JEE MAIN April'19

PAPER WITH SOLUTION

8 April 2019 _ Morning _ Maths

1. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6,(x>1)$ is equal to :

(1) 32 (2) 24 (3) 29 (4)26

Sol. **2**

$y=\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$

$y = 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$

sum of coff. of all even powers in y

$= 2[{}^6C_0-{}^6C_2+{}^6C_4+{}^6C_4-{}^6C_6-3\,{}^6C_6]$

$= 2[1-15+15+15-1-3]$

$= 2[12]$

$= 24$

2. If $\alpha=\cos^{-1}\left(\frac{3}{5}\right),\beta=\tan^{-1}\left(\frac{1}{3}\right)$, where $0<\alpha,\beta<\frac{\pi}{2}$, then $\alpha-\beta$ is equal to :

(1) $\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (2) $\tan^{-1}\left(\frac{9}{14}\right)$ (3) $\cos^{-1}\left(\frac{9}{5\sqrt{10}}\right)$ (4) $\tan^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

Sol. **1**

$\alpha=\cos^{-1}\left(\frac{3}{5}\right)$, $\beta=\tan^{-1}\left(\frac{1}{3}\right)$

$\cos(\alpha-\beta)=\frac{3}{5}\frac{3}{\sqrt{10}}+\frac{4}{5}\frac{1}{\sqrt{10}}$

$=\frac{9}{5\sqrt{10}}+\frac{4}{5\sqrt{10}}$

$=\frac{9+4}{5\sqrt{10}}$

$(\alpha-\beta)=\cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$

$(\alpha-\beta)=\sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

3. The shortest distance between the line y = x and the curve $y^2 = x-2$ is :

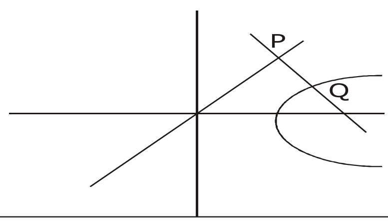
(1) $\frac{11}{4\sqrt{2}}$ (2) 2 (3) $\frac{7}{8}$ (4) $\frac{7}{4\sqrt{2}}$

Sol. **4**

Let Q: $(t^2 + 2 , t)$

for SD $\Rightarrow$ slope of tangent at Q $=1$

$\frac{1}{2t}=1$



$\left(t = \frac{1}{2}\right)$

$Q : \left(\frac{9}{4}, \frac{1}{2}\right)$

Shortest distance $PQ = \left|\frac{\frac{1}{2} - \frac{9}{4}}{\sqrt{2}}\right|$

$PQ = \frac{7}{4\sqrt{2}}$

4. If α and β be the roots of the equation $x^2-2x+2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is:

(1) 5 (2) 2 (3) 4 (4) 3

Sol. **3**

$x^2-2x+1+1 = 0$

$(x-1)^2 = -1$

$x = 1 \pm i$

$\Rightarrow \alpha = 1 + i\ ;\ \Rightarrow \beta = 1 - i$

Now $\left(\frac{\alpha}{\beta}\right)^n = 1$

$\left(\frac{1+i}{1-i}\right)^n = 1$

$\left(\frac{(1+i)^2}{2}\right)^n = 1$

$\left(\frac{2i}{2}\right)^n = 1$

$(i)^n = 1$

n = 4

5. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct ?

(1) P(A|B) = P(B)-P(A)
(2) P(A|B)$\geq$P(A)
(3) P(A|B)$\leq$P(A)
(4) P(A|B) =1

Sol. **2**

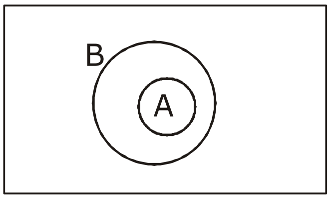
$A \subset B$

$\Rightarrow P(A \cap B) = P(A)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(A)}{P(B)} \geq P(A)$

$\Rightarrow P(A \mid B) \geq P(A)$



6. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4,10,12,14, then the product of the remaining two observations is:

(1) 45 (2) 40 (3) 48 (4) 49

Sol. **3**

$\dfrac{\sum x^2}{N} - \mu^2$ = variance & $S.D = \sqrt{\text{variance}}$

N = 7 ; variance = 16 , $\mu=8$

$\therefore \dfrac{2^2+4^2+10^2+12^2+14^2+a^2+b^2}{7} - 64 = 16$

$\Rightarrow$ a^2+b^2 = 560-460

$\Rightarrow$ a^2+b^2 = 100

$\because$ Mean = 8

$\therefore \dfrac{2+4+10+12+14+a+b}{7} = 8$

$\Rightarrow$ a+b = 56-42

$\Rightarrow$ a + b = 14

$\Rightarrow$ ab = 48

7. If $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right), |x| < 1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to :

(1) $(f(x))^2$ (2) $2f(x)$ (3) $-2f(x)$ (4) $2f(x^2)$

Sol. **2**

$f(x) = \ln\left(\dfrac{1-x}{1+x}\right)$

$f\left(\dfrac{2x}{1+x^2}\right) = \ln\left(\dfrac{1-\dfrac{2x}{1+x^2}}{1+\dfrac{2x}{1+x^2}}\right)$

$= \ln\left(\left(\dfrac{1-x}{1+x}\right)^2\right)$

$= 2\ln\left(\dfrac{1-x}{1+x}\right)$

$= 2f(x)$

8. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4+12x^3-36x^2+25, x \in R$, then :

(1) $S_1 = \{-2,1\}; S_2=\{0\}$ (2) $S_1 = \{-2\}; S_2=\{0,1\}$

(3) $S_1 = \{-2,0\}; S_2=\{1\}$ (4) $S_1 = \{-1\}; S_2=\{0,2\}$

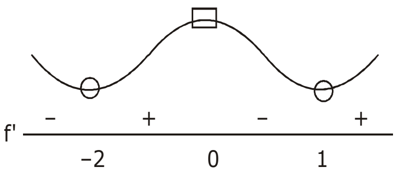
Sol. **1**

$f(X) = 9x^4 + 12x^3 - 36x^2 +25$

$f'(x) = 36x^3 + 36x^2-72x$

$= 36x(x^2+x-2)$

$= 36x(x+2)(x-1)$



local max. : $x \in \{0\} = S_2$

Local min. : $x \in \{-2,1\} = S_1$

9. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2 \frac{dy}{dx} + 2x(x^2+1)y = 1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1) = \frac{\pi}{32}$, then the value of 'a' is :

(1) 1 (2) $\frac{1}{4}$ (3) $\frac{1}{16}$ (4) $\frac{1}{2}$

Sol. **3**

$$(x^2+1)^2 \frac{dy}{dx} + 2x(x^2+1) = 1$$

$$\frac{dy}{dx} + \frac{2x}{(x^2+1)}y = \frac{1}{(x^2+1)^2} \quad \text{I}_{\text{LDE}}$$

$$\text{IF} = \; = e^{\ln(x^2+1)} = x^2+1$$

$$y.(x^2+1) = \int \frac{1}{(x^2+1)^2}.(x^2+1)dx$$

$$y(x^2+1) = \tan^{-1}(x) + C$$

For C : $0.1 = 0 + C \Rightarrow C = 0$

$$y = \frac{\tan^{-1}(x)}{(x^2+1)}$$

Now $\sqrt{a}\,y(1) = \frac{\pi}{32} \Rightarrow \sqrt{a}.\frac{\pi/4}{2} = \frac{\pi}{32} \Rightarrow \sqrt{a} = \frac{1}{4}$

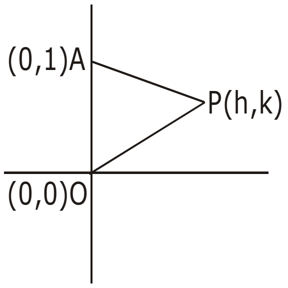
$$a = \frac{1}{16}$$

10. Let O(0,0) and A(0,1) be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is :

(1) $8x^2-9y^2 + 9y = 18$ (2) $9x^2+8y^2-8y = 16$

(3) $9x^2-8y^2 + 8y = 16$ (4) $8x^2+9y^2 - 9y = 18$

Sol. **B**



$PA+PO+OA = 4 \Rightarrow PA+PO = 3$

$\Rightarrow$ locus of P is ellipse

$PA+PO = 3 \Rightarrow 2b = 3 \Rightarrow b = 3/2$

$AO = 2be \Rightarrow e = 1/3$

Now, $e^2 = 1- a^2/b^2 \Rightarrow \frac{1}{9} = 1 - \frac{4a^2}{9}$

$1 = 9-4a^2 \Rightarrow a^2 = 2$

$E : \frac{x^2}{2} + \frac{4(y-1/2)^2}{9} = 1$

$9x^2 + 8(y-1/2)^2 = 18$

$9x^2 + 8y^2 + 2 - 8y = 18$

$9x^2 + 8y^2 - 8y-16 = 0$

11. Let $f:[0,2] \to R$ be a twice differentiable function such that $f''(x)>0$, for all $x \in (0,2)$. If $\phi(x) = f(x)+f(2-x)$, then ϕ is :

(1) decreasing on (0,1) and increasing on (1,2).

(2) increasing on (0,1) and decreasing on (1,2).

(3) decreasing on (0,2)

(4) increasing on (0,2)

Sol. **1**

$f:[0,2] \to R$, $f''(x)>0 \quad \forall\ x \in (0,2)$

$\Rightarrow f' \uparrow$

$\phi(x) = f(x) + f(2-x)$

$\phi'(x) = f'(x) - f'(2-x)$

$\phi' > 0 \Rightarrow f'(x) > f'(2-x) \Rightarrow x > 2-x \Rightarrow x > 1$

$\Rightarrow \phi \uparrow \Rightarrow 1 < x < 2$

$\Rightarrow \phi \downarrow \Rightarrow 0 < x < 1$

12. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is :

(1) $\frac{\pi}{64}$ (2) 0 (3) $\frac{\pi}{32}$ (4) $\frac{\pi}{16}$

Sol. **1**

$A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \alpha \in R$

$A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Now, $A^2 = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$

$A^3 = \begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} = \begin{pmatrix} \cos 3\alpha & -\sin 3\alpha \\ \sin 3\alpha & \cos 3\alpha \end{pmatrix}$

In gen. : $A^n = \begin{pmatrix} \cos n\alpha & -\sin n\alpha \\ \sin n\alpha & \cos n\alpha \end{pmatrix}$

Now $A^{32} = \begin{pmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$\Rightarrow \cos 32\alpha = 0$, $\sin 32\alpha = 1$

$\sin 32\alpha = 1$, $\cos 32\alpha = 0$

$\Rightarrow \alpha = \frac{\pi}{64}$

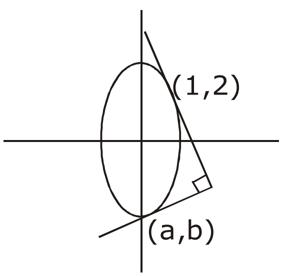
13. If the tangents on the ellipse $4x^2+y^2=8$ at the points (1,2) and (a,b) are perpendicular to each other, then a^2 is equal to :

(1) $\frac{4}{17}$ (2) $\frac{128}{17}$ (3) $\frac{64}{17}$ (4) $\frac{2}{17}$

Sol. **4**

E: $4x^2 + y^2 = 8$

$\frac{x^2}{2} + \frac{32}{8} = 1 \Rightarrow \frac{dy}{dx} = \frac{-4x}{y}$



Slope of tangent at (1,2) = m_1 = -2

Slope of tangent at (a,b) $m_2 = -4a/b$

$\because m_1 . m_2 = -1 \Rightarrow \frac{8a}{b} = 1 \Rightarrow b = 8a$

Now, (a,b) on ellipse

$4a^2 + b^2 = 8$

$4a^2 + 64a^2 = 8$

$a^2 = 8/68 = 4/34 = 2/17$

14. $\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals:

(1) 4 (2) $2\sqrt{2}$ (3) $4\sqrt{2}$ (4) $\sqrt{2}$

Sol. **3**

$$\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}} = \lim_{x \to 0} \left(\frac{\sin^2 x}{x^2}\right)\left(\frac{x^2}{1-\cos x}\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)$$

$= 1.2.2\sqrt{2} = 4\sqrt{2}$

15. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vector $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :

(1) $\sqrt{\frac{3}{2}}$ (2) $\sqrt{6}$ (3) $\frac{\sqrt{3}}{2}$ (4) $3\sqrt{6}$

Sol. **1**

$\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ & $\hat{n} = \vec{b} \times \vec{c}$

$$\Rightarrow \hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = <\hat{i} - 2\hat{j} + \hat{k}>$$

Proj. of $\vec{a}$ on $\hat{n}$ = $\left|\frac{\vec{a}.\hat{n}}{|\hat{n}|}\right|$

$= \left|\frac{2-6+1}{\sqrt{6}}\right|$

$= \left|\frac{3}{\sqrt{6}}\right| = \sqrt{\frac{3}{2}}$

16. The greatest value of $c \in R$ for which the system of linear equations, x-cy-cz = 0, cx-y+cz = 0 cx+cy-z = 0 has a non-trivial solution, is :

(1) 0 (2) 2 (3) $\frac{1}{2}$ (4) -1

Sol. **3**

For non-trinial solu. of homog. system of equation

$$\Delta = \begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

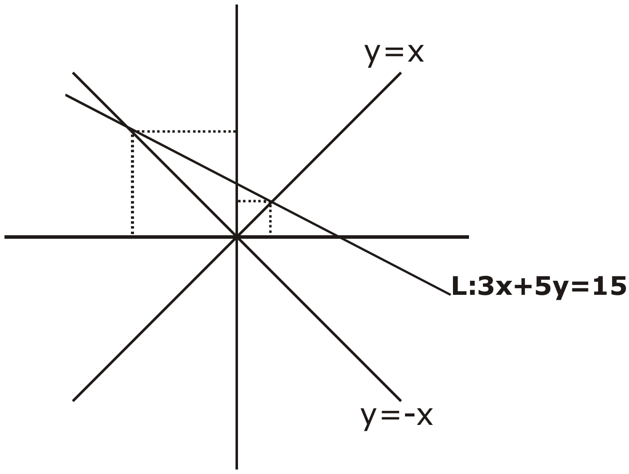
$(1-c^2) + c(-c-c^2) - c(c^2+c) = 0$
$1-c^2-c^2-c^3-c^3-c^2 = 0$
$1-3c^2-2c^3=0$
$2c^3+3c^2-1= 0$
$(2c-1)(c^2+2c+1) = 0$
$(2c-1)(c+1)^2 = 0$
$c= 1/2$ or $c= -1$
Greatest value of $c = 1/2$

17. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is :
(1) if you are not a citizen of India, then you are not born in India.
(2) if you are a citizen of India, then you are born in India.
(3) if you are not born in India, then you are not a citizen of India.
(4) if you are born in India, then you are not a citizen of India.

Sol. **1**
Contrapositive statement of $p \to q$ is $\sim q \to \sim p$

18. A point on the straight line, $3x+5y = 15$ which is equidistant from the coordinate axes will lie only in :
(1) 4^{th} quadrant
(2) 1^{st} and 2^{nd} quadrants
(3) 1^{st}, 2^{nd} and 4^{th} quadrants
(4) 1^{st} quadrant

Sol. **2**

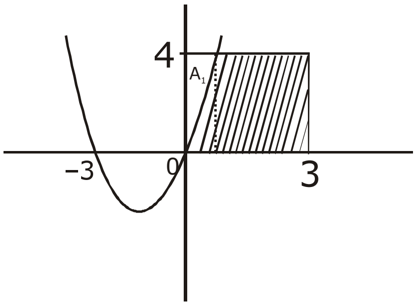


In 1^{st} & 2^{nd} quadrants according to figure then intersect in (1)& (2)

19. The area (in sq. units) of the region $A = \{(x,y) \in R \times R \mid 0 \le x \le 3, 0 \le y \le 4, y \le x^2 + 3x\}$ is :

(1) 8 (2) $\frac{59}{6}$ (3) $\frac{26}{3}$ (4) $\frac{53}{6}$

Sol. **2**



Req area = 12-(A_1)

$= 12-(4-\int_0^1 (x^2+3x)\,dx\,)$

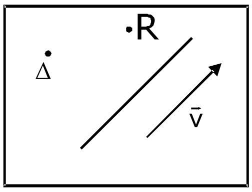
= 12-(4-(1/3 + 3/2))

$= 12-\left(4-\left(\frac{2+9}{6}\right)\right)$

= 8 + 11/6
= 59/6

20. The equation of a plane containing the line of intersection of the planes 2x-y-4= 0 and y+2z-4=0 and passing through the point (1,1,0) is :
(1) x-3y-2z= -2 (2) X+3y+z=4 (3) x-y-z=0 (4) 2x-z=2

Sol. **3**



using family of plane
P: $P_1+\lambda P_2 = 0$
P : (2)x + (-1+λ)+(2λ)-4-4λ= 0
it pass through (1,1,0)
2-1+λ-4-4λ= 0
$3\lambda = -3$
$\lambda = -1$
P: 2x-2y-2z = 0
P: x-y-z = 0

21. All possible numbers are formed using the digits 1,1,2,2,2,2,3,4,4 taken all at a time. The number of such numbers in which the odd digits occupy even places is :
(1) 162 (2) 175 (3) 160 (4) 180

Sol. **4**
4 even place & 5 odd place

$= {}^4C_3 . \frac{3!}{2!} \times \frac{6!}{4!2!}$
$= 4 . 3 . 15$
$= 180$

22. The length of the perpendicular from the point (2,-1,4) on the straight line, $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is :

(1) greater than 3 but less than 4
(2) greater than 2 but less than 3
(3) less than 2
(4) greater than 4

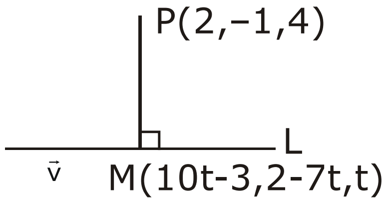
Sol. **1**

For t

$\overrightarrow{PM}.\vec{V} = 0$
(10t-5).10+(3-7t)(-7)+(t-4) = 0
100t-50-21+49t+t-4= 0
150t -75 = 0
t = 1/2

M : (2,-3/2, 1/2) $\Rightarrow \overrightarrow{PM} = (0, -1/2, -7, 2)$

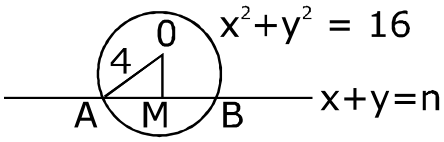
distance $|\overrightarrow{PM}| = \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \frac{5}{\sqrt{2}}$



23. The sum of the squares of the lengths of the chords intercepted on the circle, $x^2+y^2=16$, by the lines, x+y=n, $n \in N$, where N is the set of all natural numbers, is :

(1) 105 (2) 160 (3) 210 (4) 320

Sol. **3**



$\because 0 < \frac{n}{\sqrt{2}} < 4 \Rightarrow 0 < n < 4\sqrt{2} \Rightarrow 0<n<5.6$

$AB = 2AM = 2\sqrt{16 - \left(\frac{n}{\sqrt{2}}\right)^2}$

sum of square of AB $= \sum 4\left(16 - \frac{n^2}{2}\right)$

$= 4\sum\left(16-\frac{n^2}{2}\right) = 4\left(\sum_{n=1}^{5}16-\sum_{n=1}^{5}\frac{n^2}{2}\right)$

= 4(80-1/2 (1+4+9+16+25)

= 320 - 110

= 210

24. The sum of the solutions of the equation $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2,\left(x>0\right)$ is equal to :

(1) 12 (2) 9 (3) 10 (4) 4

Sol. **3**

$|\sqrt{x}-2|+\sqrt{x}\left(x-4\right)+2=0$

(i) For $\sqrt{x}\geq 2 \Rightarrow \sqrt{x}-2+x-4\sqrt{x}+2=0$

$x-3\sqrt{x}=0$

$\sqrt{x}\left(\sqrt{x}-3\right)=0$

$x=0\,|\,x=9 \Rightarrow$ x = 9 is solution

(ii) For $\sqrt{x}<2 \Rightarrow 2-\sqrt{x}+x-4\sqrt{x}+2=0$

$x-5\sqrt{x}+4=0$

$\left(\sqrt{x-4}\right)\left(\sqrt{x}-1\right)=0 \Rightarrow x=1\,|\,x=16 \Rightarrow$ x = 1 is solution

$\therefore$ sum of solution = 1+9 = 10

25. If $\cos\left(\alpha+\beta\right)=\frac{3}{5}$, $\sin\left(\alpha-\beta\right)=\frac{5}{13}$ and $0<\alpha,\beta<\frac{\pi}{4}$, then tan($2\alpha$) is equal to :

(1) $\frac{33}{52}$ (2) $\frac{21}{16}$ (3) $\frac{63}{52}$ (4) $\frac{63}{16}$

Sol. **4**

tan2α = tan $(\alpha+\beta+\alpha-\beta)$

$=\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}}$

$=\frac{48+15}{36-20}$

$=\frac{63}{16}$

26. The sum of the series $2.{}^{20}C_0+5.{}^{20}C_1+8.{}^{20}C_2+11.{}^{20}C_3+....+62.{}^{20}C_{20}$ is equal to :

(1) 2^{23} (2) 2^{24} (3) 2^{26} (4) 2^{25}

Sol. **4**

$2.{}^{20}C_0+5.{}^{20}C_1+8.{}^{20}C_2+11.{}^{20}C_3+....+60.{}^{20}C_{20}$

In Gen. $S_n = \sum_{r=0}^{20}(3r+2)^{10}C_r$

$S_n = 3\sum_{r=0}^{20} r\,{}^{20}C_r + 2\sum_{r=0}^{20} {}^{20}C_r$

$= 3.20\sum_{r=1}^{20} {}^{19}C_{r-1} + 2\left(2^{20}\right)$

$= 60.2^{19} + 2.2^{20}$
$= 2^{19}(4+60)$
$= 64.\ 2^{19}$
$= 2^6 .\ 2^{19}$
$= 2^{25}$

27. The sum of all natural numbers 'n' such that 100<n<200 and H.C.F. (91,n)>1 is :
(1) 3203 (2) 3221 (3) 3121 (4) 3303

Sol. **3**

$91 = 13 \times 7 \quad \because \text{HCF}(91,n) > 1$

sum of n = multiple of 7 + multiple of 13 - multiple of 13×7
= (105 + ...+ 196) + (104+ ....195) -182
= 7(105+196) + 4 (104 + 195) - 182
= 2107+1196-182
= 3121

28. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}} dx$ is equal to : (where c is a constant of integration.)

(1) x+2sinx+sin2x+c (2) 2x+sinx+sin2x+c
(3) x+2sinx+2sin2x+c (4) 2x+sinx+2sin2x+c

Sol. **1**

$\int \frac{2\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$

$= \int \frac{\sin(3x)+\sin(2x)}{\sin x} dx$

$= \int \frac{3\sin x - 4\sin^3 x + 2\sin x\cos x}{\sin x} dx$

$\int \left(3 - 4\sin^2 x + 2\cos x\right) dx$

$= 3x - 4\int\left(\frac{1-\cos 2x}{2}\right)dx + 2\int \cos x\, dx$

$= 3x - 2\left(x - \frac{\sin 2x}{2}\right) + 2\sin x$

= x + sin2x + 2sinx + C

29. If $f(x)=\frac{2-x\cos x}{2+x\cos x}$ and $g(x) = \log_e x$, $(x>0)$ then the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x))dx$ is :

(1) $\log_e 3$ (2) $\log_e 1$ (3) $\log_e 2$ (4) $\log_e e$

Sol. **2**

$f(x)=\frac{2-x\cos x}{2+x\cos x}$ & $g(x) = \ln x$ $(x>0)$

$$I=\int_{-\pi/4}^{\pi/4} g(f(x))dx \Rightarrow I=\int_{-\pi/4}^{\pi/4} \ln\left(\frac{2-x\cos x}{x+x\cos x}\right)dx$$

$I = 0$ $(\because g(f(x))$ is an odd function$)$

30. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$ $x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to :

(1) $2x,-\frac{\pi}{3}$ (2) $\frac{\pi}{6}-x$ (3) $x-\frac{\pi}{6}$ (4) $\frac{\pi}{3}-x$

Sol. **3**

$$2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\frac{\tan\left(\frac{\pi}{3}\right)+\tan x}{1-\tan\frac{\pi}{3}.\tan x}\right)\right)^2$$

$$2y=\left(\cot^{-1}\left(\tan\left(\frac{\pi}{3}+x\right)\right)\right)^2$$

$$2y=\left(\frac{\pi}{2}-\tan^{-1}\left(\tan\left(\frac{\pi}{3}+x\right)\right)\right)^2$$

$$2y=\left(\frac{\pi}{2}-(\frac{\pi}{3}+x)\right)^2 \quad (0<x<\pi/6)$$

$$2y = \left(\frac{\pi}{6} - x\right)^2$$

or

$$2y = \left(\frac{\pi}{2} - \left(\frac{\pi}{3} + x - \pi\right)\right)^2 \quad (\pi/6<x<\pi/2)$$

$$2y = \left(\frac{7\pi}{2} - x\right)^2$$

$$\frac{dy}{dx} = -\left(\frac{\pi}{6} - x\right) \text{ if } 0<x<\pi/6$$

Or

$$\frac{dy}{dx} = -\left(\frac{7\pi}{6} - x\right) \text{ if } \pi/6<x<\pi/2$$

मोशन ने बनाया साधारण को असाधारण

JEE Main Result Jan'19

4 RESIDENTIAL COACHING PROGRAM (DRONA) STUDENTS ABOVE 99.9 PERCENTILE

Total Students Above 99.9 percentile - **17**

Total Students Above 99 percentile - **282**

Total Students Above 95 percentile - **983**

% of Students Above 95 percentile $\frac{983}{3538}$ = **27.78%**

Scholarship on the Basis of 12th Class Result

Marks PCM or PCB	Hindi State Board	State Eng OR CBSE
70%-74%	30%	20%
75%-79%	35%	25%
80%-84%	40%	35%
85%-87%	50%	40%
88%-90%	60%	55%
91%-92%	70%	65%
93%-94%	80%	75%
95% & Above	90%	85%

New Batches for Class 11th to 12th pass
17 April 2019 & 01 May 2019

हिन्दी माध्यम के लिए पृथक बैच

Scholarship on the Basis of JEE Main Percentile

Score	JEE Mains Percentile	English Medium Scholarship	Hindi Medium Scholarship
225 Above	Above 99	Drona Free (Limited Seats)	
190 to 224	Above 97.5 To 99	100%	100%
180 to 190	Aboev 97 To 97.5	90%	90%
170 to 179	Above 96.5 To 97	80%	80%
160 to 169	Above 96 To 96.5	60%	60%
140 to 159	Above 95.5 To 96	55%	55%
74 to 139	Above 95 To 95.5	50%	50%
66 to 73	Above 93 To 95	40%	40%
50 to 65	Above 90 To 93	30%	35%
35 to 49	Above 85 To 90	25%	30%
20 to 34	Above 80 To 85	20%	25%
15 to 19	75 To 80	10%	15%

सैन्य कर्मियों के बच्चो के लिए 50% छात्रवृत्ति **प्री-मेडिकल में छात्राओं को 50% छात्रवृत्ति**