MOTION

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**Fastest Growing Institute of Kota (Raj.)**

**FOR JEE Advanced (IIT-JEE) | JEE Main (AIEEE) | CBSE | SAT | NTSE | OLYMPIADS**

# JEE MAIN EXAMINATION - 2014

# QUESTIONS WITH SOLUTIONS

# PAPER CODE - E

# [PHYSICS] Code - E

**1.** The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm 0.01$ V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(1) 0.2 mA (2) 0.02 mA

(3) 0.5 mA (4) 0.05 mA

**Sol.** **1**

$$I = \left(e^{\frac{1000V}{T}} - 1\right) mA$$

$$e^{\frac{1000V}{T}} = I + 1$$

$$\log (I + 1) = \frac{1000V}{T}$$

$$\frac{1}{I+1} \times dI = \frac{1000}{T} \times dv$$

$$dI = \frac{1000}{300} \times (5 + 1) \text{ mA} \times 0.01$$

$$= 0.2 \text{ mA}$$

**2.** From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is :

(1) $2gH = n^2u^2$ (2) $g H = (n - 2)^2u^2$

(3) $2 g H = nu^2(n - 2)$ (4) $g H = (n - 2)u^2$

**Sol.** **3**

$-H = ut - 1/2\ g\ t^2$

$t' = u/g$

Given $t = nt'$

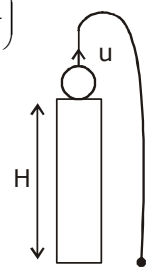
$$\Rightarrow -H = u\left(\frac{nu}{g}\right) - 1/2\ g\left(\frac{nu}{g}\right)^2$$

$$\Rightarrow -H = \frac{nu^2}{g} - 1/2\ \frac{n^2u^2}{g}$$

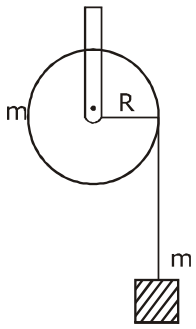
$$\Rightarrow -H = \frac{nu^2}{g}\left(1 - \frac{1}{2}n\right)$$

$\Rightarrow -2Hg = nu^2 (2 - n)$

$\Rightarrow 2Hg = nu^2 (n - 2)$




**3.** A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ?




(1) $\frac{2g}{3}$ (2) $\frac{g}{2}$ (3) $\frac{5g}{6}$ (4) g

**Sol.** **2**

$mg - T = ma$ .....(1)

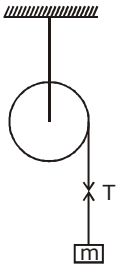
$TR = I\alpha$

$$TR = mR^2 \frac{a}{R}$$

$T = ma$ .....(2)

$mg - ma = ma$

$a = g/2$

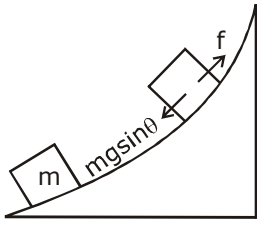



**4.** A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is :

(1) $\frac{1}{6}$m (2) $\frac{2}{3}$m

(3) $\frac{1}{3}$m (4) $\frac{1}{2}$m

**Sol.** **1**



$$y = \frac{x^3}{6}$$

$$\frac{dy}{dx} = \frac{3x^2}{6} = \frac{x^2}{2}$$

$f = \mu mg \cos\theta = mg \sin\theta$

$\mu = \tan\theta$

$$0.5 = \frac{x^2}{2} \Rightarrow x = 1$$

$$y = \frac{x^3}{6} = \frac{1}{6} \text{ m}$$

**5.** When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is :

(1) $aL^2 + bL^3$ (2) $\frac{1}{2}(aL^2 + bL^3)$

(3) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (4) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

**Sol. 3**

$$W = \int F dx$$

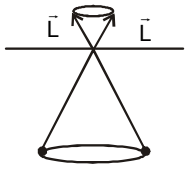
$$W = \int (ax + bx^2)\, dx$$

$$W = \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L$$

$$W = \left[\frac{aL^2}{2} + \frac{bL^3}{3}\right]$$

**6.** A bob of mass m attached to an inextensible string of length $\ell$ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed $\omega$ rad/s about the vertical. About the point of suspension :
(1) Angular momentum is conserved.
(2) Angular momentum changes in magnitude but not in direction.
(3) Angular momentum changes in direction but not in magnitude.
(4) Angular momentum changes both in direction and magnitude.

**Sol. 3**



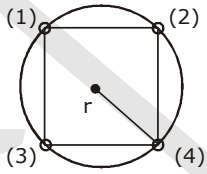

$\vec{L}$ vector rotates in direction but magnitude remains constant.

**7.** Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitiational attraction. The speed of each particle is :

(1) $\sqrt{\frac{GM}{R}}$ (2) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(3) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (4) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

**Sol. 4**




$$\frac{mv^2}{r} = F_{14} + F_{24} + F_{34}$$

$$= \frac{Gm^2}{4r^2} + \frac{Gm^2}{2r^2}\sqrt{2}$$

$$= \frac{Gm^2}{2r^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$\frac{mv^2}{r} = \frac{Gm^2}{2r^2}\left(\frac{1+2\sqrt{2}}{2}\right)$$

$$v = \sqrt{\frac{Gm}{4r}(1+2\sqrt{2})}$$

$$v = \frac{1}{2}\sqrt{\frac{Gm}{r}(1+2\sqrt{2})}$$

**8.** The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is (For steel Young's modulus is $2 \times 10^{11}$ N m$^{-2}$ and coefficient of thermal expansion is $1.1 \times 10^{-5}$ K$^{-1}$)
(1) $2.2 \times 10^8$ Pa (2) $2.2 \times 10^9$ Pa
(3) $2.2 \times 10^7$ Pa (4) $2.2 \times 10^6$ Pa

**Sol.** **1**

$y = 2 \times 10^{11}$

$l_f = l_0 (1 + \alpha\Delta t)$

$\therefore \Delta t = 100° C$

$\alpha = 1.1 \times 10^{-5} k^{-1}$

$l_f - l_0 = \alpha\Delta t$

$\Rightarrow \Delta l = 1.1 \times 10^{-5} \times 100$

$= 1.1 \times 10^{-3}$

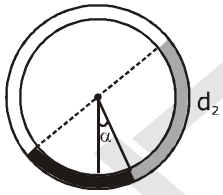
$y = \frac{p}{1.1 \times 10^{-3}}$

$2 \times 10^{11} = \frac{p}{1.1 \times 10^{-3}}$

$\Rightarrow p = 2 \times 1.1 \times 10^{11} \times 10^{-3}$

$\Rightarrow p = 2.2 \times 10^{8}$ Pa

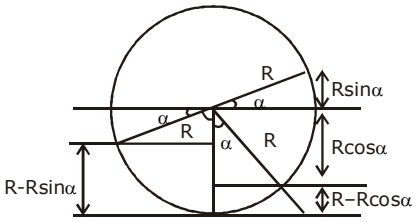
**9.** There is a circular tube in a vertical plane. Two liquids which do not mix and of densities $d_1$ and $d_2$ are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle $\alpha$ with vertical. Ratio $\frac{d_1}{d_2}$ is :




(1) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (2) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(3) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (4) $\frac{1+\sin\alpha}{1-\cos\alpha}$

**Sol.** **3**




$R - R\sin\alpha = d_1 (R - R\sin\alpha) + d_2 (R\sin\alpha + R\cos\alpha)$

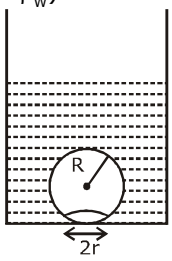
$d_1 (1 - \sin\alpha) = d_1 (1 - \cos\alpha) + d_2 (\sin\alpha + \cos\alpha)$

$d_1 - d_1 \sin\alpha = d_1 - d_1 \cos\alpha + d_2 (\sin\alpha + \text{cod}\ \alpha)$

$d_1 (\cos\alpha - \sin\alpha) = d_2 (\sin\alpha + \cos\alpha)$

$\frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{1+\tan\alpha}{1-\tan\alpha}$

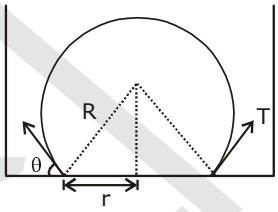
**10.** On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of the water is T, value of r just before bubbles detatch is: (density of water is $\rho_w$)




(1) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (2) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(3) $R^2\sqrt{\frac{\rho_w g}{T}}$ (4) $R^2\sqrt{\frac{3\rho_w g}{T}}$

**Sol.**




$\frac{4}{3}\pi R^3 \rho_w g = \Delta P (\pi r^2) + T (2\pi r) \sin\theta$

$\frac{4}{3}\pi R^2 \rho_w g = \frac{2T}{R} (\pi r^2) + T (2\pi r) \frac{r}{R}$

$\frac{4}{3}\pi R^3 \rho_w g = \frac{4T}{R} \pi r^2$

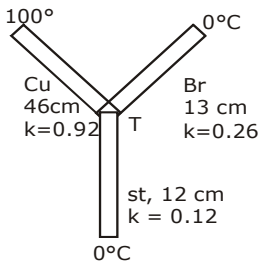
$r = R^2\sqrt{\frac{\rho_w g}{3T}}$

**11.** Three rods of Copper, Brass and Steel are welded together to form a Y - shaped structure. Area of cross - section of each rod = 4 $cm^2$. End of copper rod is maintained at 100ºC where as ends of brass and steel are kept at 0ºC. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is :

(1) 1.2 cal/s (2) 2.4 cal/s

(3) 4.8 cal/s (4) 6.0 cal/s

**Sol. 3**




By Kirchoff law

$$\frac{T-100}{R_1}+\frac{T-0}{R_2}+\frac{T-0}{R_3}=0$$

$$\Rightarrow \frac{T-100}{(46)}(0.92)+\frac{T}{13}(0.26)+\frac{T}{12}(0.12)=0$$

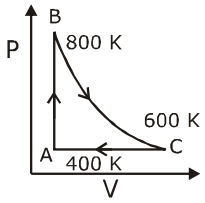
$$\frac{T-100}{50}+\frac{T}{50}+\frac{T}{100}=0$$

$2T - 200 + 2T + T = 0$

$T = 40$

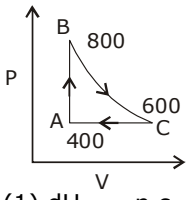
$$\left(\frac{dQ}{dt}\right)_{Cu}=\frac{(100-40)}{46}(0.92)(4)=4.8 \text{ cal/s}$$

**12.** One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperature at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement :




(1) The change in internal energy in whole cyclic process is 250 R.
(2) The change in internal energy in the process CA is 700 R.
(3) The change in internal energy in the process AB is –350 R.
(4) The change in internal energy in the process BC is –500 R.

**Sol. 4**




(1) $dU_{AB} = n\,c_v\,dT$

$$= 1 \times \frac{5R}{2}(400)$$

$= 1000\ R$

(2) $dU_{BC} = n\,c_v\,dT$

$$= 1 \times \frac{5R}{2}(-200)$$

$= -500\ R$

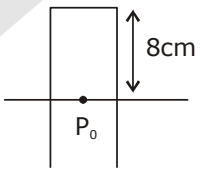
(3) $dU_{ABCA} = 0$

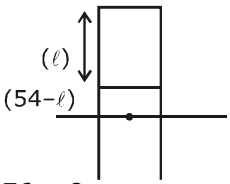
(4) $$dU_{CA} = 1 \times \frac{5R}{2}(-200)$$

$= -500\ R$

**13.** An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?
(Atmospheric pressure = 76 cm of Hg)
(1) 16 cm (2) 22 cm
(3) 38 cm (4) 6 cm

**Sol. 1**




$76 \times 8 = \ell \times p_{gas}$

$$\frac{76\times 8}{\ell} = p_{gas}$$

$$\frac{76\times 8}{\ell} + (54 - \ell) = 76$$

$54 - \ell = 76 \quad \left(1 - \frac{8}{\ell}\right)$

$= 76 \left(\frac{\ell - 8}{\ell}\right)$

$54\ell - \ell^2 = 76\ell - 608$

$\ell^2 + 22\ell - 608 = 0$

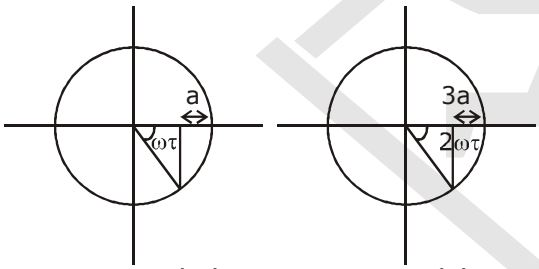
$\ell = \frac{-22 \pm \sqrt{(22)^2 + 4 \times 808}}{2}$

$= \frac{-22 + 54}{2}$

$= 16$ cm

**14.** A particle moves with simple harmonic motion in a straight line. In first $\tau$ s, after starting from rest it travels a distance a, and in nest $\tau$ s it travels 2a, in same direction, then :
(1) Amplitude of motion is 3a
(2) Time period of oscillations is $8\tau$
(3) Amplitude of motion is 4a
(4) Time period of oscillations is $6\tau$

**Sol.** **4**




$A - A\cos(\omega\tau) = a$ ....(1)

$A - A\cos(2\omega\tau) = 3a$

$2A(1 - \cos^2\omega\tau) = 3a$ ...(2)

Divide (2) By (1)

$2(1 + \cos\omega\tau) = 3$

$\cos\omega\tau = 1/2$

$\omega\tau = \pi/3$

$\frac{2\pi}{T}\tau = \pi/3$

$T = 6\tau$

**15.** A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.
(1) 12 (2) 8 (3) 6 (4) 4

**Sol.** **3**

$f = \frac{V}{4L} = \frac{340}{4 \times 85 \times 10^{-2}} = 100$ Hz

100 H, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz = 6

**16.** Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where $V_O$ is the potential at the origin and $V_A$ the potential at x = 2 m is :
(1) 120 J (2) – 120 J
(3) – 80 J (4) 80 J

**Sol.** **3**

$dv = \int \vec{E}.\vec{dr}$

$V_A - V_0 = -\int_0^2 30x^2 dx$

$= -\left[30\frac{x^3}{3}\right]_0^2$

$= -[10x^3]$

$= -10[8 - 0]$

$V_A - V_0 = -80$ J

**17.** A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is $3 \times 10^4$ V/m, the change density of the positive plate will be close to :
(1) $6 \times 10^{-7}$ C/m$^2$
(2) $3 \times 10^{-7}$ C/m$^2$
(3) $3 \times 10^4$ C/m$^2$
(4) $6 \times 10^4$ C/m$^2$

**Sol.** **1**

$Q = cv$

$Q = \frac{\varepsilon_0.\varepsilon_r A}{d} V$

$\sigma = \frac{Q}{A}$

$\sigma = \frac{\varepsilon_0.\varepsilon_r}{d} V$

$= \varepsilon_0 \varepsilon_r .E$

$= 8.85 \times 10^{-12} \times 2.2 \times 3 \times 10^4$

$= 58.41 \times 10^{-8} \simeq 6 \times 10^{-7}$ c/m$^2$

**18.** In a large building, there are 15 bulbs of 40 W, 5 bulbs 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be :

(1) 8 A (2) 10 A
(3) 12 A (4) 14 A

**Sol.** **3**

$$P = \frac{V^2}{R}$$

$$R_1 = \frac{(200)^2}{40}$$

Total current

$$= \frac{V}{R_1} \times 15 + \frac{V}{R_2} \times 5 + \frac{V}{R_3} \times 5 + \frac{V}{R_4} \times 1$$

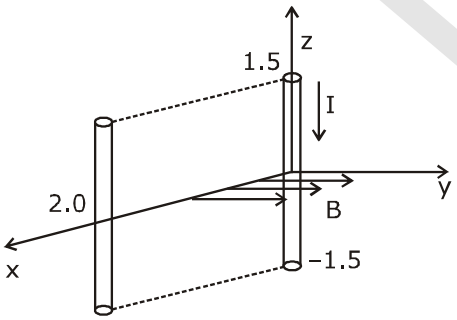
$R = V^2/P$

$$= \frac{40+15}{220} + \frac{100+5}{220} + \frac{80+5}{220} + \frac{1000}{220}$$

= 11.3626 A

Minimum value is 12 A.

**19.** A conducor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} \hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in $5 \times 10^{-3}$ s. Assume parallel motion along the x-axis.




(1) 1.57 W (2) 2.97 W
(3) 14.85 W (4) 29.7 W

**Sol.** **2**

$$w = \int 10 \times 3 \times 3 \times 10^{-4} e^{-0.2x} dx$$

$$= 4.5 \times 10^{-2} (1 - e^{-0.4})$$

$$P = \frac{w}{T} = 2.97$$

**20.** The coercivity of a small magent where the ferromagnet gets demagnetized is $3 \times 10^3$ A $m^{-1}$. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is :

(1) 30 mA (2) 60 mA
(3) 3 A (4) 6 A

**Sol.** **3**

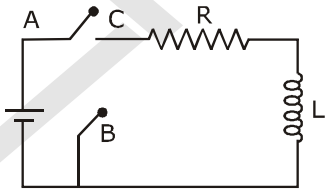
$$\text{Coercivity} \rightarrow \frac{B}{\mu_0} = 3 \times 10^3 \text{ A/m}$$

$$B = \mu_0 (3 \times 10^3) = \mu_0 nI$$

$$= \mu_0 \left(\frac{100}{0.1}\right) I$$

$$\Rightarrow I = \frac{3 \times 10^3 \times 0.1}{100} = 3 \text{ A}$$

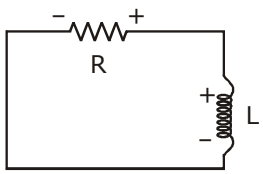
**21.** In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to :




(1) $\frac{e}{1-e}$ (2) 1 (3) –1 (4) $\frac{1-e}{e}$

**Sol.** **3**

When BC are joined only L & R are in circuit




Applying KVL, $V_R + V_L = 0$

$\Rightarrow V_R = -V_L$

$$\Rightarrow \frac{V_R}{V_L} = -1$$

**22.** During the propagation of electromagnetic waves in a medium :
(1) Electric energy density is double of the magnetic energy density.
(2) Electric energy density is half of the magnetic energy density.
(3) Electric energy density is equal to the magnetic energy density.
(4) Both electric and magnetic energy densities are zero.

**Sol.** **3**

In an EM wave

$$1/2\ \varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

Both are equal.

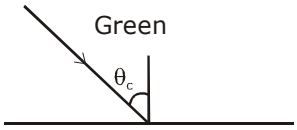
**23.** A thin convex lens made from crown grass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal length $f_1$ and $f_2$ respectively. The correct relation between the focal lengths is :
(1) $f_1 = f_2 < f$
(2) $f_1 > f$ and $f_2$ becomes negative
(3) $f_2 > f$ and $f_1$ becomes negative
(4) $f_1$ and $f_2$ both become negative

**Sol.** **2**

Since 4/3 < 3/2 thus $f_1 > f$
since 5/3 > 3/2 thus $f_2$ changes sign

**24.** A green light is incident from the water to the air - water interface at the critical angle ($\theta$). Select the correct statement.
(3) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.
(4) The entire spectrum of visible light will come out of the water at various angles to the normal.
(1) The entire spectrum of visible light will come out of the water at an angle of 90º to the normal.
(2) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

**Sol.** **2**




$\mu \propto \frac{1}{\lambda}$ $\quad f > f_{green}$

$\lambda < \lambda_{green}$

$\mu > \mu_{green} \rightarrow$ T.I.R

**25.** Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30º makes two beams appear equally bright. If the initial intensities of the two beams are $I_A$ and $I_B$ respectively, then $\frac{I_A}{I_B}$ equals :

(1) 3 (2) $\frac{3}{2}$ (3) 1 (4) $\frac{1}{3}$

**Sol.** **4**

$I_A \cos^2 30° = I_B \cos^2 60°$

$$\frac{I_A}{I_B} = \frac{\cos^2 60°}{\cos^2 30°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2}$$

$$= \frac{1/4}{3/4}$$

$= 1/3$

**26.** The radiation corresponding to 3 → 2 transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of $3 \times 10^{-4}$ T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to :
(1) 1.8 eV (2) 1.1 eV
(3) 0.8 eV (4) 1.6 eV

**Sol.** **2**

$$r = \frac{mv}{qB}$$

$$v = \frac{qBr}{m} = \frac{1.6\times10^{-19}\times3\times10^{-4}\times10\times10^{-3}}{9.1\times10^{-31}}$$

$= 5.27 \times 10^5$

$KE_{max} = 1/2\ mv^2$
$= 1/2 \times 9.1 \times 10^{-31} \times (5.27 \times 10^5)^2$
$= \frac{126.36 \times 10^{-21}}{1.6 \times 10^{-19}}$
$= 78.975 \times 10^{-2}$
$= 0.78$ eV

$$E = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= 13.6\left[\frac{1}{4} - \frac{1}{9}\right]$$

$= 1.88$ ev
$1.88 = 0.78 + \phi$
$\phi = 1.1$ eV

**27.** Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wave lengths of emitted radiation are $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$ respectively then approximately which one of the following is correct?
(1) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(2) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(3) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(4) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

**Sol. 3**

$$\frac{hc}{\lambda} = Rc\,z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} \propto z^2$$

$$\frac{1}{\lambda_1} \propto (1)^2,\ \frac{1}{\lambda_2} \propto (1)^2,\ \frac{1}{\lambda_3} \propto (2)^2,\ \frac{1}{\lambda_4} \propto (3)^2$$

$$\Rightarrow \lambda_1 : \lambda_2 : \lambda_3 : \lambda_4 = 1 : 1 : \frac{1}{4} : \frac{1}{9}$$

**28.** The forward biased diode connection is :

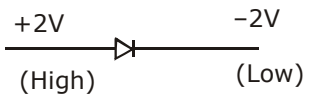
(1) 


(2) 


(3) 


(4) 


**Sol. 1**




Forward biased

**29.** Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists :

| List-I | | List-II | |
|---|---|---|---|
| (a) | Infrared waves | (i) | To treat muscular strain |
| (b) | Radio waves | (ii) | For broadcasting |
| (c) | X-rays | (iii) | To detect fracture of bones |
| (d) | Ultraviolet rays | (iv) | Absorbed by the ozone layer of the atmosphere |

| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| (1) | (iv) | (iii) | (ii) | (i) |
| (2) | (i) | (ii) | (iv) | (iii) |
| (3) | (iii) | (ii) | (i) | (iv) |
| (4) | (i) | (ii) | (iii) | (iv) |

**Sol. 4**

Knowledge based

**30.** A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(1) A meter scale
(2) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(3) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(4) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

**Sol. 2**

The vernier calliper has least count $\frac{0.1\text{cm}}{10}$

$= 0.01$ cm
Also 3.50 cm has smallest reading
0.01 cm

# [CHEMISTRY]

**31.** The correct set of four quantum numbers for the valence electrons of rubidium atom (Z = 37) is ?
(1) 5, 0, 0, + 1/2
(2) 5, 0, 1, + 1/2
(3) 5, 1, 1 + 1/2
(4) 5, 1, 0, + 1/2

**Sol.** (1)

**32.** If Z is a compressibility factor, van der waals equation at low pressure can be written as ?

(1) $Z = 1 - \frac{Pb}{RT}$

(2) $Z = 1 - \frac{a}{VRT}$

(3) $Z = 1 + \frac{RT}{Pb}$

(4) $Z = 1 + \frac{Pb}{RT}$

**Sol.** (2)
According to vander waal's equation for one mole gas

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

at low pressure volume will be high
$\therefore$ b can be neglected,
so,

$$\left(P + \frac{a}{V^2}\right)(V) = RT$$

$$\Rightarrow PV + \frac{a}{V} = RT$$

$$\Rightarrow Z = \frac{PV}{RT} = 1 - \frac{a}{VRT}$$

**33.** CsCl crystallises in body centred cubic lattice. If 'a' is its edge length then which of the following expressions is correct ?

(1) $r_{Cs^+} + r_{Cl^-} = 3a$

(2) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$

(3) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

(4) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$

**Sol.** 3
$Cl^-$ forms SC
$Cs^+$ goes to centre of the cube

$$\therefore \quad r_+ + r_- = \frac{\sqrt{3}a}{2}$$

**34.** For the estimation of nitrogen, 1.4 g of an organic compound was digested by kjeldahl method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $\frac{M}{10}$ sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is ?
(1) 3 % (2) 10 % (3) 6 % (4) 5 %

**Sol.** (2)
1.4 gm $\rightarrow$ $NH_3$

$$\equiv 60mL\,\frac{M}{10}H_2SO_4 + 20mL\,\frac{M}{10}NaOH$$

Let's Assume x % N

$$\frac{1.4 \times \frac{x}{100}}{14} \times 1 + \frac{1}{10} \times \frac{20}{1000} = \frac{1}{10} \times \frac{60}{1000} \times 2$$

$$x = (0.012 - 0.002) \times \frac{14 \times 100}{1.4}$$

$$= 10\ \%$$

**35.** Resistance of 0.2 M solution of an electrolyte is 50 $\Omega$. The specific conductance of the solution is 1.4 S $m^{-1}$. The resistance of 0.5 M solution of the same electrolyte is 280 $\Omega$. The molar conductivity of 0.5 M solution of the electrolyte in S $m^2$ $mol^{-1}$ is ?
(1) $5 \times 10^{-4}$
(2) $5 \times 10^{2}$
(3) $5 \times 10^{3}$
(4) $5 \times 10^{-3}$

**Sol.** 1

$$R = \rho\,\frac{\ell}{a}$$

$$50 = \frac{1}{1.4} \times \frac{\ell}{a}$$

$$\frac{\ell}{a} = 70\,m^{-1}$$

$$R = \rho\,\frac{\ell}{a}$$

$280 = \frac{1}{\kappa} \times 70$

$\kappa = 0.25\ Sm^{-1}$

$= 0.25 \times 10^{-2}\ \Omega^{-1}\text{-}cm^{-1}$

$\Lambda_m = \frac{1000 \times \kappa}{c}$

$= \frac{1000 \times 0.25 \times 10^{-2}}{0.5}$

$= 5\ \Omega^{-1}\text{-}cm^2\ mole^{-1}$

$= 5 \times 10^{-4}\ S\text{-}m^2\ mol^{-1}$

**36.** For complete combustion of ethanol, $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ $mol^{-1}$ at 25ºC. Assuming ideality the Enthalpy of combustion, $\Delta_C H$, for the reaction will be :
($R = 8.314$ kJ $mol^{-1}$)
(1) $-1366.95$ kJ $mol^{-1}$
(2) $-1350.50$ kJ $mol^{-1}$
(3) $-1460.50$ kJ $mol^{-1}$
(4) $-1361.95$ kJ $mol^{-1}$

**Sol.** (1)

$\Delta H = \Delta U + \Delta n_g RT$

$= -1364.47 + (-1) \times 8.314 \times 10^{-3} \times 298$

$= -1364.47 - 2.47$

$= -1366.95$ kJ $mol^{-1}$

**37.** the equivalent conductance of NaCl at concentration C and at infinite dilution are $\lambda_C$ and $\lambda_\infty$ respectively. The correct relationship between $\lambda_C$ and $\lambda_\infty$ is given as ?
(where the constant B is positive)
(1) $\lambda_C = \lambda_\infty + (B)\ C$
(2) $\lambda_C = \lambda_\infty + (B)\ \sqrt{C}$
(3) $\lambda_C = \lambda_\infty - (B)\ \sqrt{C}$
(4) $\lambda_C = \lambda_\infty - (B)\ C$

**Sol.** (3)

$\lambda_c = \lambda_\infty - B\sqrt{C}$

**38.** Consider separate solutions of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M $KBr(aq)$ and 0.125 M $Na_3PO_4(aq)$ at 25ºC. Which statement is true about these solutions, assuming all salts to be strong electrolytes?
(1) they all the have the same osmotic pressure
(2) 0.500 M $C_2H_5OH(aq)$ has the highest osmotic pressure.
(3) 0.125 M $Na_3PO_4(aq)$ has the highest osmotic pressure.
(4) 0.100 M $Mg_3(PO_4)_2(aq)$ has the highest osmotic pressure.

**Sol.** 1

| | i | ic |
|---|---|---|
| 0.5 M $C_2H_5OH$ (aq.) | 1 | 0.5 |
| 0.10 M $Mg_3(PO_4)_2$ | 5 | 0.5 |
| 0.25 M KBr | 2 | 0.5 |
| 0.125 M $Na_3PO_4$ | 4 | 0.5 |

All have same colligative prop.

**39.** For the reaction $SO_{2(s)} + \frac{1}{2}O_2(g) \rightleftharpoons SO_{3(g)}$, if $K_p = K_C(RT)^x$ where the symbols have usual meaning then the value of x is: (assuming ideality) ?
(1) 1/2
(2) – 1/2
(3) –1
(4) 1

**Sol.** (2)

$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$Kp = K_c(RT)^{\Delta ng}$

$\Delta n_g = 1 - (1 + \frac{1}{2}) = -\frac{1}{2}$

$Kp = K_c(RT)^{-1/2}$

**40.** For the non-stoichiometre reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate expreiments, all at 298 K

| Initial Concentration (A) | Initial Concentration (B) | Initial rate of formation of C ($mol L^{-} S^{-}$) |
|---|---|---|
| 0.1M | 0.1M | $1.2 \times 10^{-3}$ |
| 0.1M | 0.2M | $1.2 \times 10^{-3}$ |
| 0.2M | 0.1M | $2.4 \times 10^{-3}$ |

the rate law for the formation of C is

(1) $\frac{dc}{dt} = k[A][B]^2$

(2) $\frac{dc}{dt} = k[A]^2[B]$

(3) $\frac{dc}{dt} = k[A][B]$

(4) $\frac{dc}{dt} = k[A]$

**Sol.** (4)

$2A + B \rightarrow C + D$

(1) & (3) $\Rightarrow$ $[A]^1$

(1) & (2) $\Rightarrow$ $[B]^o$

$\therefore$ $r = k[A]^1$

**41.** Among the following oxoacids, the correct decreasing order of acid strength is :

(1) $HOCl > HClO_2 > HClO_3 > HClO_4$

(2) $HClO_2 > HClO_4 > HClO_3 > HOCl$

(3) $HClO_4 > HClO_3 > HClO_2 > HOCl$

(4) $HClO_4 > HOCl > HClO_2 > HClO_3$

**Sol.** (3)

**42.** The metal that connnot be obtained by electrolysis of an aqueous solution of its salts is

(1) Cu

(2) Ca

(3) Ag

(4) Cr

**Sol.** 2

$E^o_{H_2o/H_2} > E^o_{Ca^{2+}/Ca}$

Ca is more electro +ve so aqueous solutions cannot be use.

**43.** The octahedral complex of a metal ion $M^{3+}$ with four monodentate ligands $L_1$, $L_2$, $L_3$ and $L_4$ absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is ?

(1) $L_3 < L_2 < L_4 < L_1$

(2) $L_1 < L_3 < L_2 < L_4$

(3) $L_4 < L_3 < L_2 < L_1$

(4) $L_1 < L_2 < L_4 < L_3$

**Sol.** (2)

(a) According to spectro chemical series more absorption frequency stronger the ligand

$$VIBGYOR \xrightarrow[E\downarrow]{}$$

$L_1 < L_3 < L_2 < L_4$

**44.** Which one of the following properties is not shown by NO?

(1) It is diamagnetic in gaseous state

(2) It's bond order is 2.5

(3) It combines with oxygen to form nitrogen dioxide

(4) It is a neutral oxide

**Sol.** (1)

(1) $NO + 1/2\ O_2 \rightarrow NO_2$

(2) bond Order of NO is 2.5

(3) It is paramagnetic in gaseous state

(4) It is neutral oxide

**45.** In which of the following reactions $H_2O_2$ acts as a reducing agent?

(a) $H_2O_2 + 2H^+ + 2e^- \rightarrow 2H_2O$

(b) $H_2O_2 - 2e^- \rightarrow O_2 + 2H^+$

(c) $H_2O_2 + 2e^- \rightarrow 2OH^-$

(d) $H_2O_2 + 2OH^- - 2e^- \rightarrow O_2 + 2H_2O$

(1) (a), (c)

(2) (c), (d)

(3) (a), (b)

(4) (b), (d)

**Sol.** 4

As a reducing agent

$H_2O_2^{-1} \longrightarrow \overset{0}{O_2} + 2e^-$

(b) & (d)

**46.** The correct statement for the molecule, $CsI_3$, is ?
(1) It contains $Cs^{3+}$ and $I^-$ ions.
(2) It contains $Cs^+$ and $I_3^-$ ions.
(3) It is a covalent molecule.
(4) It contains $Cs^+$, $I^-$ and lattice $I_2$ molecule.

**Sol.** (2)

**47.** The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecule is :
(1) 1 : 8 (2) 7 : 32
(3) 1 : 4 (4) 3 : 16

**Sol.** (2)

| | $O_2$ | : | $N_2$ |
|---|---|---|---|
| mass | 1 | : | 4 |
| Moles | $\frac{1}{32}$ | : | $\frac{4}{28}$ |
| | 7 | : | 32 |

**48.** Given below are the half - cell reactions
$Mn^{2+} + 2e^- \rightarrow Mn$ : $E^o = -1.18$ V
$2(Mn^3 + e^- \rightarrow Mn^{2+})$ : $E^o = +1.51$ V
The $E^o$ for $3Mn^{2+} \rightarrow Mn + 2Mn^{3+}$ will be
(1) – 2.69 V ; the reaction will not occur
(2) – 0.33 V ; the reaction will occur
(3) – 0.33 V ; the reaction will not occur
(4) – 2.69 V ; the reaction will occur

**Sol.** (1)
$Mn^{2+} + 2e^- \rightarrow Mn$ ; $E^{o\,=} -1.18$ V
$2Mn^{3+} + 2e^- \rightarrow 2Mn^{2+}$ ; $E^o = +1.51$ V
(1) – (2)
$Mn^{2+} + 2e^- + 2Mn^{2+} \rightarrow 2Mn^{3+} + 2e^{-1} + Mn$
$E^{o\,=} -1.18 - 1.51$ ($\therefore$ n is same)
$\Rightarrow E^o = -2.69$ V
(3) The reaction will nor occur

**49.** Which series of reactions correctly represents chemical relations related to iron and its compound ?
(1) $Fe \xrightarrow{Cl_2,heat} FeCl_3 \xrightarrow{heat,\ air} FeCl_2 \xrightarrow{Zn} Fe$
(2) $Fe \xrightarrow{dil\ H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,O_2} Fe(SO_4)_3 \xrightarrow{heat} Fe$
(3) $Fe \xrightarrow{O_2,\ heat} FeO \xrightarrow{dil\ H_2SO_4} FeSO_4 \xrightarrow{heat} Fe$
(4) $Fe \xrightarrow{O_2,\ heat} Fe_3O_4 \xrightarrow{CO,\ 600^oC} FeO \xrightarrow{CO,\ 700^oC} Fe$

**Sol.** (4)
This reaction is used in blast furnace.

**50.** The equation which is balanced and represents the correct product(s) is?
(1) $[Mg(H_2O)_6]^{2+} + (EDTA)^{4-} \xrightarrow{excess\ NaOH} [Mg(EDTA)]^{2+} + 6H_2O$
(2) $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow CO^{2+} + 5NH_4^+ + Cl^-$
(3) $Li_2O + 2KCl \rightarrow 2LiCl + K_2O$
(4) $CuSO_4 + 4KCN \rightarrow K_2[Cu(CN)_4] + K_2SO_4$

**Sol.** (2)
In this reaction $H^+$ ion attacks ammonia molecule and hence decomposes the complex which results information of amonia

**51.** In $S_N2$ reactions, the correct order of reactivity for the following compounds ?
$CH_3Cl$, $CH_3CH_2Cl$, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is
(1) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl$
(2) $CH_3Cl > (CH_3)_2CHCl > CH_3CH_2Cl > (CH_3)_3CCl$
(3) $(CH_3)_2CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3CCl$
(4) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

**Sol.** (2)
$S_N2$

$$\text{Reactivity of } SN^2 \propto \frac{1}{\text{Steric hindrance}}$$

Order of reactivity towards $S_N2$.
$CH_3Cl > CH_3{-}CH_2{-}Cl > (CH_3)_2CH{-}Cl > (CH_3)_3C{-}Cl$

**52.** On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is :
(1) An alkyl cyanide
(2) An alkanediol
(3) An alkanol
(4) An alkyl isocyanide

**Sol.** (4)

$R—NH_2 + CHCl_3 + KOH \xrightarrow{\Delta} R–NC$

$1^o$-amine chloroform Alkyl isocyanide

it is carbyl amine reaction or isocyanide test.

**53.** The most suitable reagent for the conversion of $R - CH_2 - OH \rightarrow R - CHO$ is :
(1) $CrO_3$
(2) $K_2Cr_2O_7$
(3) $KMnO_4$
(4) PCC (Pyridininum Chlorochromate)

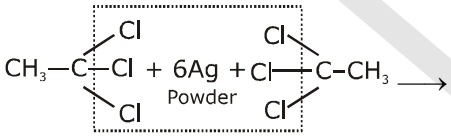
**Sol.** (4)

$R—CH_2—OH \xrightarrow{PCC} R—\overset{O}{\overset{\|}{C}}—H$

PCC (pyridinium chloro chromate) is mild oxidising agent.

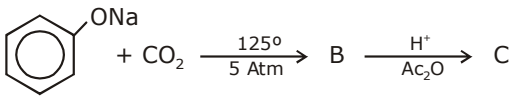
**54.** The major organic compound formed by the reaction of 1,1,1-trichloroethane with silver powder is ?
(1) Acetylene (2) 2 - Butene
(3) 2 Butyne (4) Ethene

**Sol.** (3)

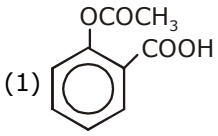


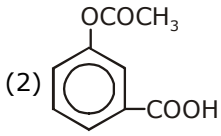

$CH_3—C\equiv C—CH_3 + 6AgCl$
2-butyne

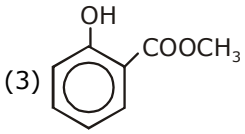
**55.** Sodium phenoxide when heated with $CO_2$ under pressure at 125° C yields a product which on acetylation produces C ?

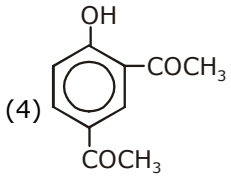



The major product C would be;

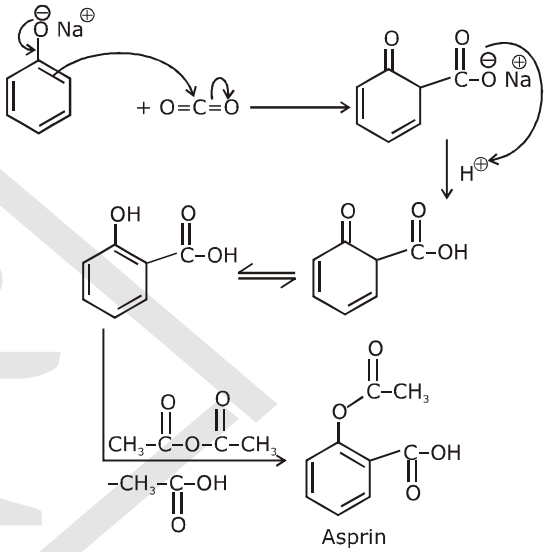
(1) 


(2) 


(3) 

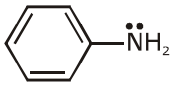

(4) 


**Sol.** (1)




**56.** Considering the basic strength of amines in aqueous solution, which one has the smallest $pK_b$ value?
(1) $(CH_3)_2NH$ (2) $C_6H_5NH_2$
(3) $(CH_3)_3N$ (4) $CH_3NH_2$

**Sol.** (1)



Delocalised lone pair electrons.

(least Basic)

Order of basic strength in aqueous solution
$\Rightarrow 2^o > 1^o > 3^o$ amine > aniline.

$$\text{Basic strength} \propto K_b \propto \frac{1}{pk_b}$$

$(CH_3)_2NH$ is most basic so it has smallest $pK_b$.

**57.** For which of the following molecule significant $\mu \neq 0$ ?

(a) 

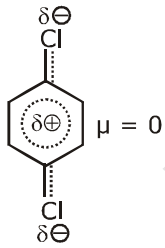
(b) 

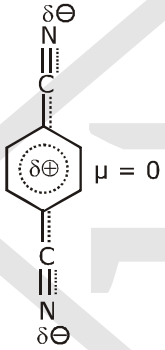
(c) 

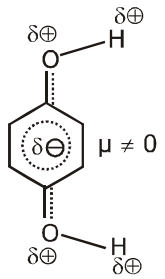
(d) 

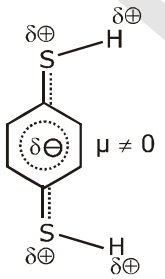
(1) Only (c)
(2) (a) and (b)
(3) Only (a)
(4) (c) and (d)

**Sol.** (4)

(a) 

(b) 

(c) 

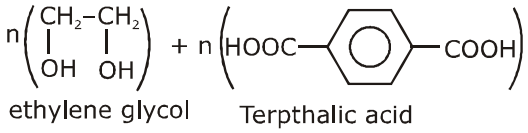
(d) 

**58.** Which one is classified as a condensation polymer ?

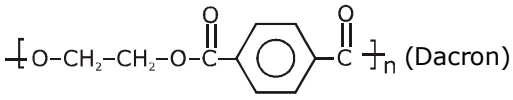
(1) Dacron (2) Acrylonitrile
(3) Teflon (4) Neoprene

**Sol.** (1)

Terylene or Dacron




$\downarrow -nH_2O$ (Condensation)

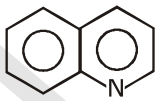

(Dacron)

Other than dacron are addition polymers.

**59.** Which one of the following bases is not present in DNA?

(1) Quinoline (2) Thymine
(3) Cytosine (4) Adenine

**Sol.** (1)


(Quinoline)

Quinoline is not present into DNA.
Cytosin, Thymine and Adenine are present in DNA.

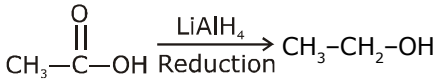
**60.** In the reaction,

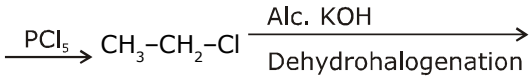
$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{\text{Alc. KOH}} C,$

the product C is ?

(1) Acetaldehyde
(2) Acetyl chloride
(3) Ethylene
(4) Acetylene

**Sol.** (3)




$CH_2=CH_2$ (Ethylene)

## SECTION – A
## Single Correct

**61.** If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to :

(1) X (2) Y

(3) N (4) Y – X

**Sol.** **[2]**

$x : \quad 4^n - 3n - 1$

$x : \quad (1+3)^n - 3n - 1$

$x : \quad 1 + 3n + {}^nC_2 3^2 + {}^nC_3 3^3 + \ldots\ldots - 3n - 1$

$x : \quad 9\{{}^nC_2 + {}^nC_3 3 + \ldots\ldots\}$ ...(1)

& $y : \quad 9(n-1)$ ...(2)

From (1) & (2) multiply of 9 Hence $X \cup Y = Y$

**62.** If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$ :

(1) is strictly greater than $\frac{5}{2}$

(2) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(3) is equal to $\frac{5}{2}$

(4) lies in the interval (1, 2)

**Sol.** **[4]**

$|z| \geq 2$

$$\left||z| - \frac{1}{2}\right| \leq \left|z + \frac{1}{2}\right| \leq |z| + \frac{1}{2}$$

$$\left|z + \frac{1}{2}\right|_{min} = \frac{3}{2}$$

**63.** If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval :

(1) (–2, –1)

(2) $(-\infty, -2) \cup (2, \infty)$

(3) $(-1, 0) \cup (0, 1)$ (4) (1, 2)

**Sol.** **[3]**

$x - [x] = \{x\} = t \in [0, 1)$

$-3t^2 + 2t + a^2 = 0$

$\Rightarrow a^2 = 3t^2 - 2t \in [0, 1)$

Since eqn cannot have integral root : $t \neq 0$

$\Rightarrow \quad a^2 \in (0, 1)$

$\Rightarrow \quad a \in (-1, 0) \cup (0, 1)$

**64.** Let $\alpha$ and $\beta$ be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

**Sol.** **[2]**

Given $\alpha + \beta = -\frac{q}{p}$ $\quad \alpha\beta = \frac{r}{p}$

p, q, r $\rightarrow$ A. P. , then

$q - p = r - q \Rightarrow \boxed{2q = r + p}$ ...(1)

again $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$$\frac{\alpha + \beta}{\alpha\beta} = 4$$

$\boxed{-q = 4r}$ ...(2)

(1) & (2) gives

$-8r = r + p$

$\Rightarrow \boxed{p = -9r}$ ...(3)

Now, $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$$= \frac{\sqrt{q^2 - 4pr}}{|p|}$$

$$= \frac{\sqrt{16r^2 + 36r^2}}{|9r|}$$

$$= \frac{\sqrt{52}}{9} = \frac{2\sqrt{13}}{9}$$

**65.** If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2 (1-\beta)^2 (\alpha-\beta)^2$, then K is equal to :

(1) 1 (2) −1

(3) $\alpha\beta$ (4) $\frac{1}{\alpha\beta}$

**Sol.** **[1]**

$f(n) = \alpha^n + \beta^n$

$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$= (1-\alpha)^2 (\alpha-\beta)^2 (\beta-1)^2$

$\Rightarrow \boxed{k = 1}$

**66.** If A is an 3 × 3 non - singular matrix such that AA' = A'A and B = $A^{-1}$ A', then BB' equals:

(1) $B^{-1}$ (2) $(B^{-1})'$

(3) I + B (4) I

**Sol.** **[4]**

AA' = A'A

B = $A^{-1}A'$

B' = $A(A^{-1})'$

B . B' = $A^{-1}(A'A)(A^{-1})'$

= $A^{-1}(AA')(A^{-1})^1$

= $(A^{-1}A)(A')(A')^{-1}$

= I

**67.** If the coefficients of $x^3$ and $x^4$ in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to :

(1) $\left(14, \frac{272}{3}\right)$ (2) $\left(16, \frac{272}{3}\right)$

(3) $\left(16, \frac{251}{3}\right)$ (4) $\left(14, \frac{251}{3}\right)$

**Sol.** **[2]**

$(1 + ax + bx^2)(1 - 2x)^{18}$

coeff. of $x^3 = {}^{18}C_3 (-2)^3 + a.\,{}^{18}C_2 (-2)^2 + b.\,{}^{18}C_1 (-2)^1$

$= \boxed{-6528 + a612 - 36b = 0}$

coeff. of $x^4 = {}^{18}C_4 (-2)^4 + a.\,{}^{18}C_3 (-2)^3 + b\,{}^{18}C_2 (-2)^2$

$= \boxed{48960 + a.(-6528) + b612 = 0}$

We get $(16, \frac{272}{3})$

**68.** If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + \ldots\ldots + 10(11)^9 = k(10)^9$, then k is equal to

(1) 100 (2) 110

(3) $\frac{121}{10}$ (4) $\frac{441}{100}$

**Sol.** **[1]**

$10^9 + 2.11^1\, 10^8 + 3.11^2\, 10^7$ ---- $+ 10.11^9$

$$S = 10^9 \left(1 + 2\left(\frac{11}{10}\right)^1 + 3.\left(\frac{11}{10}\right)^2 + \ldots\ldots + 10\left(\frac{11}{10}\right)^9\right)$$

$$S\left(\frac{11}{10}\right)^1 = 10^9 \left(\left(\frac{11}{10}\right)^1 + 2\left(\frac{11}{10}\right)^2 + 3\left(\frac{11}{10}\right)^3 + \ldots\ldots 10\left(\frac{11}{10}\right)^{10}\right)$$

$$s\left(1 - \frac{11}{10}\right) = 10^9 \left(1 + \left(\frac{11}{10}\right)^1 + \left(\frac{11}{10}\right)^2 + - - - - + \left(\frac{11}{10}\right)^9 - 10\left(\frac{11}{10}\right)^{10}\right)$$

$$= 10^9 \left(\frac{1\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1} - 10\left(\frac{11}{10}\right)^{10}\right)$$

$$S\left(\frac{-1}{10}\right) = 10^9\left(\left(\left(\frac{11}{10}\right)^{10} - 1\right)10 - 10\left(\frac{11}{10}\right)^{10}\right)$$

$$= 10^9\, 10\left(\left(\frac{11}{10}\right)^{10} - 1 - \left(\frac{11}{10}\right)^{10}\right)$$

$$S \cdot \frac{-1}{10} = 10^9\,(-10) \Rightarrow S = 100.\ 10^9$$

So K = 100

**69.** Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is :

(1) $2 - \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $\sqrt{2} + \sqrt{3}$ (4) $3 + \sqrt{2}$

**Sol.** **[2]**

a, ar, $ar^2$ in GP

a, 2ar, $ar^2$ in AP

$$2ar = \frac{a + ar^2}{2}$$

$4r = 1 + r^2$

$r^2 - 4r + 1 = 0$

$$r = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$r = \frac{4 \pm 2\sqrt{3}}{2}$$

$$r = 2 \pm \sqrt{3}$$

so $r = 2 + \sqrt{3}$ as In GP

**70.** $\lim\limits_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to :

(1) $-\pi$ (2) $\pi$

(3) $\frac{\pi}{2}$ (4) 1

**Sol.** **[2]**

$$\lim_{x\to 0} \frac{\sin(\pi - \pi c' x)}{x^2}$$

$$\lim_{x\to 0}\left[\frac{\sin(\pi - \pi c^2 x)}{(\pi - \pi c^2 x)}\right]\pi\left[\frac{(1 - cx)}{x^2}\right](1 + cx)$$

$$= 1 \cdot \pi \cdot \frac{1}{2} \cdot 2$$

$$= \boxed{\pi}$$

**71.** If g is the inverse of a function f and $f'(x) = \frac{1}{1 + x^5}$, then g'(x) is equal to :

(1) $\frac{1}{1 + \{g(x)\}^5}$ (2) $1 + \{g(x)\}^5$

(3) $1 + x^5$ (4) $5x^4$

**Sol.** **[2]**

$f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(x) = \frac{1}{\frac{1}{1 + (g(x))^5}}$$

$1 + (g(x))^5$

**72.** If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some $c \in ]0, 1[$ :

(1) f'(c) = g'(c)

(2) f'(c) = 2g'(c)

(3) 2f'(c) = g'(c)

(4) 2f'(c) = 3g'(c)

**Sol.** **[2]**

2g'(c) = f'(c)

$$= 2\left(\frac{g(1) - g(0)}{1 - 0}\right) = \left(\frac{f(1) - f(0)}{1 - 0}\right)$$

$$= 2\left(\frac{2 - 0}{1}\right) = \left(\frac{6 - 2}{1}\right) \Rightarrow 4 = 4$$

**73.** If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then :

(1) $\alpha = 2, \beta = -\frac{1}{2}$ (2) $\alpha = 2, \beta = \frac{1}{2}$

(3) $\alpha = -6, \beta = \frac{1}{2}$ (4) $\alpha = -6, \beta = -\frac{1}{2}$

**Sol.** **[1]**

$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

at $x = -1, 2 \Rightarrow f'(x) = 0$

$\Rightarrow \quad -\alpha - 2\beta + 1 = 0 \quad ....(1)$

$\frac{\alpha}{2} + 4\beta + 1 = 0 \quad .... (2)$

$\alpha = 2, \beta = -\frac{1}{2}$

**74.** The integral $\int \left(1 + x - \frac{1}{x}\right) e^{x+\frac{1}{x}}\, dx$ is equal to:

(1) $(x+1)e^{x+\frac{1}{x}} + c$ (2) $-xe^{x+\frac{1}{x}} + c$

(3) $(x-1)\, e^{x+\frac{1}{x}} + c$ (4) $xe^{x+\frac{1}{x}} + c$

**Sol.** **[4]**

$\int \left(e^{x+\frac{1}{x}} + \left(x - \frac{1}{x}\right)e^{x+\frac{1}{x}}\right) dx \quad ...(1)$

$e^{x+\frac{1}{x}} = f(x)$

$e^{x+\frac{1}{x}} \left(1 - \frac{1}{x^2}\right) dx = f'(x)$

$\Rightarrow \int \left( \underbrace{e^{x+\frac{1}{x}}}_{f(x)} + \underbrace{x.e^{x+\frac{1}{x}}\left(1 - \frac{1}{x^2}\right)}_{xf'(x)} \right) dx$

$\Rightarrow x\, f(x) = x\, .\, e^{x+\frac{1}{x}} + C$

**75.** The integral $\int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\, dx$ equals :

(1) $4\sqrt{3} - 4$ (2) $4\sqrt{3} - 4 - \frac{\pi}{3}$

(3) $\pi - 4$ (4) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

**Sol.** **[2]**

$\int_0^{\pi} \left|\left(1 - 2\sin\frac{x}{2}\right)\right| dx$

$= \int_0^{\frac{\pi}{3}} \left|\left(1 - 2\sin\frac{x}{2}\right)\right| dx - \int_{\frac{\pi}{3}}^{\pi} \left|\left(1 - 2\sin\frac{x}{2}\right)\right| dx$

$= \left(x + 4\cos\frac{x}{2}\right)_0^{\frac{\pi}{3}} - \left(x + 4\cos\frac{x}{2}\right)_{\frac{\pi}{3}}^{\pi}$

$= \frac{\pi}{3} + 4\cos\frac{\pi}{6} - 0 - 4 - \left(\pi + 4\cos\frac{\pi}{2} - \frac{\pi}{3} - 4\cos\frac{\pi}{6}\right)$

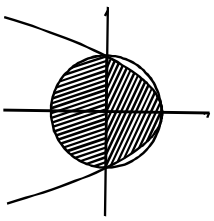
$= -\frac{\pi}{3} + 4\sqrt{3} - 4$

**76.** The area of the region described by $A = \{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\}$ is :

(1) $\frac{\pi}{2} - \frac{2}{3}$ (2) $\frac{\pi}{2} + \frac{2}{3}$

(3) $\frac{\pi}{2} + \frac{4}{3}$ (4) $\frac{\pi}{2} - \frac{4}{3}$

**Sol.** **[3]**



$A_1 = 2\left|\int_0^1 \sqrt{1-x}\,dx\right|$

$A_1 = 2\left|\int_1^0 2t^2 dt\right|$

$A_1 = 4 \cdot \frac{1}{3}$

$\boxed{\text{Area} = \frac{\pi}{2} + \frac{4}{3}}$

**77.** Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200.$

If p(0) = 100, then p(t) equals :

(1) $600 - 500\,e^{t/2}$ (2) $400 - 300\,e^{-t/2}$

(3) $400 - 300\,e^{t/2}$ (4) $300 - 200\,e^{-t/2}$

**Sol.** **[3]**

$\int \frac{2dp(t)}{p(t)-400} = \int dt$

$2\log|p(t) - 400| = t + c$ ...(1)

$t = 0, p = 100$

$2\log(300) = C$

From (1)

$2\log|p(t) - 400| = t + 2\log(300)$

$|p(t) - 400| = e^{t/2} \cdot e^{\log(300)}$

$\boxed{p(t) = 400 - e^{t/2}(300)}$

**78.** Let PS be the median of the triangle with vertices P(2, 2), Q(6, –1) and R(7, 3). The equation of the line passing through (1, –1) and parallel to PS is :

(1) $4x + 7y + 3 = 0$ (2) $2x - 9y - 11 = 0$

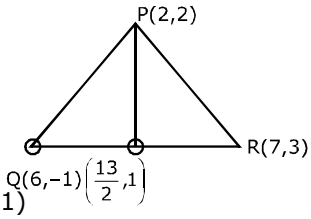
(3) $4x - 7y - 11 = 0$ (4) $2x + 9y + 7 = 0$

**Sol.** **[4]**

Slope of PS

$m = \frac{2-1}{2-\frac{13}{2}} = \frac{1}{-\frac{9}{2}}$

$m = -\frac{2}{9}$




Eqⁿ of line

$y + 1 = -\frac{2}{9}(x - 1)$

$9y + 9 = 2x + 2$

$2x + 9y + 7 = 0$

**79.** Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then :

(1) $3bc - 2ad = 0$ (2) $3bc + 2ad = 0$

(3) $2bc - 3ad = 0$ (4) $2bc + 3ad = 0$

**Sol.** **[1]**

$4ax + 2ay + c = 0$

$5bx + 2by + d = 0$

let intersection point be $(\alpha, -\alpha)$

$4a\alpha - 2a\alpha + c = 0$

$\alpha = -\frac{c}{2a}$ ....(1)

$5b\alpha - 2b\alpha + d = 0$

$\alpha = -\frac{d}{3b}$ ....(2)

$-\left(-\frac{c}{2a}\right) = -\frac{d}{3b}$

$\boxed{3bc = 2ad}$

$3bc - 2ad = 0$

**80.** The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is :
(1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$
(2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$
(3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$
(4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

**Sol.** **[1]**

Ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$

If foot of $\perp^r$ is (h,k) then equation of line $xh + ky = k^2 + h^2$

is $y = \left(\frac{-h}{k}\right) x + \left(\frac{k^2 + h^2}{k}\right)$

Now if this tangent then condition of tangency
$c^2 = a^2m^2 + b^2$

$$\left(\frac{k^2 + h^2}{k}\right) = 6\left(\frac{-h}{k}\right)^2 + 2$$

$$\frac{\left(k^2 + h^2\right)^2}{k^2} = \frac{6h^2}{k^2} + 2$$

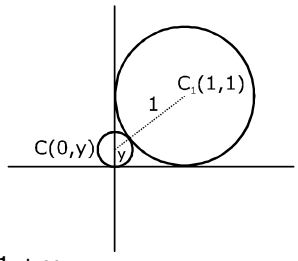
$(k^2 + h^2)^2 = 6h^2 + 2k^2$
$(x^2 + y^2)^2 = 6x^2 + 2y^2$

**81.** Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

**Sol.** **[2]**




$C_1C = 1 + y$
$(1 - 0)^2 + (1 - y)^2 = (1 + y)^2$
$1 + 1 - 2y + y = 1 + y^2 + 2y$
$4y = 1$

$$\boxed{y = \frac{1}{4}}$$

**82.** The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is :

(1) $\frac{1}{8}$ (2) $\frac{2}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{2}$

**Sol.** **[3]**

$y^2 = 4x$, $x^2 = -32y$

Equation of Tangent, Equation of Tangent

$y = mx + \frac{1}{m}$, $y = mx + 8m^2$

$$\frac{1}{m} = 8m^2$$

$$m = \frac{1}{2}$$

**83.** The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line :

(1) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

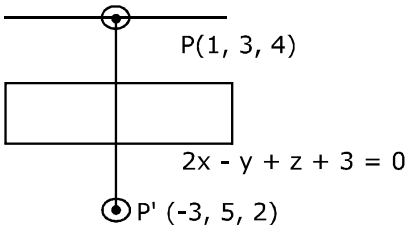
(2) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(3) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(4) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

**Sol.** **[3]**

$$\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$$




$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=-2\frac{(2-3+4+3)}{4+1+1}$$

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=-2$$

x= 1 – 4, y = 3 + 2, z = 4 – 2
P' (–3, 5 ,2)
Line is || to the plane
Equation of image line

$$\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$$

**84.** The angle between the lines whose direction cosines satisfy the equations $\ell + m + n = 0$ and $\ell^2 = m^2 + n^2$ is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

**Sol.** **[3]**

$l + m + n = 0$
$l^2 - m^2 - n^2 = 0$
$(m + n)^2 - m^2 - n^2 = 0$
$mn = 0$
$m = 0$ or $n = 0$
$1.l + 1.m + 1.n = 0$, $1.l + 1.m + 1.n = 0$
$0.l + 1.m + 0.n = 0$, $0.l + 0.m + 1.n = 0$

$$\frac{l}{\begin{vmatrix}1&1\\1&0\end{vmatrix}}=\frac{m}{\begin{vmatrix}1&1\\0&0\end{vmatrix}}=\frac{n}{\begin{vmatrix}1&1\\0&1\end{vmatrix}},\quad \frac{l}{\begin{vmatrix}1&1\\0&1\end{vmatrix}}=\frac{m}{\begin{vmatrix}1&1\\1&0\end{vmatrix}}=\frac{n}{\begin{vmatrix}1&1\\0&0\end{vmatrix}}$$

$$\frac{l}{-1}=\frac{m}{0}=\frac{n}{1} \quad \text{or} \quad \frac{l}{1}=\frac{m}{-1}=\frac{n}{0}$$

$$\text{Cos}\,\theta = \pm\left(\frac{-1+0+0}{\sqrt{2}\times\sqrt{2}}\right)$$

$$\text{Cos}\,\theta = \pm\frac{1}{2} \qquad \theta = \frac{\pi}{3}$$

**85.** If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then $\lambda$ is equal to :

(1) 0 (2) 1
(3) 2 (4) 3

**Sol.** **[2]**

Clearly = 1

**86.** Let A and B be two events such that

$P(\overline{A\cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and

$P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are :

(1) independent but not equally likely
(2) independent and equally likely
(3) mutually exclusive and independent.
(4) equally likely but not independent

**Sol.** **[1]**

$P(\overline{A\cup B}) = \frac{1}{6}$ $\qquad P(A\cap B) = \frac{1}{4}$

$P(\overline{A}) = \frac{1}{4}$

clearly $P(A) = \frac{3}{4}$

and $P(A \cup B) = \frac{5}{6}$

$P(A) + P(B) - P(A \cap B) = \frac{5}{6}$

$\frac{3}{4} + P(B) - \frac{1}{4} = \frac{5}{6}$

$P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$

$P(A \cap B) = \frac{1}{4}$

$P(A) \cdot P(B) = \frac{3}{4}\frac{1}{3} = \frac{1}{4}$ so independent

$P(A) \neq P(B)$ so not equally likely
so option (3)

**87.** The variance of first 50 even natural numbers is :

(1) 437 (2) $\frac{437}{4}$

(3) $\frac{833}{4}$ (4) 833

**Sol.** **[4]**

$$V_\infty = 4\frac{(50\times51\times101)}{6\times50} - 4\frac{(50\times51\times50\times51)}{50\times50\times4}$$

$= 17\,[202 - 153]$
$= 17 \times 49$
$= 833$

**88.** Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals :

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

**Sol.** **[2]**

$$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4x + \cos^4x) - \frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$= \frac{1}{4}(1 - 2\sin^2x\cos^2x) - \frac{1}{6}(1 - 3\sin^2x\cos^2x)$$

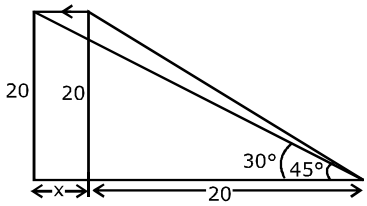
$$= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}$$

**89.** A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is 45º. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to 30º. Then the speed (in m/s) of the bird is :

(1) $20\sqrt{2}$ (2) $20(\sqrt{3} - 1)$

(3) $40(\sqrt{2} - 1)$ (4) $40(\sqrt{3} - \sqrt{2})$

**Sol.** **[2]**




$$\tan30° = \frac{20}{x+20} = \frac{1}{\sqrt{3}}$$

$x + 20 = 20\sqrt{3}$

$x = 20(\sqrt{3} - 1)$

**90.** The statement $\sim (P \leftrightarrow \sim q)$ is :
(1) a tautology
(2) a fallacy
(3) equivalent to $p \leftrightarrow q$
(4) equivalent to $\sim p \leftrightarrow q$

**Sol.** **[3]**

| p | q | ~ q | p ↔ ~ q | ~ (p ↔ ~ q) | p ↔ q |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | T | T | F | F |
| F | T | F | T | F | F |
| F | F | T | F | T | T |