^2}\right)dt$

$I = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{1}{1-t(1-t)}\right)dt$

$I = \frac{1}{2}\int_0^1 \tan^{-1}\left(\frac{(1-t)+t}{1-t(1-t)}\right)dt$

$I = \frac{1}{2}\int_0^1 [\tan^{-1}(1-t) + \tan^{-1}(t)]dt$

$I = \frac{1}{2}\int_0^1 \tan^{-1}(1-t)dt + \frac{1}{2}\int_0^1 \tan^{-1}(t)dt$

$I = \frac{1}{2}\int_0^1 \tan^{-1}(1-t)dt + \frac{1}{2}\int_0^1 \tan^{-1}(1-t)dt$

$I = \int_0^1 \tan^{-1}(1-t)dt$

put 1-t = y
-dt = dy
$t \to 0;\ y \to 1$
as $t \to 1;\ y \to 0$

$I = -\int_1^0 \tan^{-1} y dy$

$I = \int_0^1 \tan^{-1} y dy$

using by parts
I = [y.$\tan^{-1}$y-1/2ln(1+y^2)]
I = 1.$\tan^{-1}$(1) - 1/2 ln(2) = 0

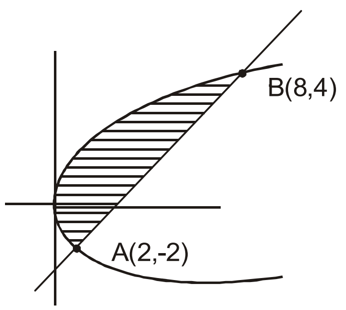
$I = \frac{\pi}{4} - \frac{1}{2}\ln 2$

9. The area (in sq. units) of the region A = $\left\{(x,y) : \frac{y^2}{2} \le x \le y+4\right\}$ is :

(1) $\frac{53}{3}$ (2) 16 (3) 18 (4) 30

Sol. 3

$\frac{y^2}{2} \le x \le y+4$
$y^2 \le 2x$ & $x \le y+4$
$y^2 - 2x \le 0$...(1) &
$x-y-4 \le 0$....(2)
Solve 1 & 2
$y^2=2x$ & x = y+4
$\therefore$ y^2 = 2(y+4)
y^2-2y-8 = 0
y(y+2) −4(y+2) = 0
(y + 2) (y − 4) = 0
Y = -2 & y = 4
$\therefore$ x = 2 & x = 8
A (2,-2) & B (8,4)
$\therefore$ Required area is



area = $\int_{-2}^{4}(\text{line}-\text{parabola})dy$

$\int_{-2}^{4}\left[(y+4)-\frac{y^2}{2}\right]dy$

$= \left(\frac{y^2}{2}+4y-\frac{1}{6}.y^3\right)_{-2}^{4}$

area $\left[\frac{(4)^2}{2}+4(4)-\frac{1}{6}(4)^3\right]-\left[\frac{(-2)^2}{2}+4(-2)\frac{1}{6}(-2)^3\right]$

area = 54/3
area = 18 sq. unit

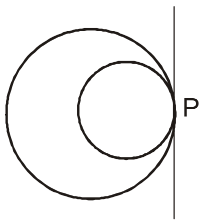
10. the common tangent to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 + 6x + 8y - 24 = 0$ also passes through the point :
(1) (–4,6) (2) (–6, 4) (3) (6,–2) (4) (4, – 2)

Sol. **3**

$x^2+y^2=4 \rightarrow C_1 = (0,0) r_1 = 2$
$x^2+y^2+6x+8y-24 = 0 \rightarrow C_2 = (-3,-4), r_2 = 7$
distance $C_1C_2 = 5$ & $r_1+r_2 = 9$

$|r_1 - r_2| = 5$

as $C_1C_2 = |r_1 - r_2|$
∴ Circle touches internally
∴ equation of comman tangent will be same as common chord
∴ S_1-S_2 = 0
$(x^2+y^2+6x+8y-24)-(x^2+y^2-4) = 0$
$6x + 8y - 20 = 0$
3x + 4y – 10 = 0 common tangent
point (6,-2) satisfy this



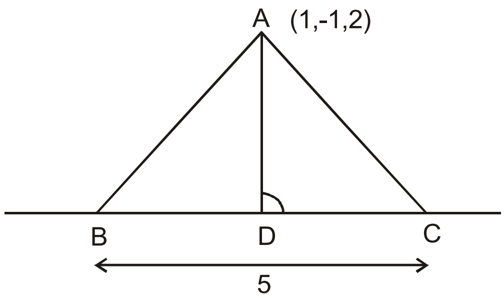
11. The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that BC = 5 units. Then the area (in sq. units) of this triangle, given that the point A(1, – 1, 2), is :
(1) 6 (2) $\sqrt{34}$ (3) $2\sqrt{34}$ (4) $5\sqrt{17}$

Sol. **2**

area of $\Delta ABC = \frac{1}{2}(AD)(BC)$

let D is any point on line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$

$D = (3\lambda - 2, 1, 4\lambda)$
Direction ratio of AD are
$3\lambda - 3, 2, 4\lambda - 2$



as AD perpendicular to line

$\therefore (3\lambda - 3)(3) + 0(2) + 4(4\lambda - 2) = 0$

$25\lambda = 17$

λ **= 17/25**

Point $D \equiv \left(\frac{51}{25} - 2, 1, \frac{68}{25}\right)$

$D \equiv \left(\frac{1}{25}, 1, \frac{68}{25}\right)$

$AD = \sqrt{\left(1 - \frac{1}{25}\right)^2 + (-1-1)^2 + \left(\frac{68}{25} - 2\right)^2}$

$AD = \sqrt{\left(\frac{24}{25}\right)^2 + (4)^2 + \left(\frac{18}{25}\right)^2}$

$AD = \frac{\sqrt{(24)^2 + (50)^2 + (18)^2}}{25}$

$AD = \frac{\sqrt{546 + 2500 + 324}}{25}$

$= \frac{\sqrt{3400}}{25}$

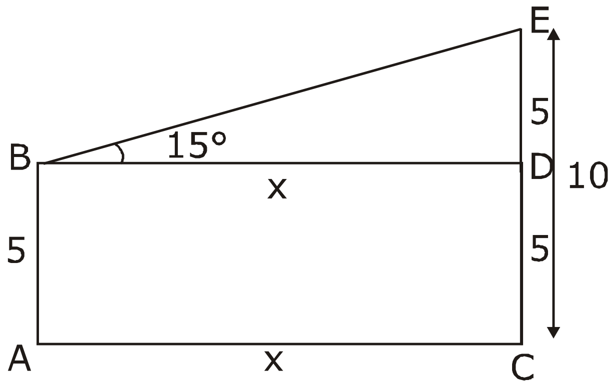
so area of $\Delta ABC = \frac{1}{2}(5).\frac{\sqrt{3400}}{25}$

$= \sqrt{34}$

12. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is :

(1) $5(\sqrt{3} + 1)$ (2) $5(2 + \sqrt{3})$ (3) $\frac{5}{2}(2 + \sqrt{3})$ (4) $10(\sqrt{3} - 1)$

Sol. 2



In ΔBDE

$\tan 15° = \frac{5}{x}$

$x = 5.\cot 15°$

$x = \frac{5.(\sqrt{3}+1)}{(\sqrt{3}-1)}$

$x = 5(2+\sqrt{3})$

13. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 +.... upto 11th tems is :
(1) 915 (2) 945 (3) 916 (4) 946

Sol. **4**

S = 1 + (2×3) + (3×5) + (4 ×7) + ...upto 11

$T_r = r(2r - 1)$

$\therefore S_n = \sum T_r$

$S_n = \sum r(2r-1)$

$S_n = 2\sum r^2 - \sum r$

$S_n = 2\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{n(n+1)}{2}$

$S_n = \frac{n(n+1)(2n+1)}{3} - \frac{n(n+1)}{2}$

$= n(n+1)\left[\frac{2n+1}{3} - \frac{1}{2}\right]$

$= n(n+1)\left[\frac{4n+2-3}{6}\right]$

$S_n = \frac{n(n+1)(4n-1)}{6}$

put n = 11 for sum of 11 terms

$S_{11} = \frac{11(12)(43)}{6}$

$S_{11} = 946$

14. If the function $f(x) = \begin{cases} a|\pi - x| + 1, & x \le 5 \\ b|x - \pi| + 3, & x > 5 \end{cases}$ is continuous at x = 5, then the value of a – b is :

(1) $-\frac{2}{\pi+5}$ (2) $\frac{2}{\pi-5}$ (3) $\frac{2}{5-\pi}$ (4) $\frac{2}{\pi+5}$

Sol. **2**

We have to check at x = 5

$f(5) = a|\pi - 5| + 1 = a(5-\pi) + 1$

$f(5^+) = b|5 - \pi| + 3$

$= b(5-\pi)+3$

$f(5^-) = a|5 - \pi| + 1$

as f(x) is continous at x = 5

$\therefore a(5-\pi) + 1 = b(5-\pi) + 3$

(a-b)(5-π)= 2

(a-b) = $\frac{2}{5-\pi}$

15. If the tangent to the parabola $y^2 = x$ at a point (α,β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then, α is equal to :

(1) $2\sqrt{2}-1$ (2) $\sqrt{2}+1$ (3) $\sqrt{2}-1$ (4) $2\sqrt{2}+1$

Sol. **2**

$y^2 = x$

tangent at $P(\alpha, \beta)$ is T = 0

$\beta y - \left(\frac{x+\alpha}{2}\right) = 0$

$2\beta y - x - \alpha = 0$

$2\beta y = x + \alpha$

$y = \frac{1}{2\beta}x + \frac{\alpha}{2\beta}$

ellipse is

$x^2 + 2y^2 = 1$

$\frac{x^2}{1} + \frac{y^2}{\frac{1}{2}} = 1$

if a line

y = mx + c is a tangent

then $C^2 = a^2 m^2 + b^2$

$\left(\frac{\alpha}{2\beta}\right)^2 = (1)\left(\frac{1}{2\beta}\right)^2 + \frac{1}{2}$

$\frac{\alpha^2}{4\beta^2} = \frac{1}{4\beta^2} + \frac{1}{2}$

also point P(α,β) is on $y^2 = x$

$\therefore \beta^2 = \alpha$

$\therefore \frac{\alpha^2}{4\alpha} = \frac{1}{4\alpha} + \frac{1}{2}$

$\alpha^2 = 1 + 2\alpha$

$\alpha^2 - 2\alpha - 1 = 0$

$\alpha = \frac{2 \pm \sqrt{4+4}}{2}$

$\alpha = 1 \pm \sqrt{2}$

$\alpha = 1 + \sqrt{2}$ & a = $1 - \sqrt{2}$

16. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non - trivial solution (x, y , z) , then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to :

(1) $\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) -4 (4) $\frac{3}{4}$

Sol. **1**

$2x + 3y - z = 0$

$x + ky - 2z = 0$

$2x - y + z = 0$

for non - trivial solutions, $\Delta = 0$

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$2(k - 2) - 3(1 + 4) - 1(-1 - 2k) = 0$

$2k - 4 - 15 + 1 + 2k = 0$

$4k = 18$

$k = 9/2$

Now, $2x - y + z = 0$

$2x + z = y$

$\frac{2x}{y} + \frac{z}{y} = 1$

$2.\frac{x}{y} + \frac{z}{y} - 1 = 0$....(1)

also $2x - z = -3y$

$\frac{2x}{y} - \frac{z}{y} + 3 = 0$ ....(2)

add (1) and (2)

$4.\frac{x}{y} + 2 = 0$

$\frac{x}{y} = \frac{-1}{2}$

$2x + 3y = z$

$\frac{2x}{z} + \frac{3y}{z} = 1$

$2.\frac{x}{z} + 3.\frac{y}{z} - 1 = 0$.....(3)

also $2x - y + z = 0$

$2x - y = -z$

$\frac{2x}{z} - \frac{y}{z} + 1 = 0$ ....(4)

from (3) – (4)

$\frac{y}{z} = \frac{1}{2}$

$2x + 3y - z = 0$

$3x - z = -2x$

$\frac{3y}{x} - \frac{z}{x} + 2 = 0$ ...(5)

put $\frac{y}{x} = -2$ in (5) $\frac{z}{x} = -4$

$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{1}{2}$

17. If $\int e^{\sec x}\left(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)\right)dx = e^{\sec x}f(x) + C$, then a possible choice of f(x) is :

(1) $\sec x - \tan x - \frac{1}{2}$ (2) $\sec x + \tan x + \frac{1}{2}$ (3) $x\sec x + \tan x + \frac{1}{2}$ (4) $\sec x + x\tan x - \frac{1}{2}$

Sol. **2**

$\int e^{\sec x}\left(\sec x \tan x f(x) + (\sec x \tan x + \sec^2 x)\right)dx = e^{\sec x}f(x) + C$

Differentiating both sides

$e^{\sec x}(\sec x.\tan x.f(x)) + e^{\sec x}.(\sec x \tan x + \sec^2 x) = e^{\sec x}\sec x.\tan x.f(x) + e^{\sec x}.f'(x)$

$e^{\sec x}(\sec x.\tan x + \sec^2 x) = e^{\sec x}.f'(x)$

$f'(x) = (\sec x \tan x + \sec^2 x)$

integrating both sides

$\int f'(x)dx = \int(\sec x \tan x + \sec^2 x)dx$

$f(x) = \sec x + \tan x + C$

18. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is :

(1) $8\sqrt{5}$ (2) $4\sqrt{3}$ (3) $10\sqrt{5}$ (4) $8\sqrt{3}$

Sol. **1**

$(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$

$\alpha + \beta = \frac{3}{m^2+1}$ & $\alpha\beta = \frac{(m^2+1)^2}{(m^2+1)} = (m^2 + 1)$

sum of roots is greatest of $(m^2 +1)$ is minimum when m = 0

$\therefore$ equation is $x^2 - 3x + 1 = 0$

$\therefore \alpha + \beta = 3$

& $\alpha\beta = 1$

$|\alpha^3 - \beta^3| = \left|(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)\right|$

$= \left|\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}\left((\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta\right)\right|$

$= \left|\left(\sqrt{(3)^2 - 4(1)}\right)\left((3)^2 - 1\right)\right|$

$= \left|\sqrt{5}(8)\right|$

$= 8\sqrt{5}$

19. If a unit vector $\vec{a}$ makes angles π/3 with $\hat{i}$, π/4 with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{5\pi}{12}$

Sol. **1**

$\alpha = \frac{\pi}{3}$, $\beta = \frac{\pi}{4}$ & $\gamma = ?$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$

$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

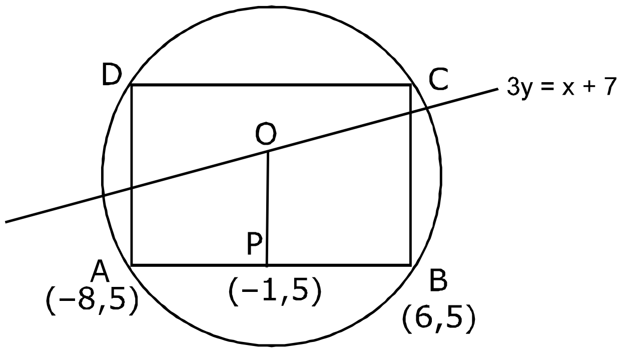
$\cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore \gamma = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

20. A rectangle is inscribed in a circle with a diameter lyaing along the line 3y = x + 7. If the two adjacent vertices of the rectangle are (–8,5) and (6,5), then the area of the rectangle (in sq. units) is :

(1) 98 (2) 84 (3) 72 (4) 56

Sol. **2**



AB is paralle to x - axis
∴ OP is parallel to y - axis
∴ x - coordinate an OP will be constant
i.e. x = – 1
put x = – 1 in line 3y = x + 7
3y = – 1 + 7
y = 2
∴ O ≡ (−1,2)
OP = 3
∴ area of rectangle ABCD = (AB)(BC)
= (14)(2(OP)
=(14)(2 × 3)
14 × 6 = 84

21. The value of sin10° sin30° sin50° sin70° is :

(1) $\frac{1}{16}$ (2) $\frac{1}{32}$ (3) $\frac{1}{18}$ (4) $\frac{1}{36}$

Sol. **1**

sin10° sin30° sin50° sin70°
sin30°(sin50° sin10° sin70°)

$$\frac{1}{2}\left[\sin(60^\circ-10^\circ)\sin 10^\circ \sin(60^\circ+10^\circ)\right]$$

$$=\frac{1}{2}\left[\frac{1}{4}\sin 3(10)\right] = \frac{1}{8}\sin 30^\circ = \frac{1}{16}$$

22. The total number of matrices A = $\begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in R, x \neq y)$ for which $A^TA = 3I_3$ is :

(1) 6 (2) 4 (3) 3 (4) 2

Sol. **2**

$$A = \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}$$

$A^T. A = 3I_3$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\therefore 6y^2 = 3$ & $8x^2 = 3$

$y^2 = \frac{1}{2}$

$x^2 = \frac{3}{8}$

$\therefore y \pm \frac{1}{\sqrt{2}} \quad x = \pm\sqrt{\frac{3}{8}}$

$\therefore$ 4 marices are possible

23. If f : R $\to$ R is a differentiable function and f(2) = 6, then $\lim_{x\to 2} \int_6^{f(x)} \frac{2tdt}{(x-2)}$ is :

(1) 0 (2) 24f'(2) (3) 2f'(2) (4) 12f'(2)

Sol. **4**

$$\lim_{x\to 2} \frac{\int_6^{f(x)} 2tdt}{x-2}$$

as f(2) = 6 therefore it is $\frac{0}{0}$ form, using newton Leibnitz rule

$$\lim_{x\to 2} \frac{2.f(x).f'(x)-0}{1}$$

= 2f(2) . f'(2)

= 2.(6). f'(2) $\Rightarrow$ 12f'(2)

24. If $p \Rightarrow (q \vee r)$ is false, then the truth values of p,q,r are respectively :

(1) T,T,F (2) F,T,T (3) F,F,F (4) T,F,F

Sol. **4**

p	q	r	$q \vee r$	$p \Rightarrow (q \vee r)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

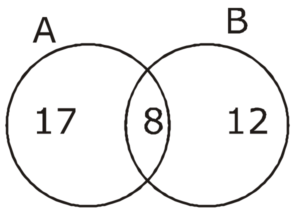
as, $p \Rightarrow (q \vee r)$ is false

$\therefore$ Truth values of p,q,r are T,F,F

25. Two newpapers A and B are published in a city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is :

(1) 13.9 (2) 13 (3) 12.8 (4) 13.5

Sol. **1**



Let x = 17, y = 8, z = 12

Total percentage of persons who look into advertisement

= (30% of x) + (40% of z) + (50% of y)

$$= \left(\frac{3}{10}\times 17\right) + \left(\frac{4}{10}\times 12\right) + \left(\frac{5}{10}\times 8\right)$$

$$= \frac{51}{10}+\frac{48}{10}+\frac{40}{10}$$

$$= \frac{139}{10} = 13.9$$

26. The domain of the defination of the function $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$ is :

(1) $(-2,-1)\cup(-1,0)\cup(2,\infty)$ (2) $(-1,0)\cup(1,2)\cup(3,\infty)$

(3) $(-1,0)\cup(1,2)\cup(2,\infty)$ (4) $(1,2)\cup(2,\infty)$

Sol. **3**

$f(x) = \frac{1}{4-x^2} + \log_{10}\left(x^3 - x\right)$

$4 - x^2 \neq 0$

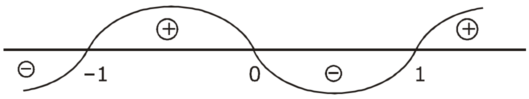
$x^2 \neq \pm 2$ ....(1)

$x^3 - x > 0$

$x(x^2 - 1) > 0$

$x(x - 1)(x + 1) > 0$

$x \in (-1,0) \cup (1,\infty)$ ...(2)



From (1) & (2)

$x \in (-1,0) \cup (1,2) \cup (2,\infty)$

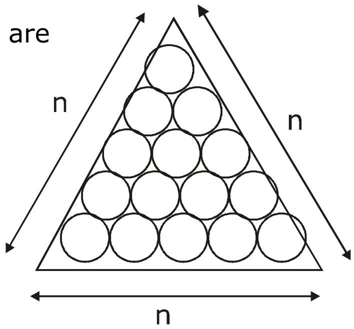
27. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total numbers of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then number of balls used to form the equailateral triangle is :

(1) 157 (2) 225 (3) 262 (4) 190

Sol. **4**

Total ball used to form equilateral trianlge are

$= \frac{n(n+1)}{2}$



Total ball used to form square = $(n - 2)^2$

but given

$\frac{n(n+1)}{2} + 99 = (n - 2)^2$

$n(n + 1) + 198 = 2(n^2 + 4 - 4n)$

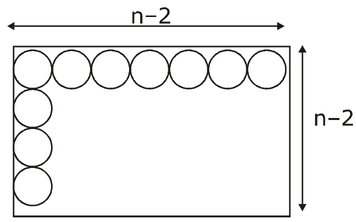
$\Rightarrow (n + 10)(n - 19) = 0$

$n = 19$

$\therefore$ Total balls used to form equilateral trianlge

$= \frac{n(n+1)}{2} = \frac{19 \times 20}{2}$

$= 190$



28. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then :

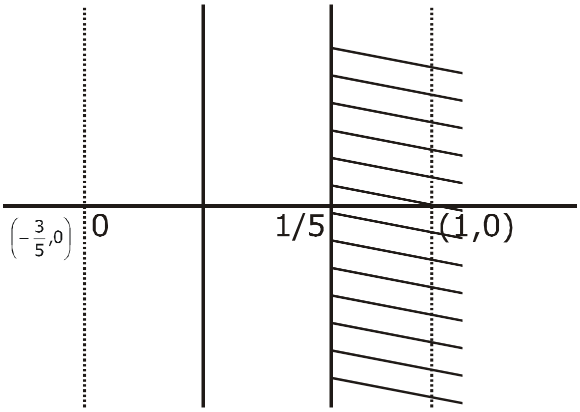
(1) $5\,\text{Re}(\omega) > 1$ (2) $5\,\text{Re}(\omega) > 4$ (3) $5\,\text{Im}(\omega) < 1$ (4) $4\,\text{Im}(\omega) > 5$

Sol. **1**

$|z| < 1$

$5\omega(1 - z) = 5 + 3z$

$5\omega - 5\omega z = 5 + 3z$



$|z| = 5\left|\frac{\omega-1}{3+5\omega}\right| < 1$

$5|\omega - 1| < |3 + 5\omega|$

$5|\omega - 1| < 5\left|\omega + \frac{3}{5}\right|$

$|\omega - 1| < \left|\omega - \left(-\frac{3}{5}\right)\right|$

$5\ \text{Re}(\omega) > 1$

29. If the two lines $x + (a - 1)y = 1$ and $2x + a^2y = 1 (a \in R - \{0,1\})$ are perpendicular, then the distance of their point of intersection from the origin is :

(1) $\frac{\sqrt{2}}{5}$ (2) $\frac{2}{5}$ (3) $\sqrt{\frac{2}{5}}$ (4) $\frac{2}{\sqrt{5}}$

Sol. **3**

$L_1 \rightarrow x + (a - 1)y - 1 = 0 \Rightarrow m_1 = -\frac{1}{a-1}$

$L_2 \rightarrow 2x + a^2 y - 1 = 0 \Rightarrow m_2 = -\frac{2}{a^2}$

as $L_1 \perp L_2$

$\therefore m_1m_2 = -1$

$\left(\frac{-1}{a-1}\right)\left(\frac{-2}{a^2}\right) = -1$

$\frac{2}{a^2(a-1)} = -1$

$a^2(a - 1) + 2 = 0$

$a^3 - a^2 + 2 = 0$

$(a + 1)$ is a factor

$\therefore (a + 1)(a^2 - 2a + 2) = 0$

$a = -1$

$\therefore L_1 \rightarrow x - 2y - 1 = 00$

$L_2 \rightarrow 2x + y - 1 = 0$

solve L_1 & L_2

$$P \equiv \left(\frac{3}{5}, -\frac{1}{5}\right)$$

distance of point P
from origin is

$$OP = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{-1}{5}\right)^2}$$

$$OP = \sqrt{\frac{10}{25}}$$

$$OP = \sqrt{\frac{2}{5}}$$

30. Let P be the plane, which contains the line of intersection of the planes, $x + y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$ and it is perpendicular to the xy - plane. Then the distance of the point (0,0,256) from P is equal to :

(1) $205\sqrt{5}$ (2) $11/\sqrt{5}$ (3) $63\sqrt{5}$ (4) $17/\sqrt{5}$

Sol. **4**

$P_1 \rightarrow x + y + z - 6 = 0$

$P_2 \rightarrow 2x + 3y + z + 5 = 0$

required plane is $p_1 + \lambda p_2 = 0$

$(x + y + z - 6) + \lambda(2x + 3y + z + 5) = 0$

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + \lambda)z + (5\lambda - 6) = 0$

This plane is $\perp$ to xy - plane

$\therefore \vec{n}$ || to xy plane

$\vec{n}.\hat{k} = 0$

$1 + \lambda = 0 \Rightarrow \lambda = -1$

$\therefore -x - 2y - 11 = 0$ required plane

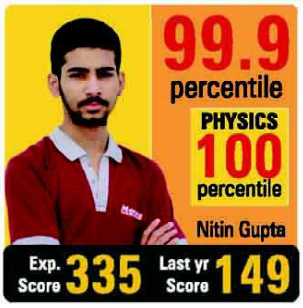
distance of this plane from (0,0,256) is

$$p = \left|\frac{0 + 0 + 11}{\sqrt{5}}\right| = \frac{11}{\sqrt{5}}$$

मोशन ने बनाया साधारण को असाधारण

JEE Main Result Jan'19

4 RESIDENTIAL COACHING PROGRAM (DRONA) STUDENTS ABOVE 99.9 PERCENTILE



Total Students Above 99.9 percentile - **17**

Total Students Above 99 percentile - **282**

Total Students Above 95 percentile - **983**

% of Students Above 95 percentile $\frac{983}{3538}$ = **27.78%**

Scholarship on the Basis of 12th Class Result

Marks PCM or PCB	Hindi State Board	State Eng OR CBSE
70%-74%	30%	20%
75%-79%	35%	25%
80%-84%	40%	35%
85%-87%	50%	40%
88%-90%	60%	55%
91%-92%	70%	65%
93%-94%	80%	75%
95% & Above	90%	85%

New Batches for Class 11th to 12th pass
17 April 2019 & 01 May 2019

हिन्दी माध्यम के लिए पृथक बैच

Scholarship on the Basis of JEE Main Percentile

Score	JEE Mains Percentile	English Medium Scholarship	Hindi Medium Scholarship
225 Above	Above 99	Drona Free (Limited Seats)	
190 to 224	Above 97.5 To 99	100%	100%
180 to 190	Aboev 97 To 97.5	90%	90%
170 to 179	Above 96.5 To 97	80%	80%
160 to 169	Above 96 To 96.5	60%	60%
140 to 159	Above 95.5 To 96	55%	55%
74 to 139	Above 95 To 95.5	50%	50%
66 to 73	Above 93 To 95	40%	40%
50 to 65	Above 90 To 93	30%	35%
35 to 49	Above 85 To 90	25%	30%
20 to 34	Above 80 To 85	20%	25%
15 to 19	75 To 80	10%	15%

सैन्य कर्मियों के बच्चो के लिए 50% छात्रवृत्ति   प्री-मेडिकल में छात्राओं को 50% छात्रवृत्ति