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JEE ADVANCED

EXAMINATION 2014

QUESTIONS WITH SOLUTIONS

PAPER - 2 [CODE - 4]

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PHYSICS

1. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and and hc= 1240 eV nm, the work function of the metal is nearly

(A) 3.7 eV (B) 3.2 eV
(C) 2.8 eV (D) 2.5 eV

Sol. A

$E_1 = \frac{12400}{2480} = 5eV$

$E_2 = \frac{12400}{3100} = 4eV$

$\therefore 5 = \phi + 4k$ .....(1)

$4 = \phi + k$ .....(2)

On solving

$\Rightarrow \phi = \frac{11}{3} = 3.66 = 3.7$ eV

2. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{M0} is the wavelength iof the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to

(A) 1.99 (B) 2.14
(C) 0.50 (D) 0.48

Sol. B

By Mosley's Law :

$\sqrt{f} = a(z-1)$

As $v = f\lambda \Rightarrow f = \frac{c}{\lambda}$

$\therefore$ Inverse relation and hence :

$\lambda \propto \frac{1}{(Z-1)^2}$

$\frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2-1}{Z_1-1}\right)^2 \Rightarrow \frac{\lambda_{\mu_1}}{\lambda_{\mu_0}} = \frac{(42-1)^2}{(24-1)^2} = 2.14$

3. Parallel rays of light of intensity I=912 Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7\times10^{-8} Wm^{-2}K^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temeprature of the black body is close to

(A) 330 K (B) 660 K
(C) 990 K (D) 1550 K

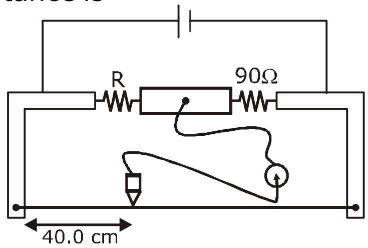
Sol. A

$912\times\pi R^2 = 5.7\times10^{-18}\times1\times[T^4-(300)^4] \times 4\pi R^2$

$\Rightarrow 10^8(121) = T^4$

$\Rightarrow T = 330$ K

4. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is



(A) $60 \pm 0.15\Omega$
(B) $135 \pm 0.56\Omega$
(C) $60 \pm 0.25\Omega$
(D) $135 \pm 0.23\Omega$

Sol. C

$R \times 60 = 90 \times 40$

$R = 90 \times \frac{\ell_2}{\ell_1}$

$\ell n R = \ell n 90 + \ell n \ell_2 + \ell n \ell_1$

$\frac{dR}{R} = \frac{d\ell_2}{\ell_2} + \frac{d\ell_1}{\ell_1}$

$\frac{dR}{60} = \frac{0.1}{60} + \frac{0.1}{40}$

$dR = 0.1 + 0.15$

$= 0.25$

$\therefore R = (60 \pm 0.25)\ \Omega$

5. A planet of radius $R = \frac{1}{10} \times$(radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius oif Earth=6×10^6m and the acceleration dur to gravity on Earth is 10 ms^{-2})

(A) 96 N (B) 108 N
(C) 120 N (D) 150 N

Sol. **B**

$R = \frac{1}{10} R_e$

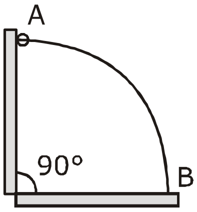
$\int dF = \int \rho.dx.\frac{9}{10}\left(1 - \frac{x}{R_e/10}\right)$

$= 10^{-3} \times 1 \left[\frac{R_e}{50} - \left(\frac{R_e}{50}\right)^2\right] \times \frac{10}{R_e \times 2}$

$= 10^{-3} \times 6 \times 10^6 \left[\frac{1}{50} - \frac{1}{900}\right]$

$= 108$

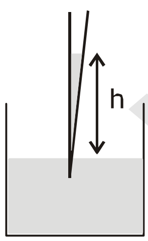
6. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is



(A) always radially outwards.
(B) always radially inwards.
(C) radially outwards initially and radially in wards later.
(D) radially inwards intially and radially out wards later.

Sol. **D**
By theory

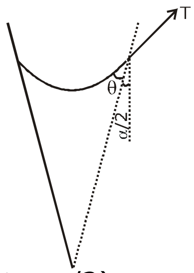
7. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. when dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)



(A) $\frac{2s}{b\rho g}\cos(\theta - \alpha)$ (B) $\frac{2s}{b\rho g}\cos(\theta + \alpha)$

(C) $\frac{2s}{b\rho g}\cos(\theta + \alpha/2)$ (D) $\frac{2s}{b\rho g}\cos(\theta + \alpha/2)$

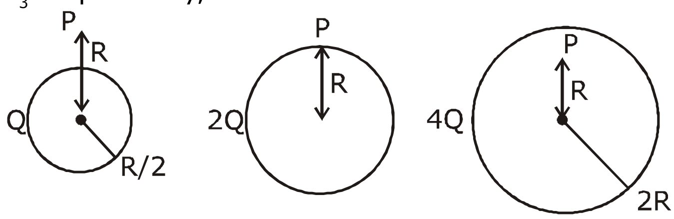
Sol. **D**



$h = T \cos(\theta + \alpha/2)$

$h = \frac{2s}{b\rho g} \cos(\theta + \alpha/2)$

8. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1,2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1,2 and 3 are E_1, E_2 and E_3 respectively, the



Sphere1 Sphere2 Sphere3

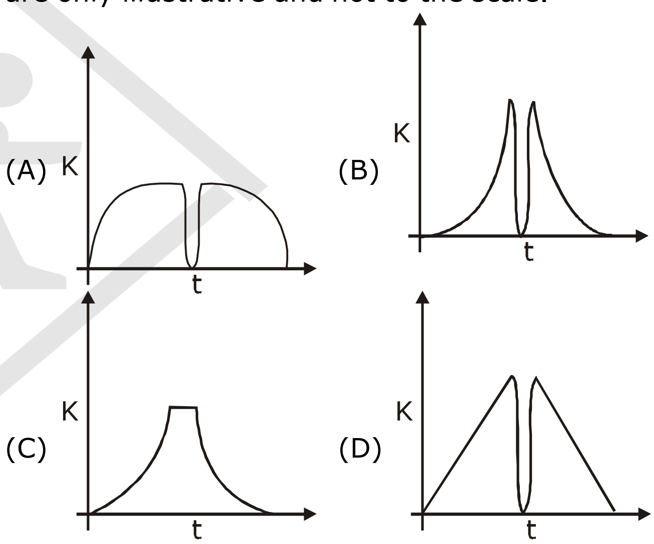
(A) $E_1>E_2>E_3$ (B) $E_3>E_1>E_2$
(C) $E_2>E_1>E_3$ (D) $E_3>E_2>E_1$

Sol. **C**

$E_1 = \frac{KQ}{R^2}$ $E_2 = \frac{2KQ}{R^2}$

$E_3 = \frac{4KQ.R}{(2R)^3} = \frac{KQ}{2R^2}$ $\therefore E_2 > E_1 > E_3$

9. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.



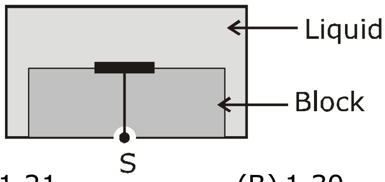
Sol. **B**

$V = gt$

$\therefore KE = \frac{1}{2} m (gt)^2$

$\therefore$ Relation is parabolic

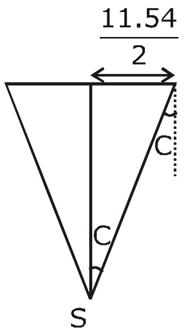
10. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as swhon in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is



(A) 1.21 (B) 1.30
(C) 1.36 (D) 1.42

Sol. **C**

$$\sin C = \frac{\frac{11.54}{2}}{\sqrt{\left(\frac{11.54}{2}\right)^2 + 100}}$$



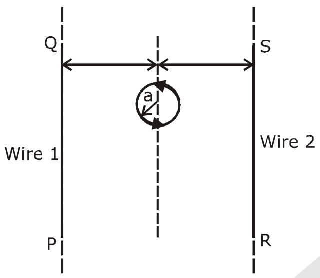
Also $\sin C = \frac{n}{2.72}$

$$\frac{5.77}{\sqrt{133.3}} = \frac{M}{2.72}$$

$n = 1.36$

Paragraph 11 & 12

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



11. When $d \approx a$ but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$

(B) current in wire 1 and wire 2 is the direction PQ and SR, respective and $h \approx a$

(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

Sol. **C**

$$\frac{\mu_0 i a^2}{2\left(a^2 + h^2\right)^{3/2}} = \frac{\mu_0 i}{2\pi\sqrt{a^2 + h^2}}\cos\theta + \frac{\mu_0 i}{2\pi\sqrt{a^2 + h^2}}\cos\theta$$

$$\frac{a}{\left(a^2 + h^2\right)^{3/2}} = \frac{2}{\pi\left(a^2 + h^2\right)}$$

$$\frac{a}{\left(a^2 + h^2\right)^{1/2}} = \frac{2}{\pi}$$

$$\frac{a^2}{\left(a^2 + h^2\right)} = \frac{4}{\pi^2}$$

$a^2 = 0.4a^2 + 0.4h^2$

$0.6a^2 = 0.4h^2$

$h^2 = \frac{3}{2} a^2$

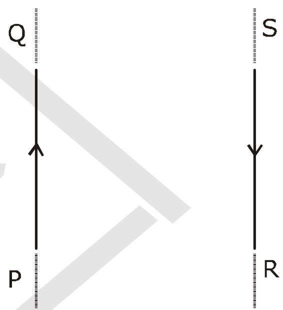
$h \approx 1.2\ a$

12. consider d>>a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown inthe figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$

(C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

Sol. **B**



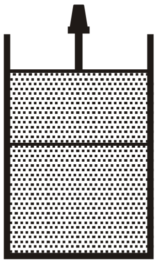
$\tau = MB \sin\theta$

$$= i\pi a^2 \times \frac{2\mu_0 i}{2\pi d} \sin 30^0$$

$$= \frac{\mu_0 i^2 a^2}{d} \times \frac{1}{2} = \frac{\mu_0 i^2 a^2}{2d}$$

Paragraph 13 & 14

In the figure a containner is shown to have a movalble (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R$, and those for an ideal diatomis gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.



13. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature to the gases will be

(A) 550 K (B) 525 K
(C) 513 K (D) 490 K

Sol. D

$2 \times \frac{3}{2} R (700 - T)$

$= 2R (T - 400) + \frac{5}{2} R (T - 400) \times 2$

T = 490 K

14. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) 250 R (B) 200 R
(C) 100 R (D) -100 R

Sol. D

$|Q_1| = |Q_2|$

$\frac{5}{2} \times 2 \times R (700 - T) = \frac{7}{2} \times 2 \times R (T - 400)$

∴ T = 525 K

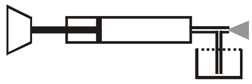
W.D. by lower gas = 2 × R × 175 = 350 R

W.D. upper gas = 2 × R × 125 = 250 R

∴ net W.D. = −100 R

Paragraph 15 & 16

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the containwer rises into the nozzle and is sdprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



15. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(A) 0.1 ms^{-1} (B) 1 ms^{-1}
(C) 2 ms^{-1} (D) 8 ms^{-1}

Sol. C

From equation of continuity

$A_1V_1 = A_2V_2$

$\pi(20)^2 \times 5 \times t = \pi(1)^2 \times v \times t$

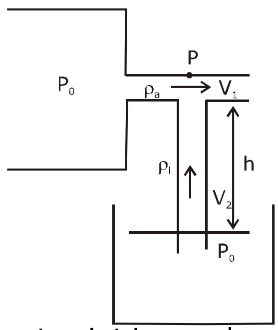
v = 400 × 5 = 2000 mm/s

= 2 m/s

16. If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_l}}$ (B) $\sqrt{\rho_a \rho l}$
(C) $\sqrt{\frac{\rho l}{\rho_a}}$ (D) ρl

Sol. A



Pressure at point in nozzle ⇒ P

$P = P_0 - 1\ \rho_a V_1^2$ .....(1)

$P = P_0 - \frac{1}{2} \rho_l V_2^2 - \rho_l gh$ ......(2)

equation (1) and (2)

$\frac{1}{2} \rho_a V_1^2 = \frac{1}{2} \rho_l V_2^2 + \rho_l gh$

Neglecting the term $r_e gh$

$V_2 = \sqrt{\frac{\rho_a}{\rho_l}} V_1$

Rate of liquid flow = AV_2

$= AV_1 \sqrt{\frac{\rho_a}{\rho_l}}$ Rate of liquid flow $\propto \sqrt{\frac{\rho_a}{\rho_l}}$

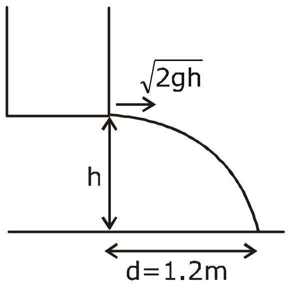
17. A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in list I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those is List II and select the correct answer using the code given belsow the lists.

List I	List II
P. Lift is acceleration vertically up.	1. d = 1.2 m
Q. Lift is accelerating vertically with an acceleration less than the gravitational acceleration.	2. d> 1.2 m
R. Lift is moving vertically up with constant speed.	3. d < 1.2 m
S. Lift is falling freely.	4. No water leaks out of the jar

Code :

(A) P-2, Q-3, R-2, S-4
(B) P-2, Q-3, R-1, S-4
(C) P-1, Q-1, R-1, S-4
(D) P-2, Q-3, R-1, S-1

Sol. **C**

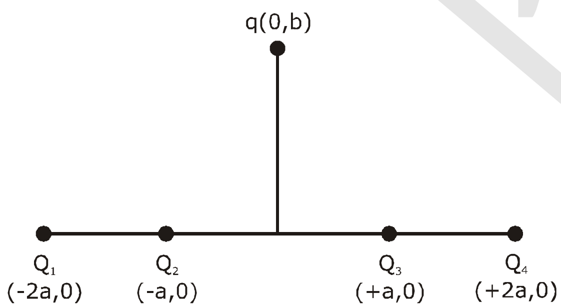


$$d = vt = \sqrt{2gh}\sqrt{\frac{2h}{g}} = 2h \text{ which is independent}$$

of g.

But when the lift falls freely no water leaks out of the jar as $g_{eff} = 0$.

18. For charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = -2a, -a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four potions of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists.

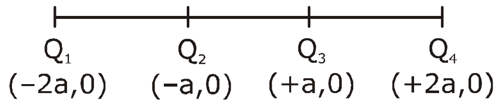


List I	List II
P. Q_1, Q_2, Q_3, Q_4 all positive	1. +x
Q. Q_1, Q_2 positive; Q_3, Q_4 negative	2. -x
R. Q_1, Q_4 positive; Q_2, Q_3 negative	3. +y
S. Q_1, Q_3 positive; Q_2, Q_4 negative	4. -y

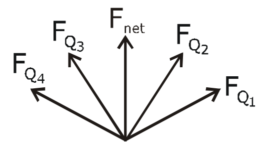
Code :

(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

Sol. **A**

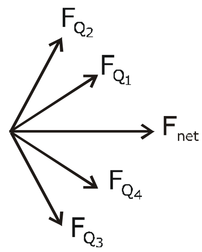


(P) All +ve ⇒

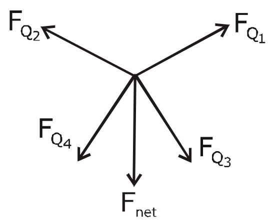


(Q) Q_1, Q_2 +ve ⇒

Q_3, Q_4 −ve ⇒

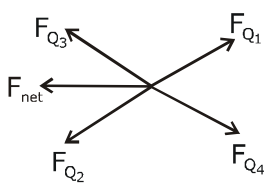


(R) Q_1, Q_3 −ve ⇒



Q_2, Q_3 −ve

(S) Q_1, Q_3 +ve ⇒



Q_2, Q_4 −ve

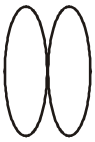
19. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surface is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.

List I	List II
P. 	1. 2r
Q. 	2. r/2
R. 	3. -r
S. 	4. r

Code :

(A) P-1, Q-2, R-3, S-4
(B) P-2, Q-4, R-3, S-1
(C) P-4, Q-1, R-2, S-3
(D) P-2, Q-1, R-3, S-4

Sol. **D**

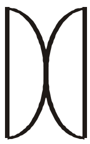


$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{r} + \frac{1}{r}\right) \qquad \frac{1}{f_1} = \left(\frac{1}{2}\right)\frac{2}{r}$$

$$\frac{1}{f_1} = \frac{1}{r} \qquad \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{r} \qquad f_{eq} = \frac{r}{2}$$

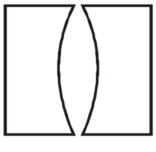
$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\frac{1}{r}$$



$$\frac{1}{f_1} = \frac{1}{2r}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \frac{1}{f_{eq}} = \frac{1}{r}$$

$$f_{eq} = r$$

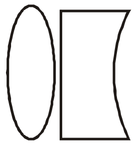


$$\frac{1}{f_1} = -\frac{1}{2r}$$

$$\frac{1}{f_2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{r}\right)$$

$$\frac{1}{f_{eq}} = \frac{-1}{2r} \frac{-1}{2r}$$

$$f_{eq} = -r$$



$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{r} + \frac{1}{r}\right)$$

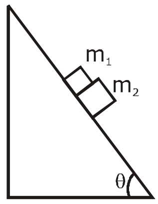
$$= \frac{1}{r}$$

$$\frac{1}{f_2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{r}\right) \qquad = \frac{-1}{2r}$$

$$\frac{1}{f_{eq}} = \frac{1}{r} - \frac{1}{2r} \qquad f_{eq} = 2r$$

P - 2 Q - 4 R - 3 S - 1

20. A block of mass m_1 = 1 kg another mass m_2=2kg are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in list I. The coefficient of static and dinamic friction between the block m_2 and the plane are equal to μ=0.3. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.



List I	List II
P. θ = 5°	1. $m_2 g \sin\theta$
Q. θ = 10°	2. $(m_1+m_2) g \sin\theta$
R. θ = 15°	3. $\mu m_2 g \cos\theta$
S. θ = 20°	4. $\mu(m_1+m_2) g \cos q$

Code :

(A) P-1, Q-1, R-1, S-3
(B) P-2, Q-2, R-2, S-3
(C) P-2, Q-2, R-2, S-4
(D) P-2, Q-2, R-3, S-3

Sol. **D**

There will be no slipping if friction balances the net force acting downwards along the incline

$f_{s_{max}} = \mu\, m_2\, g \cos\theta$

Force in the downward direction is $(m_1 + m_2)\, g \sin\theta$ thus angle at which slipping starts

$(m_1 + m_2)\, g \sin\theta = \mu\, m_2\, g \cos\theta$

$$\tan\theta = \frac{0.3\times 2}{3} = 0.2$$

given tan 11.5° = 0.2

Thus θ = 11.5°

Thus for angles less than 11.5° there won't be any slipping hence friction is static and equal to $(M_1 + M_2)\, g \sin\theta$ for greater than 11.5° the friction is dynamic and is equal to $\mu\, m_2\, g \cos\theta$.

21. For the identification of β-naphthol using dye test, it is necessary to use

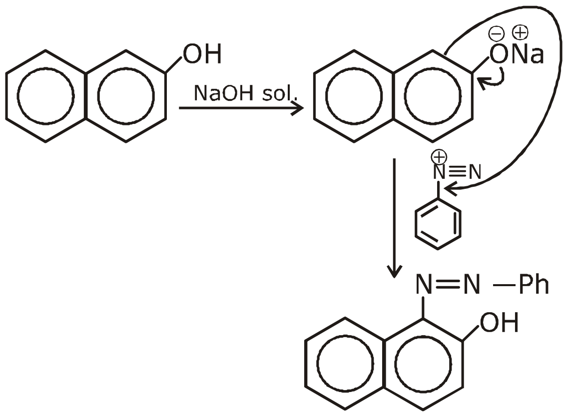
(A) dichloromethane solution of β-napthol

(B) acidic solution of β-naphthol

(C) neutral solution of β-naphthol

(D) alkaline solution of β-naphthol

Sol. **D**



22. For the elementary reaction **M** $\rightarrow$ **N**, the rate of disappearance of **M** increases by a factor of 8 upon doubling the concentration of **M**. The order of the reaction with respect to **M** is :

(A) 4 (B) 3

(C) 2 (D) 1

Sol. **(B)**

$r = K[M]^x$

$8r = K[2M]^x$

$\therefore \quad x = 3$

23. For the process

$H_2O\,(l) \rightarrow H_2O\,(g)$

at T=100ºC and 1 atomsphere pressure, the correct choice is :

(A) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} > 0$

(B) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} < 0$

(C) $\Delta S_{system} < 0$ and $\Delta S_{surroundings} > 0$

(D) $\Delta S_{system} > 0$ and $\Delta S_{surroundings} < 0$

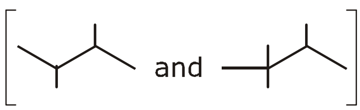
Sol. **(B)**

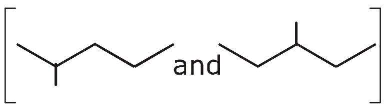
$H_2O\,(l) + \text{Heat} \rightarrow H_2O\,(g)$

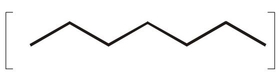
$\therefore \quad \Delta S_{system} > 0$

$\& \quad \Delta S_{surroundings} < 0$

24. Isomers of hexane, based on their branching, can be divided into three distinct classes as shown in the figure.

(I) 

(II) 

(III) 

The correct order of their boiling point is :

(A) I > II > III (B) III > II > I

(C) II > III > I (D) III > I > II

Sol. **B**

Boiling point α surface area α $\dfrac{1}{\text{branching}}$

25. Assuming 2s-2p mixing is NOT operative, the paramagnetic species among the following is:

(A) Be_2 (B) B_2

(C) C_2 (D) N_2

Sol. **(C)**

if 2s-2p mixing is not allowed then MOD will be like O_2.

$$\sigma 1s\ \sigma^* 1s\ \sigma 2s\ \sigma^* 2s\ \sigma 2p_x \begin{bmatrix} \pi 2p_y \\ || \\ \pi 2p_z \end{bmatrix} \begin{bmatrix} \overset{*}{\pi} 2p_y \\ || \\ \overset{*}{\pi} 2p_z \end{bmatrix} \sigma^* 2p_x.$$

26. The product formed in the reaction of $SOCl_2$ with white phosphorous is :

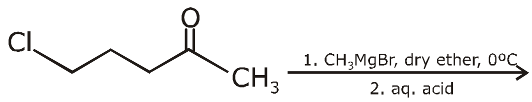
(A) PCl_3 (B) SO_2Cl_2

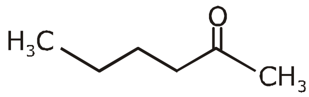
(D) SCl_2 (D) $POCl_3$

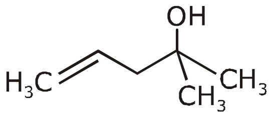
Sol. **(A)**

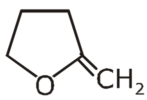
$\underset{\text{white}}{P_4} + SOCl_2 \longrightarrow PCl_3 + SO_2 + S_2Cl_2$

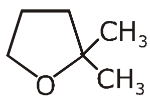
27. The major product in the following reaction is



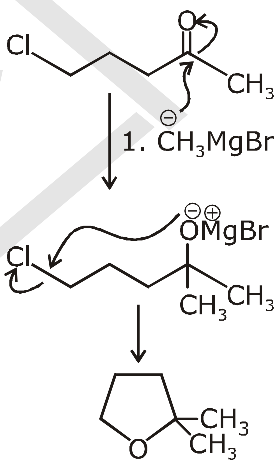
(A) 

(B) 

(C) 

(D) 

Sol. **D**

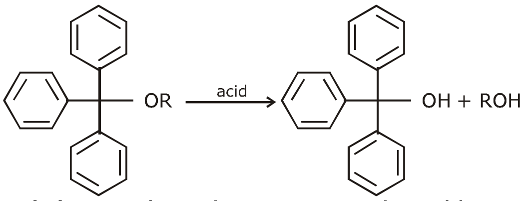


28. Hydrogen peroxide in its reaction with KIO_4 and NH_2OH respectively, is acting as a

(A) reducing agent, oxidising agent
(B) reducing agent, reducing agent
(C) oxidising agent, oxidising agent
(D) oxidising agent, reducing agent

Ans. (A)

29. The acidic hydrolysis of ether (X), shown below is fastest when

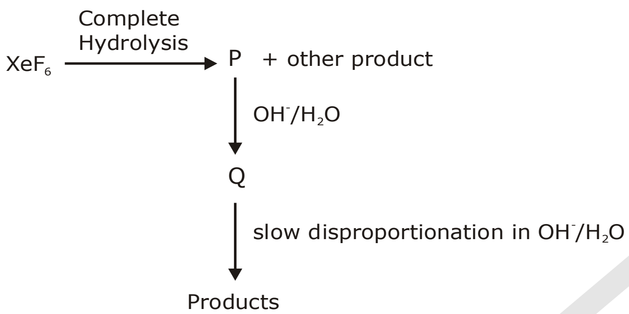


(A) one phenyl group is replaced by a methyl group.
(B) one phenyl group is replaced by a para-methoxyphenyl group
(C) two phenyl groups are replaced by two para-methoxyphenyl groups
(D) no structural change is made to X

Ans. C

Rate of SN_1 reaction α stability of carbocation. If two phenyl group is replaced by two p-methoxy phenyl group then it gives most stable carbocation so fastest reaction.

30. Under ambient conditions, the total number of gases released as products in the final step of the reaction scheme shown below is



(A) 0 (B) 1
(C) 2 (D) 3

Ans. (C)

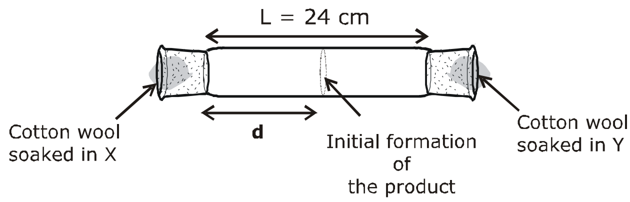
$$XeF_6 + H_2O \xrightarrow[\text{Hydrolysis}]{\text{Complete}} XeO_3 + HF$$

$$XeO_3 + OH^- \text{ (basic medium)} \rightleftharpoons HXeO_4^-$$

$$HXeO_4^- + OH^- \xrightarrow[\text{Disproportionation}]{\text{Slow}} XeO_6^{4-} + Xe\uparrow + O_2\uparrow + H_2O$$

Paragraph For Question 31 and 32

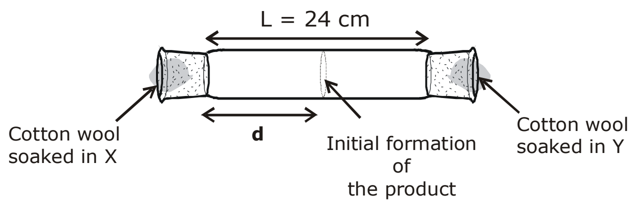
X and Y are two volatile liquids with molar weights of 10g mol^{-1} and 40g mol^{-1} respectively. Two cotton plugs, one soaked in X and the other soaked in Y, are simultaneously placed at the ends of a tube of length L = 24cm, as shown in the figure. The tube is filled with an inert gas at 1 atmosphere pressure and a temperature of 300 k. Vapurs of X and Y react to form a product which is first observed at a distance d cm from the plug soaked in X. Take X and Y to have equal molecular diameters and assume ideal behaviour for the inert gas and the two vapous.



31. The value of d in cm (shown in the figure), as estimated from Graham's law, is

(A) 8 (B) 12
(C) 16 (D) 20

Sol. (C)



$$\frac{rate_x}{rate_y} = \sqrt{\frac{MM_y}{MM_x}}$$

$$\Rightarrow \quad \frac{d_x/t}{d_y/t} = \sqrt{\frac{MM_y}{MM_x}}$$

$$\Rightarrow \quad \frac{d}{24-d} = \sqrt{\frac{40}{10}}$$

$\Rightarrow$ d = 48 – 2d

$\Rightarrow$ d = 16

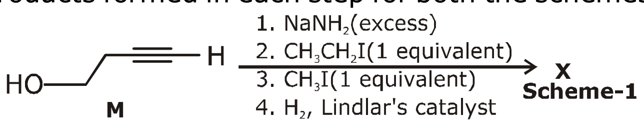
32. The experimental value of d is found to be smaller than the estimate obtained using Graham's law. This is due to

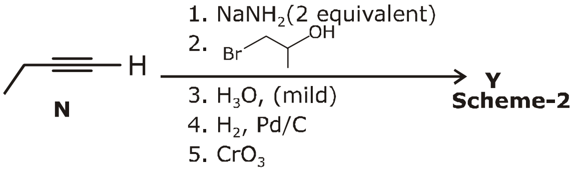
(A) larger mean free path for X as compared to that of Y.
(B) larger mean free path for Y as compared to that of X.
(C) increased collision frequency of Y with the inert gas as compared to that of X with the inert gas.
(D) increased collision frequency of X with the inert gas as compared to that of Y with the inert gas.

Sol. (B, D)

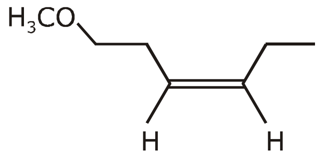
Paragraph For Questions 33 and 34

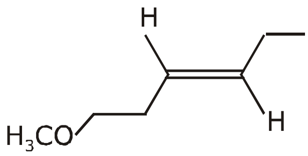
Schemes 1 and 2 describe sequential transformation of alkynes M and N. Consider only the major products formed in each step for both the schemes.

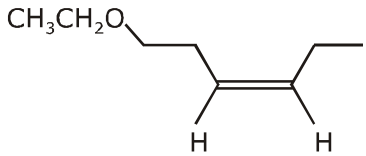


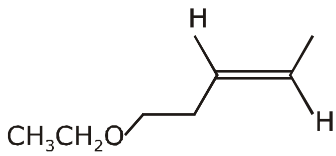


33. The product X is

(A)


(B)


(C)


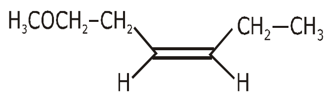
(D)


Sol. **A**

$HO-CH_2-CH_2-C\equiv CH \xrightarrow{NaNH_2} HO-CH_2-CH_2-CH$

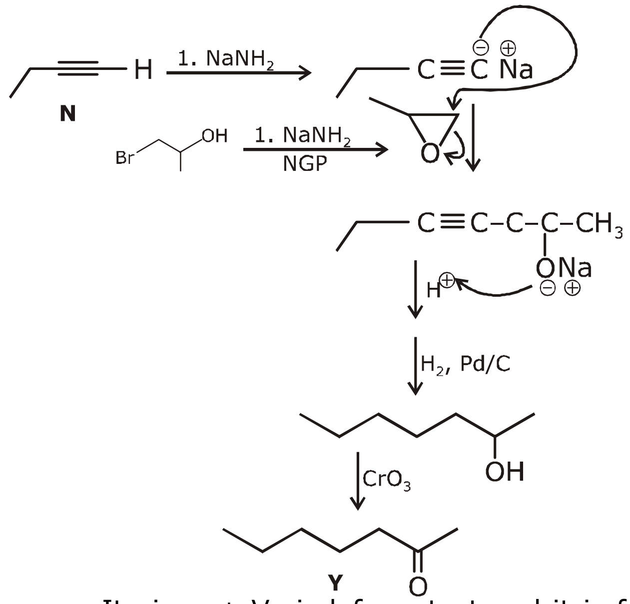
$\equiv C^-Na^+ \xrightarrow{CH_3-CH_2I} HO-CH_2-CH_2C\equiv C-CH_2-CH_3$

$\xrightarrow{CH_3-I} CH_3-O-CH_2-CH_2-C\equiv C-CH_2-CH_3$

$\xrightarrow[\text{catalst}]{H_2/\text{Lindlar's}}$


34. The correct statement with respect to product Y is
(A) It gives a positive Tollens test and is a functional isomer of X.
(B) It gives a positive Tollens test and is a geometrical isomer of X.
(C) It gives a positive iodoform test and is a functional isomer of X.
(D) It gives a positive iodoform test and is a geometrical isomer of X.

Sol. **C**

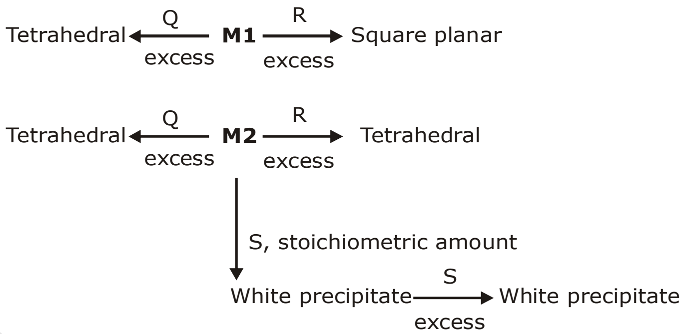


It gives + Ve iodoform test and it is functional isomer if X .

Paragraph For Questions 35 and 36

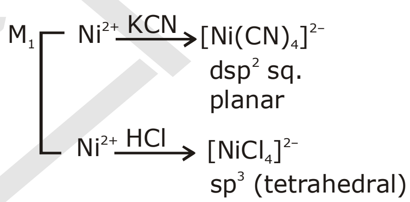
An aqueous solution of metal ion M1 reacts separately with reagents Q and R in excess to give tetrahedral and square planar complexes, respectively. An aqueous solution of another metal ion M2 always forms tetrahedral complexes with these reagents. Aqueous solution of M2 on reaction with reagent S gives white precipitate which dissolves in excess of S. The reactions are summarized in the scheme given below:

SCHEME:



35. M1, Q and R, respectively are
(A) Zn^{2+}, kCN and HCl (B) Ni^{2+}, HCl and KCN
(C) Cd^{2+}, KCN and HCl (D) Co^{2+}, HCl and KCN

Ans. **(B)**



M_2 ┌ $Zn^{2+} + KCN \longrightarrow [Zn(CN)_4]^{2-}$ (Tetrahedral sp^3)

└ $Zn^{2+} + HCl \longrightarrow [Zn(Cl_4]^{2-} + H^+$ (Tetrahedral sp^3)

$Zn^{2+} + KOH \longrightarrow Zn(OH)_2\downarrow$ (white ppt.) $\xrightarrow{OH^-}$ $[Zn(OH)_4]^{2-}$ (color less solution)

36. Reagent S is

(A) $K_4[Fe(CN)_6]$ (B) Na_2HPO_4

(C) K_2CrO_4 (D) KOH

Ans. (D)

37. Match each coordination compound in List-I with an appropriate pair of characteristics from List-II and select the correct answer using the code given below the lists.

{en = $H_2NCH_2CH_2NH_2$; atomic numbers : Ti = 22, Cr = 24, Co = 27; Pt = 78}

List-I

(P) $[Cr(NH_3)_4Cl_2]Cl$

(Q)$[Ti(H_2O)_5Cl\ (NO_3)_2]$

(R) $[Pt(en)(NH_3)Cl]NO_3]$

(S) $[Co(NH_3)_4(NO_3)_2]NO_3$

List-II

(1) Paramagnetic and exhibits ionisation isomerism

(2) Diamagnetic and exhibits cis-trans isomerism

(3) Paramagnetic and exhibits cis-trans isomerism

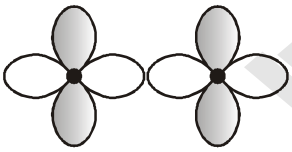
(4) Diamagnetic and exhibits ionisation isomerism

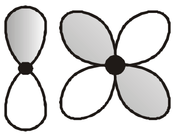
	P	Q	R	S
(A)	4	2	3	1
(B)	3	1	4	2
(C)	2	1	3	4
(D)	1	3	4	2

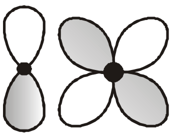
Ans. B

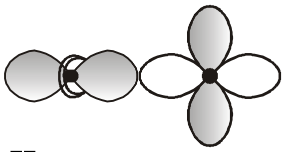
38. Match the orbital overlap figure shown in List-I with the description given in List-II and select the correct answer using the code given below the lists.

List-I

(P) 

(Q) 

(R) 

(S) 

List-II

(1) p–d π antibonding

(2) d–d σ bonding

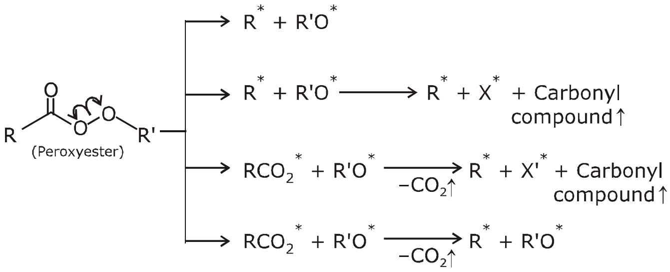
(3) p–d π bonding

(4) d–d σ antibonding

	P	Q	R	S
(A)	2	1	3	4
(B)	4	3	1	2
(C)	2	3	1	4
(D)	4	1	3	2

Ans. C

39. Different possible thermal decomposition pathways for perosysters are shown below. match each opathway form List with an appropriate structure from List II and select the correct answer using the code given below the lists.



List-I

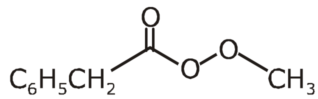
(P) Pathway P

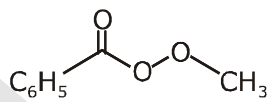
(Q) Pathway Q

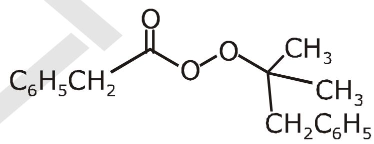
(R) Pathway R

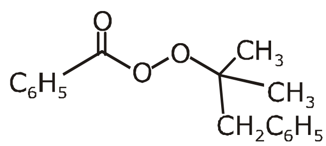
(S) Pathway S

List-II

(1) 

(2) 

(3) 

(4) 

Code :

	P	Q	R	S
(A)	1	3	4	2
(B)	2	4	3	1
(C)	4	1	2	3
(D)	3	2	1	4

Sol. **A**

Compound (I) follow the pathway (P) and form formaldehyde as carbonyl. It remove CO_2 in I step

Compound (II) follow the pathway (S) and form formaldehyde as carbonyl. It remove CO_2 in II step

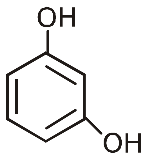
Compound (III) follow the pathway (Q) and does not form carbonyl. It remove CO_2 in I step

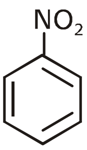
Compound (IV) follow the pathway (R) and does not form carbonyl. It remove CO_2 in II step

40. Match the four starting materials (P, Q, R, S) given in List I with the corresponding reaction schemes (I, II, III, IV) provided in List-II and select the correct answer using the code given below the lists.

List-I

(P) 

(Q) 

(R) 

(S) 

List-II

1. Scheme I

(i) $KMnO_4$, HO^-, heat (ii) H^+, H_2O
(iii) $SoCl_2$ (iv) NH_3

$\longrightarrow C_7H_6N_2O_3$

2. Scheme II

(i) Sn/HCl (ii) CH_3COCl (iii) Conc. H_2SO_4
(iv) HNO_3 (v) dil. H_2SO_4, heat (vi) HO^-

$\longrightarrow C_6H_6N_2O_2$

3. Scheme III

(i) red hot iron, 873 K
(ii) fuming HNO_3, H_2SO_4 heat
(iii) H_2S. NH_3 (iv) $NaNO_2$, H_2SO_4
(v) hydrolysis

$\longrightarrow C_6H_5NO_3$

4. Scheme IV

(i) conc. H_2SO_4, 60ºC
(ii) conc. HNO_3, conc. H_2SO_4
(iii)dil. H_2SO_4, heat

Code :

	P	Q	R	S
(A)	1	4	2	3
(B)	3	1	4	2
(C)	3	4	2	1
(D)	4	1	3	2

Sol. **C**

(P) Acetylene $\xrightarrow{\text{Red hot Fe}}$ Benzene

$\downarrow$ Mixed acid, Δ

Dinitrobenzene

$\downarrow$ H_2S, NH_3

m-nitro aniline

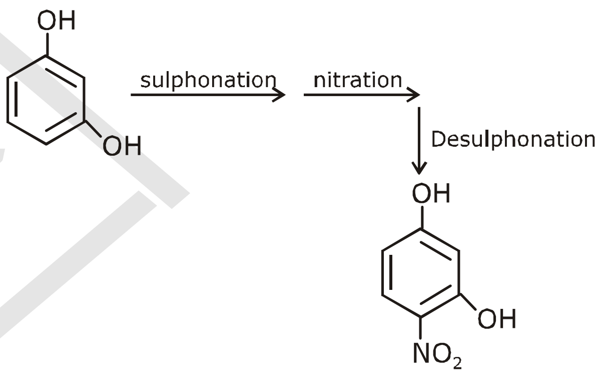
$\downarrow$ $NaNO_2$ + H_2SO_4

m-nitro benzene diazonium ion

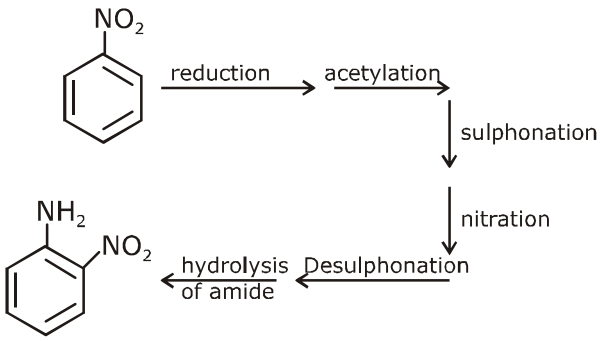
$\downarrow$ Hydrolysis

m-nitro phenol
($C_6H_5NO_3$)

(Q)

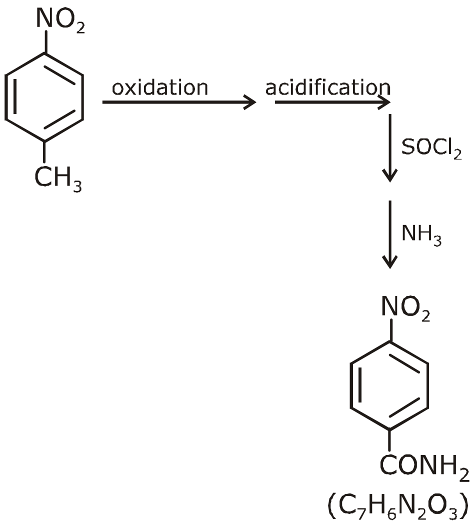


(R)



NO2
reduction
acetylation
sulphonation
nitration
NH2
NO2
hydrolysis
of amide
Desulphonation

(S)



NO2
CH3
oxidation
acidification
SOCl2
NH3
NO2
CONH2
(C7H6N2O3)

SECTION – A
Single Correct

41. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is

(A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$

(C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$

Sol. **B**

$a + b = x, \quad ab = y$

$(a + b)^2 - c^2 = ab$

$\Rightarrow \cos C = \frac{-1}{2}$

$\Rightarrow C = 120°$

also $x - C = \frac{y}{x+c}$

Now $\frac{r}{R} = \frac{(s-c)\tan 60°}{\frac{c}{2\sin 120°}} = \frac{(s-c)\sqrt{3}\cdot\sqrt{3}}{c}$

$= 3\cdot\frac{y}{2(x+c)}\cdot\frac{1}{c}$

42. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

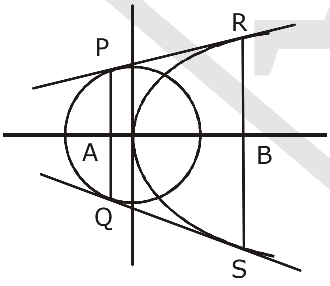
(A) 3 (B) 6

(C) 9 (D) 15

Sol. **D**

$R\left(\frac{a}{m^2}, \frac{2a}{m}\right)$, $S\left(\frac{a}{m^2}, \frac{-2a}{m}\right)$

$y = mx + \frac{2}{m}$



$\frac{\left|0 - 0 + \frac{2}{m}\right|}{\sqrt{m^2+1}} = \sqrt{2}$

$\frac{4}{m^2(m^2+1)} = 2$

$m^4 + m^2 - 2 = 0$

$\Rightarrow (m^2 - 1)(m^2 + 2) = 0$

$\Rightarrow m = \pm 1$

R(2, 4) & S(2, −4)

P(−1, 1) Q(−1, −1)

Area $= \frac{1}{2}$ (PQ + RS) · AB $= \frac{1}{2}(2 + 8)\cdot 3$

43. The quadratic equation p(x) = 0 with real coefficients has purely imaginary roots. Then the equation

p(p(x)) = 0 has

(A) only purely imaginary roots

(B) all real roots

(C) two real and two purely imaginary roots

(D) neither real nor purely imaginary roots

Sol. **A**

p(p(x)) = 0

$\Rightarrow$ p(x) = purely imaginary roots

$\Rightarrow$ No real value of x will satisfy

$\because$ For real x, LHS is purely real

44. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265

(C) 53 (D) 67

Sol. **C**

Required ways

$= \frac{6!\left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)}{5}$

$= \frac{360 - 120 + 30 - 6 + 1}{5} = \frac{265}{5} = 53$

45. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Sol. **A**

BGBGB BGBBG BBGGB BBGBG BBBGG

Probability $= \frac{5(3! \times 2!)}{5!} = \frac{1}{2}$

46. Let $f : [0, 2] \to R$ be a function which is continuous on [0, 2] and is differentiable on (0, 2) with f(0) = 1. Let $F(x) = \int_0^{x^2} f(\sqrt{t})\,dt$ for $x \in [0, 2]$. If F'(x) = f'(x) for all $x \in (0, 2)$, then F(2) equals

(A) $e^2 - 1$ (B) $e^4 - 1$
(C) $e - 1$ (D) e^4

Sol. **B**

$F(x) = \int_0^{x^2} f(\sqrt{t})\,dt$

differentiating

$f'(x) = f(x) \cdot 2x$

$\int \frac{f'(x)}{f(x)} = \int 2x$

$\ell n\ f(x) = x^2 + k$

$f(0) = 1 \Rightarrow k = 0$

$f(x) = e^{x^2}$

$F(x) = \int_0^{x^2} e^t\,dt$

$F(x) = [e^{x^2} - 1]$

$F(2) = \lim_{x \to 2} (e^{x^2} - 1) = e^4 - 1$

47. The function y = f(x) is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$ in (–1, 1) satisfying f(0) = 0. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)\,dx$ is

(A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$
(C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Sol. **B**

$\frac{dy}{dx} - \left(\frac{x}{1 - x^2}\right) y = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$

$I.F. = e^{\frac{1}{2}\int \frac{-2x}{1 - x^2} dx}$

$I.F. = e^{+\frac{1}{2}\ln(1 - x^2)} = \sqrt{1 - x^2}$

$y\sqrt{1 - x^2} = \int \frac{x^4 + 2x}{\sqrt{1 - x^2}} \sqrt{1 - x^2}\,dx$

$y\sqrt{1 - x^2} = \frac{x^5}{5} + x^2$

$y = \frac{x^5 + 5x^2}{5\sqrt{1 - x^2}}$

$y = \frac{x^5 + 5x^2}{5\sqrt{1 - x^2}}$

$y = \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{x^5}{5\sqrt{1 - x^2}}\,dx + \int_{-\sqrt{3}/2}^{\sqrt{3}/2} \frac{5x^2}{5\sqrt{1 - x^2}}\,dx$

$= 2\int_0^{\sqrt{3}/2} \frac{5x^2}{5\sqrt{1 - x^2}}\,dx$

$= 2\int_0^{\sqrt{3}/2} \frac{x^2 + 1 - 1}{\sqrt{1 - x^2}}\,dx$

$= 2\left[-\int_0^{\sqrt{3}/2} \sqrt{(1 - x^2)}\,dx + \int_0^{\sqrt{3}/2} \frac{1}{\sqrt{1 - x^2}}\,dx\right]$

$= 2\left[-\int_0^{\sqrt{3}/2} \sqrt{(1 - x^2)}\,dx + \frac{\pi}{3}\right]$

$= 2\left[-\left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_0^{\sqrt{3}/2} + \frac{\pi}{3}\right]$

$= 2\left[-\left[\frac{\sqrt{3}}{4}\frac{1}{2} + \frac{\pi}{6}\right] + \frac{\pi}{3}\right]$

$= -\frac{\sqrt{3}}{4} - \frac{\pi}{3} + \frac{2\pi}{3}$

$\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

48. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has

(A) infinitely many solutions
(B) three solutions
(C) one solution
(D) no solution

Sol. **D**

$\sin x + 4 \sin x \cos x - 3 \sin x + 4 \sin^3 x - 3 = 0$

$4 \sin x (\cos x + 1 - \cos^2 x) - 2 \sin x - 3 = 0$

$$\Rightarrow \quad \left(\cos x - \frac{1}{2}\right)^2 + \frac{3}{4}(\text{cosec} x - 1) = 0$$

Hence No solution

49. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\,\text{cosec}\,x)^{17}\,dx$ is equal to

(A) $\int_0^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$

(B) $\int_0^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

(C) $\int_0^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$

(D) $\int_0^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

Sol. **A**

$\cot \frac{x}{2} = e^u$

$x = 2 \cot^{-1} e^u$

$$dx = \frac{-2e^u}{1+e^{2u}} du$$

$$\text{cosec } x = \frac{1+\tan^2 \frac{x}{2}}{2\tan\frac{x}{2}} = \frac{1+\frac{1}{e^{2u}}}{\frac{2}{e^u}}$$

$$= \frac{1+e^{2u}}{2e^u}$$

$$I = \int_{\ln(\sqrt{2}+1)}^{0} \left(\frac{1+e^{2u}}{e^u}\right)^{17} . \left(\frac{-2e^u\,du}{1+e^{2u}}\right)$$

$$= \int_0^{\log(\sqrt{2}+1)} 2(e^u + e^{-u})^{16}\,du$$

50. Coefficient of x^{11} in the expansion of $(1 + x^2)^4 (1 + x^3)^7 (1 + x^4)^{12}$ is

(A) 1051 (B) 1106
(C) 1113 (D) 1120

Sol. **C**

Given expression is

$(1 + 4x^2 + 6x^4 + 4x^6 + x^8).$
$(1 + {}^7C_1\,x^3 + {}^7C_2\,x^6 + {}^7C_3 x^9 + ...).$
$(1 + {}^{12}C_1\,x^4 + {}^{12}C_2\,x^8 + ...)$

Coefficient of

$x^{11} = 1 \,.\, {}^7C_1 \,.\, {}^{12}C_2 + 4 \,.\, {}^7C_3 \,.\, 1 + 6 \,.\, {}^7C_1 \,.\, {}^{12}C_1 + 1.\, {}^7C_1.1$

$= 462 + 140 + 504 + 7$

$= 1113$

Paragraph

Paragraph for Question Nos. 51 to 52

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing number 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, i = 1, 2, 3.

51. The probability that $x_1 + x_2 + x_3$ are in an arithmetic progression, is

(A) $\frac{29}{105}$ (B) $\frac{53}{105}$

(C) $\frac{57}{105}$ (D) $\frac{1}{2}$

Sol. **B**

$\underbrace{|1,\ 2,\ 3|}_{\text{Box (1)}}$ $\underbrace{|1,\ 2,\ 3,\ 4,\ 5,|}_{\text{Box (2)}}$

$\underbrace{|1,\ 2,\ 3,\ 4,\ 5, 6, 7|}_{\text{Box (3)}}$

A card is drawn from each Box

OOO + OEE + EOE + EEO

$$\frac{2}{3}.\frac{3}{5}.\frac{4}{7} + \frac{2}{3}.\frac{2}{5}.\frac{3}{7} + \frac{1}{3}.\frac{3}{5}.\frac{3}{7} + \frac{1}{3}.\frac{2}{5}.\frac{4}{7}$$

$$= \frac{53}{105}$$

52. The probability that x_1, x_2, x_3 are in an arithmetic progression, is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$

(C) $\frac{11}{105}$ (D) $\frac{7}{105}$

Sol. **C**

The only cases are

111, 123, 135, 147, 222, 234, 246, 321, 333, 345, 357

$$\text{Required Probability} = \frac{11}{3 \times 5 \times 7} = \frac{11}{105}$$

Paragraph for Question Nos. 53 to 54

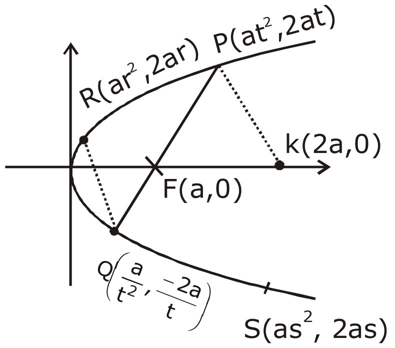
Let a, r, s, t be nonzero real number. Let $P(at^2, 2at)$, $Q(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that PQ is the focal chord and lines QR and PK are parallel, where K is the point (2a, 0).

53. The value of r is

(A) $-\frac{1}{t}$ (B) $\frac{t^2+1}{t}$

(C) $\frac{1}{t}$ (D) $\frac{t^2-1}{t}$

Sol. **D**



$m_{QR} = \frac{2}{\frac{-1}{t}+r}$, $m_{PK} = \frac{0-2at}{2a-at^2} = \frac{2t}{t^2-2}$

$$\frac{2}{\frac{-1}{t}+r} = \frac{2t}{t^2-2}$$

$$\frac{\frac{-1}{t}+r}{2} = \frac{t^2-2}{2t}$$

$$\frac{-1}{t}+r = t-\frac{2}{t}$$

$$r = t - \frac{1}{t}$$

$$r = \frac{t^2-1}{t}$$

54. If st = 1, then the tangent at P and the normal at S to the parabola meet at a point whose ordinate is

(A) $\frac{(t^2+1)^2}{2t^3}$ (B) $\frac{a(t^2+1)^2}{2t^3}$

(C) $\frac{a(t^2+1)^2}{t^3}$ (D) $\frac{a(t^2+2)^2}{t^3}$

Sol. **C**

Tangent at P is $y = \frac{x}{t} + at$ .....(1)

Normal at S is $y = -sx + 2as + as^3$

Now st = 1 so $s = \frac{1}{t}$

we get

$y = -\frac{x}{t} + \frac{2a}{t} + \frac{a}{t^3}$ .....(2)

Adding (1) & (2)

$$y = at + \frac{2a}{t} + \frac{a}{t^3}$$

$$= \frac{at^4+2at^2+a}{t^3} = \frac{a(t^4+2t^2+1)}{t^3}$$

$$= \frac{a(t^2+1)^2}{t^3}$$

Paragraph for Question Nos. 55 to 56

Given that for each a ∈ (0, 1),

$$\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$$

exists. Let this limit be g(a). In addition, it is given that the function g(a) is differentiable on (0, 1).

55. The value of $g\left(\frac{1}{2}\right)$ is

(A) π (B) 2π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Sol. **A**

$$g(a) = \lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$$

$$g(1/2) = \lim_{h\to 0^+} \int_h^{1-h} t^{-1/2}(1-t)^{-1/2}\,dt$$

$$= \lim_{h \to 0^+} \int_h^{1-h} \frac{1}{\sqrt{t(1-t)}} dt$$

$$= \int_0^1 \frac{dt}{\sqrt{t(1-t)}}$$

$t = \sin^2 \theta$

$dt = 2 \sin \theta \cos \theta \, d\theta$

$$= \int_0^{\pi/2} \frac{2 \sin\theta \cos\theta \, d\theta}{\sin\theta \cos\theta} = \int_0^{\pi/2} 2 \, d\theta$$

$$= [2\theta]_0^{\pi/2} = \pi$$

56. The value of $g'\left(\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$ (B) π

(C) $-\frac{\pi}{2}$ (D) 0

Sol. **D**

$$g(a) = \lim_{h \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1} dt$$

$$= \int_0^1 t^{-a}(1-t)^{a-1} dt$$

$$g'(a) = \int_0^1 (-t^{-a}.\ln t \ (1-t)^{a-1}$$

$$+ t^{-a} (1-t)^{a-1} \ell n (1-t)) dt$$

$$g'(1/2) = \int_0^1 (-t^{-1/2}.\ln t \ (1-t)^{-1/2}$$

$$+ t^{-1/2} (1-t)^{-1/2} \ell n (1-t)) dt$$

$$= \int_0^1 t^{-1/2} (1-t)^{-1/2} (-\ln t + \ln (1-t))dt$$

........(1)

$$g'(1/2) = \int_0^1 (1-t)^{-1/2} t^{-1/2} (-\ln (1-t) + \ln t) dt$$

........(2)

Adding (1) and (2)

$g'(1/2) = 0$

Matrix Match Type

Matching List Type (Only One Option Correct)

57.

	List I	List II
(P)	The number of polynomials f(x) with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	1. 8
(Q)	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value is	2. 2
(R)	$\int_{-2}^{2} \frac{3x^2}{(1+e^x)} dx$ equals	3. 4
(S)	$\frac{\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_0^{\frac{1}{2}} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$ equal	4. 0

	P	Q	R	S
(A)	3	2	4	1
(B)	2	3	4	1
(C)	3	2	1	4
(D)	2	3	1	4

Sol. **D**

(P) $f(x) = ax^2 + bx$

$$\left(\frac{ax^3}{3} + \frac{bx^2}{2}\right)_0^1 = 1$$

$$\frac{a}{3} + \frac{b}{2} = 1$$

$$\frac{a}{3} = 1 - \frac{b}{2}$$

$$\frac{a}{3} = \frac{2-b}{2} \qquad \frac{2a}{3} = 2 - b$$

$$b = 2 - \frac{2a}{3}$$

$$N = \frac{b-2a}{3}$$

$$\boxed{\begin{array}{l} a = 0 \Rightarrow b = 2 \\ a = 3 \Rightarrow b = 0 \end{array}}$$

(Q) $[-\sqrt{13}, \sqrt{13}]$: $f(x) = \sin x^2 + \cos x^2$

$x^2 = \frac{\pi}{4}, \frac{9\pi}{4}$

$x = \pm\sqrt{\frac{\pi}{4}}, \pm\sqrt{\frac{9\pi}{4}}$

(R) $I = \int_{-2}^{2} \frac{3x^2}{1 + e^{-x}} = \int_{-2}^{2} \frac{3e^x x^2}{e^x + 1}$

$2I = \int_{-2}^{2} 3x^2 \, dx$

$I = 2\int_0^2 3x^2$

$I = \left(\frac{3x^3}{3}\right)_0^2 = 8$

(S) numerator is odd function
$= 0$

58. **List I** **List II**

(P) Let $y(x) = \cos(3\cos^{-1} x)$, $x \in [-1,1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then $\frac{1}{y(x)}\left\{(x^2 - 1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equal — 1. 1

(Q) Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \times \overrightarrow{a_{k+1}}\right)\right| = \left|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \cdot \overrightarrow{a_{k+1}}\right)\right|$, then the minimum value of n is — 2. 2

(R) If the normal from the point P(h, 1) on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is — 3. 8

(S) Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is — 4. 9

	P	Q	R	S
(A)	4	3	2	1
(B)	2	4	3	1
(C)	4	3	1	2
(D)	2	4	1	3

Sol. **A**

(P) $y = \sin(3\cos^{-1}x)\left(\frac{3}{\sqrt{1-x^2}}\right)$

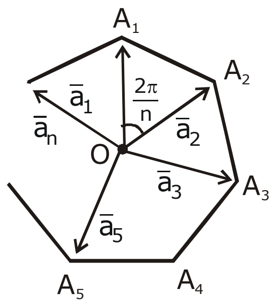
$\sqrt{1-x^2}\ y' = 3\sin(3\cos^{-1}x)$

$\sqrt{1-x^2}\ y'' - \frac{2x}{2\sqrt{1-x^2}}y' = 9\frac{\cos(3\cos^{-1}x)}{\sqrt{1-x^2}}$

$\frac{(1-x^2)y'' - xy'}{\sqrt{1-x^2}} = \frac{-9y}{\sqrt{1-x^2}}$

$(x^2 - 1)\, y'' + y' = 9y$

(Q)



$$\left|\sum_{k=1}^{n-1}(\vec{a}_k \times \vec{a}_{k+1})\right| = \left|\sum_{k=1}^{n-1}(\vec{a}_k \cdot \vec{a}_{k+1})\right|$$

$(n - 1)\,|\vec{a}_k||\vec{a}_{k+1}|\sin\frac{2\pi}{n}$

$= (n-1)\,|\vec{a}_k||\vec{a}_{k+1}|\cos\frac{2\pi}{n}$

$\Rightarrow \tan\frac{2\pi}{n} = 1$

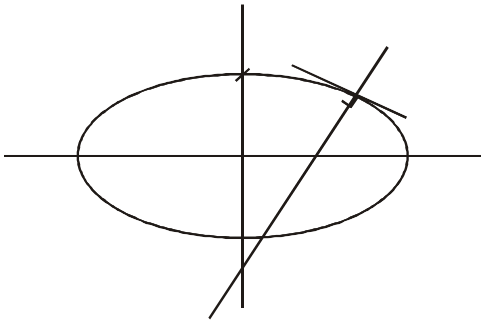
$\frac{2\pi}{n} = \frac{\pi}{4}$

$\boxed{n = 8}$

(R) $y + x = 8$

slope of Normal = + 1

$$\frac{dy}{dx} = -\frac{3x}{6y}$$



Normal (h, 1)

$\Rightarrow$ slope at (h, 1)

is $\frac{6}{3h} = 1$

$\Rightarrow$ $h = 2$

(S) $$\tan^{-1}\left(\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{2x+1}\times\frac{1}{4x+1}}\right)$$

$$\frac{4x+1+2x+1}{8x^2+6x+1-x} = \frac{2}{x^2}$$

$$\frac{6x+2}{2x(4x+3)} = \frac{2}{x^3}$$

$$\frac{3x+1}{4x+3} = \frac{2}{x} \Rightarrow 3x^2 + x = 8x + 6$$

$3x^2 - 7x - 6 = 0$

$3x^2 - 9x + 2x - 6 = 0$

$3x(x - 3) + 2(x - 3) = 0$

$x = 3, x = -\frac{2}{3}$

$x = 3$

$$\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)=\tan^{-1}\left(\frac{13+7}{90}\right)$$

$$\tan^{-1}\left(\frac{2}{9}\right)$$

R.H.S. $\tan^{-1}\left(\frac{2}{9}\right)$

$$x = -\frac{2}{3}\ \tan^{-1}\left(\frac{1}{-\frac{4}{3}+1}\right)+\tan^{-1}\left(\frac{1}{-\frac{8}{3}+1}\right)$$

$$\tan^{-1}\left(\frac{3}{-1}\right)+\tan^{-1}\left(\frac{3}{-5}\right)$$

-5

59. Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \ge 0 \end{cases}$$

$f_2(x) = x^2$;

$$f_3(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \ge 0 \end{cases}$$

(P)	f_4 is	1. onto but not one-one
(Q)	f_3 is	2. neither continuous nor one-one
(R)	f_2of_1 is	3. differentiable but not one-one
(S)	f_2 is	4. continuous and one-one

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

Sol. **D**

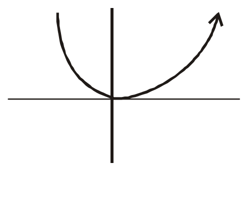
$f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$

$f_4 : R \to [0, \infty)$

$$f_1(x) = \begin{Bmatrix} -x & x < 0 \\ e^x & x \ge 0 \end{Bmatrix}$$

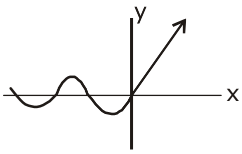
(P) $$f_4(x) = \begin{Bmatrix} f_2(f_1(x)) & x < 0 \\ f_2(f_1(x)) - 1 & x \ge 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} x^2 & x < 0 \\ e^{2x} - 1 & x \ge 0 \end{Bmatrix}$$



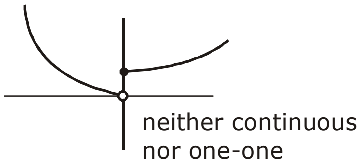
onto but not one-one

(Q) $f_3(x) =$

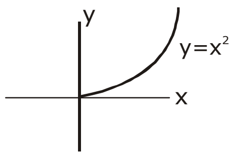


$f_3 \rightarrow$ differentiable but not one-one.

(R) $f_2 \, 0 \, f_1(x) = \begin{cases} x^2 & x < 0 \\ e^{2x} & x \geq 0 \end{cases}$



(S) $f_2(x) = x^2 \qquad x \geq 0$



continuous and one-one

60. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1,2,...,9$.

	List I	**List II**
(P)	For each z_k there exists a z_j such that $z_k \cdot z_j = 1$	1. True
(Q)	There exists a $k \in \{1,2,...,9\}$ such that $z_1 \cdot z = z_k$ has no solution z in the set of complex numbers.	2. False
(R)	$\frac{\|1 - z_1\|\|1 - z_2\|...\|1 - z_9\|}{10}$ equal	3. 1
(S)	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equal	4. 2

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

Sol. **C**

(P) $Z_k = e^{i\frac{2k\pi}{10}}$

$Z_k = 10^{th}$ roots of unity

$\Rightarrow$ P is true

(Q) $k \in \{1, 2, 3, ... 9\}$

$$Z = \frac{Z_k}{Z_1} = \frac{e^{i\frac{2k\pi}{10}}}{e^{i\frac{2\pi}{10}}} = e^{i\frac{2\pi}{10}(k-1)}$$

Z obuiously has a solution in the set of complex numbers. False

(R) $(x^{10}-1) = (x-1)(x-z_1)(x-z_2) ...... (x - z_9)$

$$(x - z_1)(x-z_2)(x-z_3)......(x-z_9) = \frac{x^{10}-1}{x-1}$$

$= 1 + x + x^2 + x^3 + ... + x^9$

Now put x = 1

$(1 - z_1)(1 - z_2) .... (1 - z_9)$

$= 1 + 1 + 1 .... + 1$

$\Rightarrow |1 - z_1| \, |1 - z_2| .... |1 - z_9| = 10$

$$\Rightarrow \frac{|1-z_1||1-z_2|.......|1-z_9|}{10} = 1$$

(S) $1 - \sum_{k=1}^{9} \cos\frac{2k\pi}{10}$

$$1 - \left(\cos\frac{2\pi}{10} + \cos\frac{4\pi}{10} + \cos\frac{6\pi}{10} + ... + \cos\frac{18\pi}{10}\right)$$

$$1 - \left(\frac{\sin\frac{9\pi}{10}\cos\pi}{\sin\frac{\pi}{10}}\right) = 1 + 1 = 2$$