MOTION

Nurturing potential through education

JEE ADVANCED

EXAMINATION 2014

QUESTIONS WITH SOLUTIONS

PAPER - 1 [CODE - 9]

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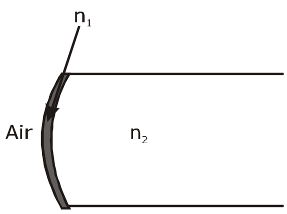
PHYSICS

1. A transparent thin film of uniform thickness and refractive index n_1=1.4 is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index n_2 = 1.5, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$

(B) $|f_1| = 2.8R$

(C) $|f_2| = 2R$

(D) $|f_2| = 1.4R$

Sol. **A,C**

Air to Glass

$$\frac{1.4}{V_1} - \frac{1}{\infty} = \frac{1.4-1}{R} \quad .....(1)$$

$$\frac{1.5}{V} - \frac{1.4}{V_1} = \frac{1.5-1.4}{R} \quad .....(2)$$

From (1) and (2)

$$\frac{1.5}{f_1} = \frac{0.5}{R} \Rightarrow f_1 = 3R$$

Glass to Air

$$\frac{1.4}{V_1} - \frac{1.5}{\infty} = \frac{1.4-1.5}{(-R)} = \frac{0.1}{R} \quad ....(3)$$

$$\frac{1}{V} - \frac{1.4}{V_1} = \frac{1-1.4}{(-R)} = \frac{0.4}{R} \quad .....(4)$$

$$\frac{1}{f_2} = \frac{0.5}{R}$$

$\Rightarrow$ f_2 = 2R

2. A sudent is perfarming an expernment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum hight at which resonance occurs is (0.350 $\pm$ 0.005)m, the gas in the tube is

(useful information : $\sqrt{167RT} = 640\ J^{1/2}$ mole$^{-1/2}$, $\sqrt{140RT} = 590\ J^{1/2}$ mole$^{-1/2}$. The molar masses M is garms are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M=20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M=28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M=32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M=36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Sol. **D**

f = 244 Hz

$$\frac{\lambda}{4} = 0.35 \Rightarrow \lambda = 1.4 \text{ m}$$

$$V = 244 \times 1.4 = 314.6 = \sqrt{\frac{\gamma(RT)}{M}}$$

$$\text{For Neon : } V = \sqrt{\frac{5}{3}\left(\frac{(RT)}{M}\right)}$$

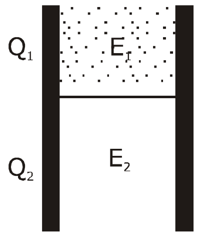
$$= \sqrt{\frac{RT}{12}} \Rightarrow \sqrt{\frac{10}{M}.\frac{RT}{6}} = \frac{7}{10}\sqrt{\frac{RT}{6}}$$

$$\text{For } N_2 : V = \sqrt{\frac{RT}{20}}$$

$$\text{For } O_2 : V = \sqrt{\frac{7}{5}\frac{RT}{32}} = \sqrt{\frac{7RT}{160}}$$

$$\text{For argon : } V = \sqrt{\frac{5}{3}\frac{RT}{36}} = \sqrt{\frac{5RT}{108}}$$

3. A parallel plate capacitor has a dielectric slab of dielectric constant K between it plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between C_1. When the capacitor is charged, the plate area covered by the dielectric get charge Q_1 and the rest of the area gets charge Q_z. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options. ignoring edge effects.

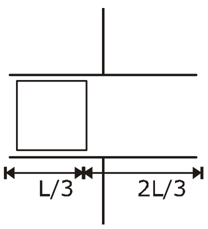


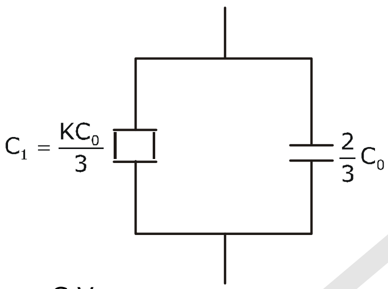
(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$

(C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

Sol. **A,D**

$C_0 \rightarrow \frac{\varepsilon_0 A}{d}$





$q = C_0V$
$q \propto C_0$

$\frac{q_1}{q_2} = \frac{k}{2}$

$\frac{E_1}{E_2} = 1$

$C = \frac{k}{3}C_0 + \frac{2}{3}C_0 = \frac{(K+2)}{3}C_0$

$\frac{C}{C_1} = \frac{(K+2)}{K}$

4. One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the direction so that stationary waves are set up in the string. the possible waveform of these stationary waves is (are)

(A) $y(t) = A \sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A \sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A \sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A \sin\frac{5\pi x}{2}\cos 250\pi t$

Sol. **A,C,D**

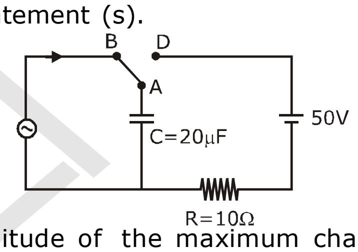
V = 100 m/s
L = 3m
At x = 0 ⇒ node
At x = 3 ⇒ antinode
Satisfying the condition in the equations on putting x = 3, sinkx should be equal to as it is an antinode.

$\sin\frac{\pi x}{6} \rightarrow \left[\frac{50\pi}{3}\times\frac{6}{\pi}\right]$

$\sin\frac{5\pi x}{6} \rightarrow \left[\frac{250}{3}\times\frac{6}{5}\right]$

$\sin\frac{5\pi x}{2} \rightarrow \left[\frac{250}{5}\times 2\right]$

5. At time t = 0, terminal A is the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$ with I_0=1A and ω=500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t=\frac{7\pi}{6\omega}$. the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C=20μF, R=10Ω and the battery is ideal with emf of 50V, identify the correct statement (s).



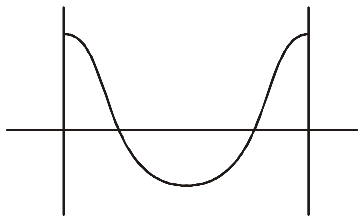
(A) Magnitude of the maximum charge on the capacitor before $t=\frac{7\pi}{6\omega}$ is 1 × 10^{-3} C.

(B) The current in the left part of the circuit just before $t=\frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10A.

(D) Q = 2×10^{-3}C.

Sol. **C,D**



$I = I_0 \cos \omega t$

$V_0 = \frac{I_0}{\omega C} = \frac{1}{50.0 \times 20 \times 10^{-6}} = \frac{1}{10 \times 10^{-3}}$

$= \frac{1}{10^{-2}} = 100$

$V = 100 \cos (\omega t - \pi/2)$

$V = 100 \sin \omega t$

$q_0 = CV = 20 \times 10^{-6} \times 100$

$= 2 \times 10^{-3} \sin \omega \times \frac{7\pi}{6\omega}$

$= 2 \times 10^{-3} \sin \frac{7\pi}{6}$

$= 2 \times 10^{-3} \left[-\sin\left(\frac{\pi}{6}\right)\right]$

$= -10^{-3}$ C

6. A light source, which emits two wavelengths λ_1 = 400nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes from them within a distance y on one side of the central maximum are m_1 and m_2 respectively, then

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes from λ_1 is greater than λ_2.

Sol. **A, B, C**

$\lambda_1 = 400$ nm $\quad \beta_1 = (400)\frac{D}{d}$

$\lambda_2 = 600$ nm $\quad \beta_2 = (600)\frac{D}{d}, \beta_2 > \beta_1$

no. of fringes $= \frac{y}{\beta}$

Angular separation : $\lambda = \frac{B}{D} = \frac{\lambda}{d}$

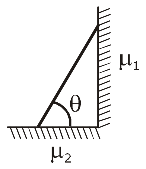
Hence $\lambda_1 < \lambda_2$

Checking (C) $\Rightarrow 3 \times 600 \times \frac{D}{d} = 9 \times 400 \times \frac{D}{2d}$

$\Rightarrow 1800 \frac{D}{d} = 1800 \frac{D}{d}$

$\therefore$ The given option is true

7. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



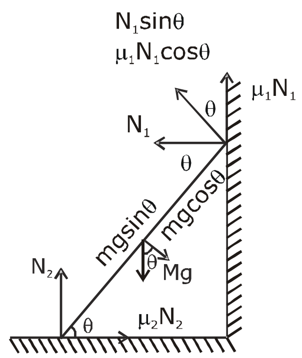
(A) $\mu_1 = 0 \; \mu_2 \neq 0$ and $N_2 \tan \theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0 \; \mu_2 = 0$ and $N_1 \tan \theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0 \; \mu_2 \neq 0$ and $N_2 = \frac{mg}{1 + \mu_1 \mu_2}$

(D) $\mu_1 = 0 \; \mu_2 \neq 0$ and $N_1 \tan \theta = \frac{mg}{2}$

Sol. **C, D**



$N_1 = \mu_2 N_2$

$N_2 = mg + \mu_1 N_1$

By torque balance:

$\frac{l}{2} mg \cos \theta = (\mu_1 N_1 \cos \theta + N_1 \sin \theta)\, l$

$\frac{mg}{2} \cos \theta = \mu_1 N_1 \cos \theta + N_1 \sin \theta$

$\mu_1 = 0$

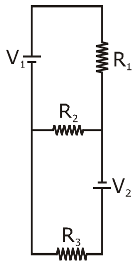
$\frac{mg}{2} \cos \theta = N_1 \sin \theta$

$\Rightarrow N_1 \tan \theta = \frac{mg}{2}$

$\mu_2 = 0$

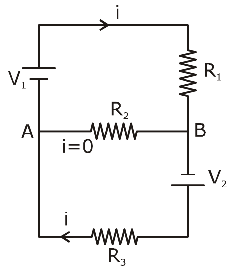
$\therefore N_1 = 0$

8. Two ideal batteries of emf V_1 and V_2 and these resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Sol. **A,B,D**



$V_A = V_B$
$V_1 = iR_1$ ....(i)
$V_2 = iR_3$ .....(ii)
(i)/(ii)

$$\frac{V_1}{V_2} = \frac{R_1}{R_3}$$

9. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, Then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$

(D) $E_2(r_0/2) = 4E_3(r_0/2)$

Sol. **C**

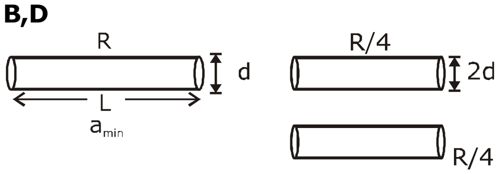
$$E_1\left(\frac{r_0}{2}\right) = \frac{KQ}{\left(\frac{r_0}{2}\right)^2} = \frac{4KQ}{r_0^2}$$

$$= \frac{4K}{r_0^2} \times 2r_0\lambda = \frac{8K\lambda}{r_0}$$

10. Heater of an electric kettle is made of a wire of lenght L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. Howe much time in munutes will it take to raise the temperature of the same amout of water by 40 K?

(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in series
(D) 0.5 if wires are in parallel

Sol. **B,D**

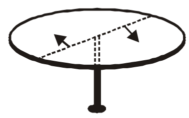


Since $P = \frac{V^2}{R}$

Series : $R_{eq} = \frac{R}{2}$ time $\to$ half

Parallel : $R_{eq} = \frac{R}{8}$ time $\to$ $\frac{1}{8}$ times

11. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25b m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

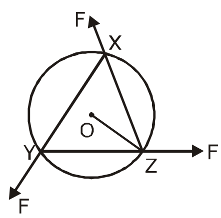


Sol. **0004**

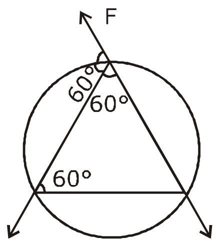
$L_i = L_f$
$0 = I\omega - 2mvr$
$I\omega = 2mvr$

$$\omega = \frac{2mvr}{I} = \frac{2 \times 0.09 \times 0.9}{0.45 \times 0.5 \times 0.5} \times 2 = 4$$

12. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F=0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with it vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc is rad s^{-1} is



Sol. **0002**



$$\alpha = \frac{2}{I} = \frac{3F\cos 60^\circ \times R}{I} = \frac{3F}{2} \cdot \frac{2}{(MR^2)}$$

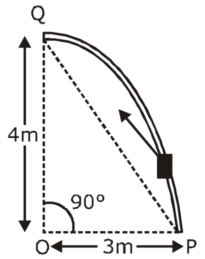
$$\alpha = \frac{3F}{MR^2} = \frac{3(0.5)}{(1.5)(0.5)^2}$$

$\alpha = 2$

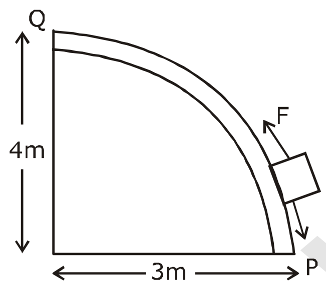
$\omega = \alpha t$

$\omega = 2 \times 1 = 2$

13. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n in (take acceleration due to gravity = $10 ms^{-2}$)



Sol. **0005**



$w_{mg} + w_F = \Delta KE$

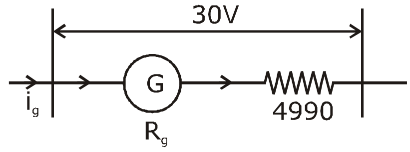
$-1\times4\times10+18\times5 = \frac{1}{2}\times1\times v^2$

$-40+90 = \frac{V^2}{2} \Rightarrow KE = 50$

$\therefore$ n = 0005

14. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{246}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is

Sol. **0005**

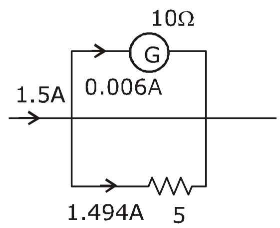


$i_g = 0.006$ Amp.

$$30 = \frac{6}{1000}[4990 + R_g]$$

$\Rightarrow 4990 + R_g = 5000$

$\Rightarrow R_g = 10\,\Omega$

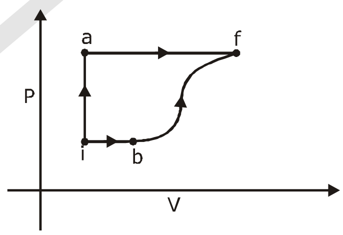


$$\frac{s}{10} = \frac{0.006}{1.494} = \frac{6}{1494}$$

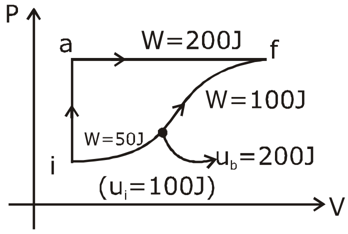
$$s = \frac{60}{1494} = \frac{10}{249}\Omega$$

$\Rightarrow$ n = 5

15. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f slong two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf}/Q_{id} is



Sol. **0002**



$\frac{Q_{bf}}{Q_{ib}}$ = ??

$w_{iaf} = 0 + 200 = 200$

$Q_{iaf} = 500$ Thus $\Delta U_{iaf} = 300$

$\Rightarrow \Delta U_{ibf} = 300$

But $\Delta U_{ib} = 100$ Thus $\Delta U_{bf} = 200$

$\Rightarrow Q_{ib} = W_{ib} + \Delta U_{ib} = 50 + 100 = 150$

$Q_{bf} = W_{bf} + \Delta U_{bf} = 100 + (200) = 300$

Ratio $\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$

16. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

Sol. **0003**

$L \propto [ML^{-3}]^a [MT^{-3}]^b [T^{-1}]^c$

$L \propto [M^{a+b}] [L^{-3a}] [T^{-3b-c}]$

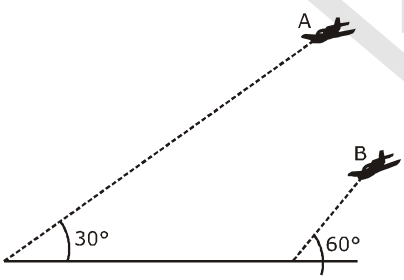
$a + b = 0$ .....(1)

$-3a = 1$

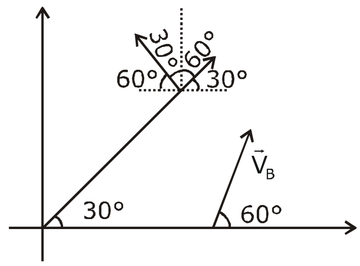
$a = -1/3$ and $b = 1/3$ ......(2)

$\therefore n = 0003$

17. Airplanes A and B are flying constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal repectively as shown in the figure. The speed of A is $100\sqrt{3}\,ms^{-1}$. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is



Sol. **0005**



$\vec{V}_A = 100\sqrt{3}\cos 30\,\hat{i} + 100\sqrt{3}\sin 30°\,\hat{j}$

$= 100\hat{i} + 50\sqrt{2}\hat{j}$

$\vec{V}_B = x\cos 60°\,\hat{i} + x\sin 60°\,\hat{j}$

$= \frac{x}{2}\hat{i} + \frac{x\sqrt{3}}{2}\hat{j}$

$\vec{V}_B - \vec{V}_A = \left(\frac{x}{2} - 150\right)\hat{i} + \left(\frac{x\sqrt{3}}{2} - 50\sqrt{3}\right)\hat{j}$

As A sees B at 90° to its line of motion hence the angle between – x axis and $\vec{V}_{BA}$ = 60°

$\tan 60° = \frac{\frac{x\sqrt{3}}{2} - 50\sqrt{3}}{150 - \frac{x}{2}}$

$150 - \frac{x}{2} = \frac{x}{2} - 50$

$x = 200$

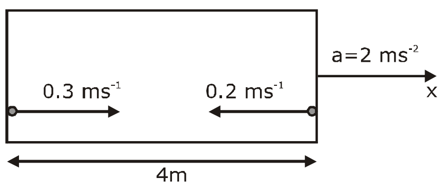
Hence $\vec{V}_{BA} = -50\hat{i} + 50\sqrt{3}\hat{j}$

$|\vec{V}_{BA}| = \sqrt{(50)^2 + (50\sqrt{3})^2}$

$= 50 \times 2 = 100$ m/s

Thus time to collide $= \frac{500}{100} = 5$ sec

18. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber is +x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in -x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Sol. **0002 & 0008**

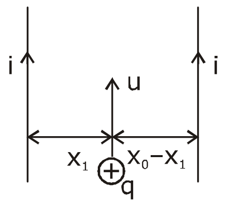
$4 = 0.2 \times t \times 1/2 \times 2 \times t^2$

$\Rightarrow t = 1.9$

$\simeq 2$ sec.

19. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, it the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{x_1} = 3$, the value of $\frac{R_1}{R_2}$ is

Sol. **0003**



Same direction:

$$B_1 = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x_1} - \frac{1}{x_0 - x_1}\right)$$

$$= \frac{\mu_0 i (x_0 - 2x_1)}{2\pi x_1 (x_0 - x_1)}$$

$$qVB = \frac{Mv^2}{r} \Rightarrow r = \frac{MV}{qB}$$

Opp. direction :

$$B_2 = \frac{\mu_0 i}{2\pi}\left(\frac{1}{x_1} + \frac{1}{x_0 - x_1}\right)$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{x_0}{x_1(x_0 - x_1)}\right]$$

$$\text{Thus } \frac{r_1}{r_2} = \frac{B_2}{B_1} = \frac{x_0}{x_1(x_0 - x_1)} \times \frac{(x_0 - x_1)x_1}{(x_0 - 2x_1)}$$

$$\frac{r_1}{r_2} = \frac{x_0}{(x_0 - 2x_1)} = \frac{3x_1}{(3x_1 - 2x_1)} = 3$$

20. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

Sol. **0004**

Acc. to S7earle's experiment:

$$\frac{mg}{A} = \frac{y\Delta\ell}{\ell}$$

$$\Rightarrow y = \frac{mg\ell}{A\Delta\ell}$$

$$\frac{dy}{y} = \frac{d(\Delta\ell)}{d\ell}$$

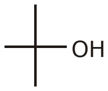
$$\Rightarrow \frac{dy}{y} = \frac{1\times10^{-5}}{25\times10^{-5}} \times 100$$

= 4 %

= 0004

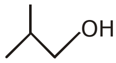
21. The correct combination of names of isomeric alcohols with molecular formula $C_4H_{10}O$ is/are:
(A) tert-butanol and 2-methylpropan-2-ol
(B) tert-butanol and 1, 1-dimethylethan-1-ol
(C) n-butanol and butan-1-ol
(D) isobutyl alcohol and 2-methylpropane-1-ol

Sol. **ACD**
isomeric alcohol of $C_4H_{10}O$.



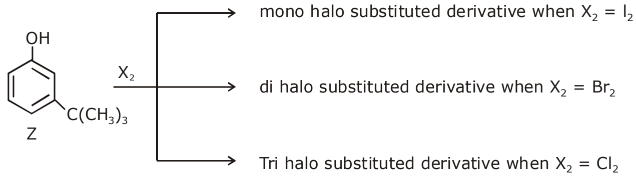
is ter. butanol or 2-methyl propan-2-ol

is n-butanol or butan-1-ol



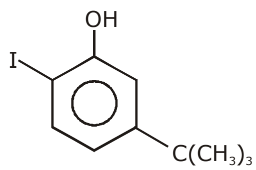
is iso butyl alcohol or 2-methyl propan-1-ol.

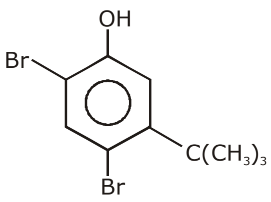
22. The reactivity of compound Z with different halogens under appropriate conditions is given below :

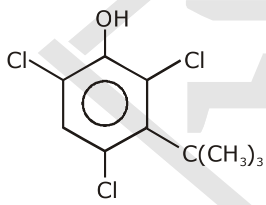


The observed pattern of electrophilic substitution can be explain by :
(A) the steric effect of the halogen
(B) the steric effect of the tert-butyl group
(C) the electronic effect of the phenolic group
(D) the electronic effect of the tert-butyl group

Sol. **ABC**

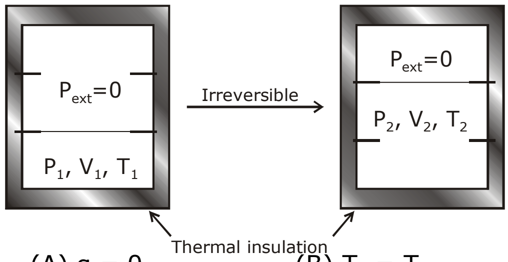
Products




and


are explained by steric effect of halogen & t-butyl group and electronic effect of phenolic group.

23. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$ (B) $T_2 = T_1$
(C) $P_2V_2 = P_1V_1$ (D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

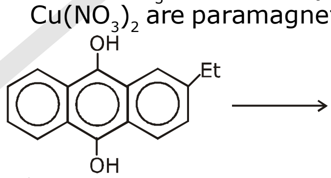
Sol. **ABC**
(A) $q = 0$ adiabatic process
(B) $T_2 = T_1$
since $P_{ext} = 0$, $w = 0$
$\Delta U = w + q = 0 + 0$
(C) $P_2V_2 = P_1V_1$
since n and T are constant
(D) PV^{γ} = constant
is applicable only for ideal gas in **reversible** adiabatic process.

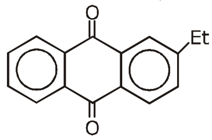
24. The correct statement(s) for the orthoboric acid is/are :
(A) It behaves as a weak acid in water due to self ionization.
(B) Acidity of its aqueous solution increases upon addition of ethylene glycol.
(C) It has a three dimensional sructure due to hydrogen bonding.
(D) It is a weak electrolyte in water.

Sol. **BD**

25. The pair(s) of reagents that yield paramagnetic species is/are :
(A) Na and excess of NH_3
(B) K and excess of O_2
(C) Cu and dilute HNO_3
(D) O_2 and 2-ethylanthraquinol

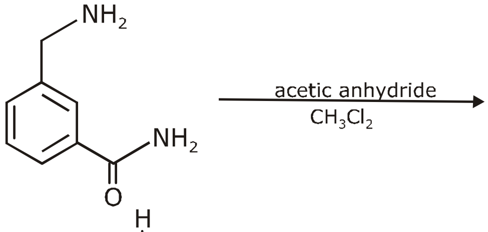
Sol. **ABC**
$Na + NH_3 \rightarrow$ Paramagnetic solution due to presence of solvated electron
$K + O_2 \rightarrow KO_2$ (O_2^- ion is paramagnetic, MOT)
$Cu + HNO_3 \rightarrow NO + Cu(NO_3)_2$ (both NO and $Cu(NO_3)_2$ are paramagnetic)

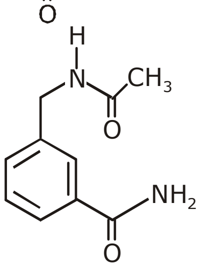
O_2 +


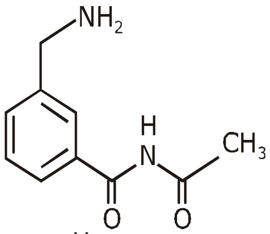


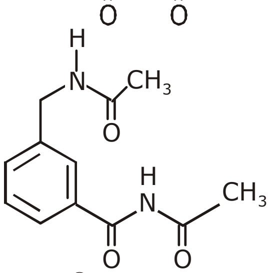
+ H_2O_2 (both are dimagnetic)

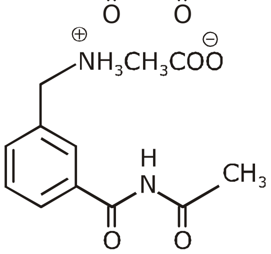
26. In the reaction shown, below, the major product(s) formed is/are :



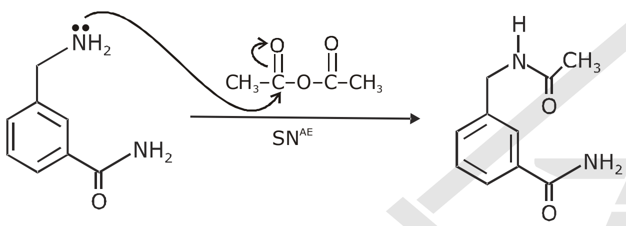
(A)  + CH_3COOH

(B)  + CH_3COOH

(C)  + H_2O

(D) 

Sol. **A**



+ CH_3COOH

27. In a galvanic cell, the salt bridge
(A) does not participate chemically in the cell reaction.
(B) stops the diffusion of ions from one electrode to another.
(C) is necessary for the occurrence of the cell reaction.
(D) ensures mixing of the two electrolytic solutions.

Sol. **AD**

28. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are :
(A) $CuFeS_2$ (B) CuO
(C) Cu_2O (D) $CuSO_4$

Sol. **C**
$Cu_2O + Cu_2S \rightarrow Cu + SO_2$ (self reduction)

29. Hydrogen bonding plays a central role in the following phenomena :
(A) ice floats in water.
(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions.
(C) Formic acid is more acidic than acetic acid.
(D) Dimerisation of acetic acid is benzene.

Sol. **AD**

30. For the reaction :
$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$
The correct statement(s) in the balanced equation is/are :
(A) Stoichiometric coefficient of HSO_4^- is 6.
(B) Iodide is oxidized
(C) Sulphur is reduced
(D) H_2O is one of the products.

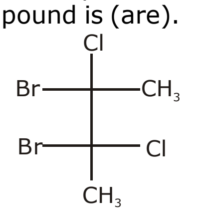
Sol. **ABD**

$$6I^- + \underset{nf\,=\,1}{ClO_3^-} + \underset{nf\,=\,6}{6H_2\overset{+6}{S}O_4} \longrightarrow Cl^- + 6H\overset{+6}{S}O_4^- + 3I_2 + 3H_2O$$

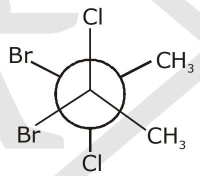
31. A list of species having the formula XZ_4 is given below.
XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$
Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is.

Sol. **4**
XeF_4, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[PtCl_4]^{2-}$

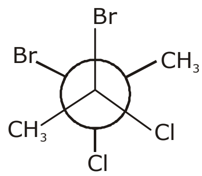
32. The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are).



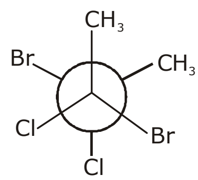
Sol. **3**



$\mu \neq 0$,



$\mu \neq 0$,



$\mu \neq 0$.

33. Consider the following list of reagents : Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$. The total number of reagents that can oxidise aqueous iodide to iodine is.

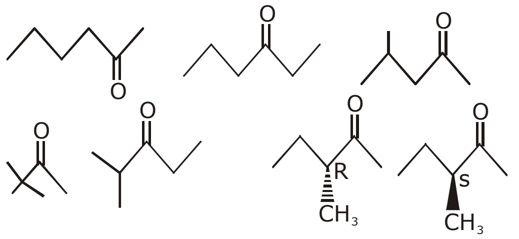
Sol. **6**
$K_2Cr_2O_7$, $CuSO_4$, H_2O_2 , Cl_2, O_3, $FeCl_3$
These compound liberats iodine from iodide ion.

34. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total numer of BLACK coloured sulphides is :

Sol. **7**
PbS, CuS, HgS, Ag_2S, NiS, CoS, Bi_2S_3
These compound are black in colour.

35. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (NOTE : stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is/are.

Sol. **7**
Ketone with Mw = 100 is $C_6H_{12}O$



36. MX_2 dissociated into M^{2+} and X^- ions in an aqueous solution with a degree of dissociation (at of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is .

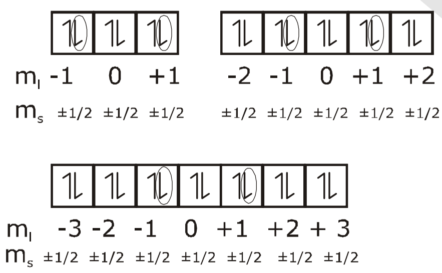
Sol. **2**

$$\begin{array}{lll} MX_2 & \longrightarrow & M^{2+} + 2X^- \\ 1 & & - \quad\quad - \\ 1-0.5 & & \\ = 0.5 & & 0.5 \quad\quad 1 \end{array}$$

$$\frac{(\Delta T_f)_{obs}}{(\Delta T_f)_{th}} = i = 2$$

37. In an atom, the total number of electrons having quantum numbers n = 4, $|m_l|$ = 1 and $m_s = -\frac{1}{2}$ is.

Sol. **6**
n = 4
l = 0, 1, 2 , 3
$|m_l|$ = 1 (only in p, d & f orbital)
m_s = –1/2



38. A compound H_2X with molar weight of 80 g is dissolved in a solvent having density of 0.4 g m^{-1} . Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is.

Sol. **8**
Let's take 1 litre of solution since its molarity is 3.2 it should contain 3.2 moles of solute.
According to question on mixing 3.2 moles solute to one litre (1000 ml) solvent above solution can be prepared since no change in volume upon dissolution.

$$m = \frac{n}{W/1000}$$

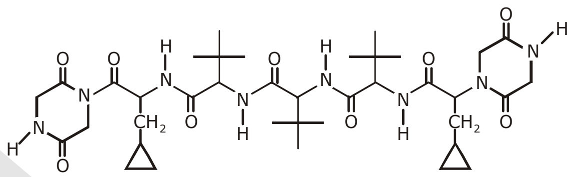
here n = moles of solute
W = weight of solvent in grams
W = V × d
= 1000 ml × 0.4 gm/ml
= 400 gm

$$m = \frac{3.2}{400/1000} = \mathbf{8}$$

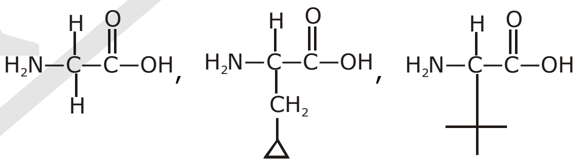
39. If the value of Avogadro number is 6.023 × 10^{23} mol^{-1} and the value of Boltzmann constant is 1.380 × 10^{-23} J K^{-1}, then number of significant digits in the calculated value of the universal gas constant is.

Sol. **4**
k = 1.380 × 10^{-23} J K^{-1} has 4 significant digits and N_A = 6.023 × 10^{23} mol^{-1} has 4 significant digits
hence R = k × N_A has also **four** significant digits

40. The total number of distinct naturally occurring amino acids obtained by complete acidic hydrolysis fo the peptide shown below is :



Sol. **3**
Distinct naturally occuring amino acids mean α-amine acids.



MATHEMATICS

41. Let f : $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by

$$f(x) = (\log(\sec x + \tan x))^3$$

Then
(A) f(x) is an odd function
(B) f(x) is an one-one function
(C) f(x) is an onto fucntion
(D) f(x) is an even function

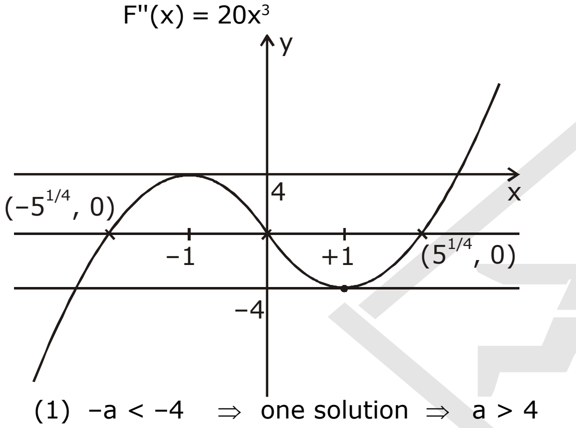
Sol. **A,B,C**

f : $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\} \to R$

$f(x) = (\log(\sec x + \tan x))^3$
$f'(x) = 3(\log(\sec x + \tan x))^2 \sec x$
$f'(-x) = 3(\log(\sec x - \tan x))^2 \sec x = f'(x)$
$\Rightarrow$ f'(x) is even $\Rightarrow$ f(x) is odd

42. Let $a \in R$ and let $f : R \to R$ be given by
$f(x) = x^5 - 5x + a$
Then
(A) f(x) has three real roots if a > 4
(B) f(x) has only one real root if a > 4
(C) f(x) has three real roots if a < − 4
(D) f(x) has three real roots if −4 < a < 4

Sol. **B,D**
$f(x) : R \to R$ $a \in R$
$f(x) = x^5 - 5x + a$
Consider $x^5 - 5x + a = 0$
or $x^5 - 5x = -a$
let $f(x) = x^5 - 5x$ $g(x) = -a$
$= x(x^4 - 5)$
$f'(x) = 5x^4 - 5 = 5(x^2 + 1)(x^2 - 1)$
$F''(x) = 20x^3$



(1) $-a < -4$ $\Rightarrow$ one solution $\Rightarrow$ $a > 4$
(2) $-4 < -a < 4$ $\Rightarrow$ 3 solution $\Rightarrow$ $4 > a > -4$
(3) $-a = 4, -4$ $\Rightarrow$ 2
(4) $-a > 4$ $\Rightarrow$ one solution $\Rightarrow$ $a < -4$

43. Let $f:(0, \infty) \to R$ be given by

$$f(x) \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}.$$

Then
(A) f(x) is monotonically increasing on $[1, \infty)$
(B) f(x) is monotonically decreasing on (0, 1)
(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
(D) $f(2^x)$ is an odd function of x on R

Sol. **A,C,D**

$$f'(x) = \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} - e^{-\left(x+\frac{1}{x}\right)}\left(x\left(-\frac{1}{x^2}\right)\right)$$

$$= \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} + \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} = \frac{2.e^{-\left(x+\frac{1}{x}\right)}}{x} > 0$$

(A) $f(2^x) = \int_{2^{-x}}^{2^x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$

$f(2^x) = -f(2^x)$ $\Rightarrow$ f is odd (D)

44. For every pair of continuous functions f, g : $[0, 1] \to R$ such that $\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0,1]\}$, the correct statement(s) is (are):
(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$
(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0,1]$
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0,1]$
(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0,1]$

Sol. **A,D**
Let Max. f(x) at $x = c_1$
& Max. g(x) at $x = c_2$
then $f(c_1) - g(c_1) \geq 0$ &
$f(c_2) - g(c_2) \leq 0$
So $f(c) - g(c) = 0$ where $c \in [c_1, c_2]$
By IVT

45. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then
(A) determinant of $(M^2 + MN^2)$ is 0
(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(C) determinant of $(M^2 + MN^2) \geq 1$
(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

Sol. **A,B**

$M^2 - N^4 = 0$

$\Rightarrow \quad (M + N^2)(M - N^2) = 0 \quad (\because\ MN = NM)$

as $\quad M - N^2 \neq 0$

$\Rightarrow \quad |M + N^2| = 0$

So A & B are correct options

46. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) Radius of S is 8 (B) radius of S is 7

(C) centre of S is (–7, 1)

(D) centre of S is (–8, 1)

Sol. **B,C**

Let S : $x^2 + y^2 + 2gx + 2fy + c = 0$

pass (0, 1)

$1 + 2f + c = 0$

S & S_1 orthogonal

$2(-1)(g) + 2(0)(f) = c - 15$

$-2g = c - 15$

S & S_2 or thogonal

$2(0)(g) + 2(0)(f) = c - 1$

$c - 1 = 0$

$c = 1, g = 7 \Rightarrow f = -1$

S : $x^2 + y^2 + 14x - 2y + 1 = 0$

r = 7

C : (–7, 1)

47. Let M be a 2 × 2 symmetric matrix with integer entries. Them M is invertible if

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Sol. **C,D**

(A) M $\begin{bmatrix} a & a \\ a & a \end{bmatrix} \Rightarrow$ M is not invertible

(B) M = $\begin{bmatrix} a & a \\ a & a \end{bmatrix} \Rightarrow$ M is not invertible

(C) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \Rightarrow$ M is invertible

(D) M = $\begin{bmatrix} b & a \\ a & c \end{bmatrix} \Rightarrow |M| = bc - a^2$ clear M is invertible.

48. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

(A) $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$

(B) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$

(C) $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$

(D) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{z} - \vec{y})$

Sol. **A,B,C**

Given $\vec{x}.\vec{y} = 1$

$\vec{y}.\vec{z} = 1$

$\vec{z}.\vec{x} = 1$

Now $\vec{a} = \lambda(\vec{x} \times (\vec{y} \times \vec{z}))$

$\vec{a} = \lambda((\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z})$

$\vec{a} = \lambda(\vec{y} - \vec{z})$

Now $\vec{a} \cdot \vec{y} = \lambda(y^2 - \vec{y} \cdot \vec{z})$

$\vec{a} \cdot \vec{y} = \lambda(2 - 1)$

$\lambda = \vec{a} \cdot \vec{y}$

$\Rightarrow \quad \vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$ (B)

Similarly $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$ (A)

$\vec{a} \cdot \vec{b} = (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})\{(\vec{y} - \vec{z}) \cdot (\vec{z} - \vec{x})\}$

$= (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})\{1 - 2 - 1 + 1\}$

$\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$ (C)

49. Let f : [a, b] → [1, ∞) be a continuous function and let g : R → R be defined as

$$f(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t)dt & \text{if } a \le x \le b, \\ \int_a^b f(t)dt & \text{if } x > b \end{cases}$$

Then

(A) g(x) is continuous but not differentiable at a

(B) g(x) is differentiable on R

(C) g(x) is continuous but not differentiable at b

(D) g(x) is continuous and differentiable at either a or b but not both

Sol. **A,C**

f : [a, b] → [1, ∞)

(A) g(x) is continuous

at x = a

Clearly not derivable

as f(a) ≥ 1 which is RHD

while LHD is 0

is (B) in wrong

again (C) is correct as at x = b

continous But not derivable

(D) is wrong

So, A & C

50. From a point P($\lambda, \lambda, \lambda$), perpendiculars PQ and PR are drawn respectively on the lines y = x, z = 1 and y = – x, z = – 1. If P is such that $\angle$QPR is a right angle, then the possible value (s) of λ is(are)

(A) $\sqrt{2}$ (B) 1

(C) –1 (D) $-\sqrt{2}$

Sol. **C**

y = x, z = 1

y = – x, z = – 1

$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = \mu$

Q(μ, μ, 1)

$\frac{x}{-1} = \frac{y}{1} = \frac{z+1}{0} = \upsilon$

R (–υ, υ, –1)

$\overrightarrow{QP} \cdot \overrightarrow{PR} = 1 \left[(\lambda-\mu)\hat{i} + (\lambda-\mu)\hat{j} + (\lambda-1)\hat{k}\right]$

$\left[(\lambda+\upsilon)\hat{i} + (\lambda-\upsilon)\hat{j} + (\lambda+1)\hat{k}\right] = 0$

$(\lambda - \mu)(\lambda + \upsilon) + (\lambda - \mu)(\lambda - \upsilon) + \lambda^2 - 1 = 0$

$\lambda^2 + \upsilon\lambda - \mu\lambda - \upsilon\mu + \lambda^2 - \upsilon\lambda - \mu\lambda + \mu\upsilon + \lambda^2 - 1 = 0$

$3\lambda^2 - 2\mu\lambda - 1 = 0$ ...(i)

$\overrightarrow{PQ} \cdot (\hat{i}+\hat{j}) = 0$

$\lambda - \mu + \lambda - \mu = 0$

$\lambda = \mu$ ...(ii)

and $\overrightarrow{PR} \cdot (-\hat{i}+\hat{j}) = 0$

$-(\lambda + \upsilon) + \lambda - \upsilon = 0$

$\upsilon = 0$ ...(iii)

from (i) (ii) and (iii)

(i) $\Rightarrow 3\lambda^2 - 2\lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$

But at $\lambda = 1$

P $\equiv$ Q

So answer is (C)

51. The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}(1-x^2)^5\right\} dx$ is

Sol. **0002**

$\int_0^1 \underbrace{4x^3}_{I} \underbrace{\left(\frac{d^2}{dx^2}(1-x^2)^5\right)}_{II} dx$

$= 4x^3 \cdot \frac{d}{dx}(1-x^2)^5 \Big|_0^1 - \int_0^1 12x^2 \cdot \frac{d}{dx}(1-x^2)^5 dx$

$= -\int_0^1 12x^2(-2x)\, 5(1-x^2)^4\, dx$

$= 0+60 \int_1^0 2x\,(x^2)(1-x^2)^4\, dx$ (put $1 - x^2 = t$)

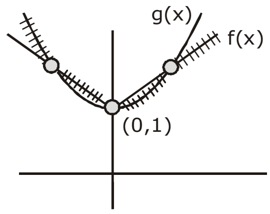
$= -60 \int_1^0 (1-t)(t)^4\, dt = 60\left[\frac{1}{30}\right] = 2$

52. Let f : R $\to$ R and g : R $\to$ R be respectively given by f(x) = [x] + 1 and g(x) = x^2 + 1. Define h : R $\to$ R by

$h(x) = \begin{cases} \max \{f(x), g(x)\} & \text{if } x \le 0, \\ \min \{f(x), g(x)\} & \text{if } x > 0 \end{cases}$

The number of points at which h(x) is not differentiable is

Sol. **0003**



f(x) = | x | + 1

g(x) = x^2 + 1

$h(x) = \begin{bmatrix} \max(f(x), f(g)) & x \le 0 \\ \min(f(x), g(x)) & x > 0 \end{bmatrix}$

No of non differentiable points = 3.

53. Let n $\ge$ 2 be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of of n is

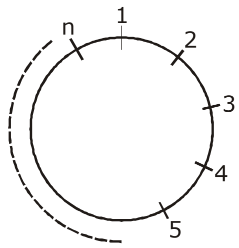
Sol. **0005**

n $\ge$ 2

n-Blue lines

total lines = ${}^nC_2$

Red lines = ${}^nC_2$ – n



given

${}^nC_2$ – n = n

$\frac{n(n-1)}{2} = 2n$

$n^2 - n = 4n$

$n^2 = 5n$

n = 5.

54. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is

Sol. **0008**

$(y - x^5)^2 = x(1 + x^2)^2$ (1, 3)

$2(y - x^5).\left(\frac{dy}{dx} - 5x^4\right) = x.2(1 + x^2).2x + (1 + x^2).1$

$2(y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)(5x^2 + 1)$

$2(3 - 1)\left(\frac{dy}{dx} - 5\right) = (1 + 1)(5 + 1) = 12$

$\frac{dy}{dx} - 5 = 3$

$\frac{dy}{dx} = 8$

Similar to teaching notes

55. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

Sol. **0004**

$(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) = p\vec{a} + q\vec{b} + r\vec{c}$

Taking dot product with $\vec{a}, \vec{b}$ & $\vec{c}$

$0 = \frac{p}{2} + q + \frac{r}{2} \quad \Rightarrow \quad p + 2q + r = 0$

$0 + \frac{1}{\sqrt{2}} = p + \frac{q}{2} + \frac{r}{2} \Rightarrow 2p + q + r = \sqrt{2}$

$\frac{1}{\sqrt{2}} + 0 = \frac{p}{2} + \frac{q}{2} + r \Rightarrow p + q + 2r = \sqrt{2}$

$\Rightarrow p = \frac{1}{\sqrt{2}}, q = -\frac{1}{\sqrt{2}}, r = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{p^2 + 2q^2 + r^2}{q^2} = \frac{\frac{1}{2} + 1 + \frac{1}{2}}{\frac{1}{2}} = 4$

56. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

Sol. **0004**

a, ar, ar^2

$\frac{a(1 + r + r^2)}{3} = ar + 2$

$a + ar + ar^2 = 3ar + 6$

$a + ar^2 = 2ar + 6$

$a(r - 1)^2 = 6 \Rightarrow a = 6, r - 1 = 1$

$\Rightarrow \frac{a^2 + a - 14}{a + 1} = \frac{36 + 6 - 14}{7} = 4.$

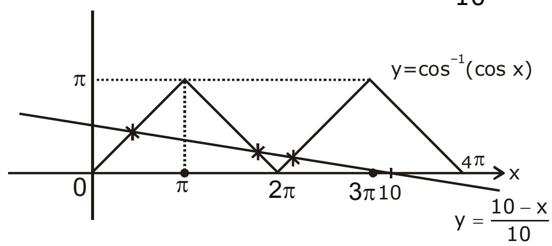
57. Let $f:[0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation.

$f(x) = \frac{10 - x}{10}$ is

Sol. **0003**

$f : [0, 4\pi] \to [0, \pi]$

$f(x) = \cos^{-1}(\cos x) \qquad f(x) = \frac{10 - x}{10}$



Number of solution = number of points of intersection = 3

58. The largest value of the non-negative integer a for which

$\lim_{x \to 1}\left\{\frac{-ax + \sin(x - 1) + a}{x + \sin(x - 1) - 1}\right\}^{\frac{1 - x}{1 - \sqrt{x}}} = \frac{1}{4}$ is

Sol. **0002**

$\lim_{x \to 1}\left\{\frac{-a(x - 1) + \sin(x - 1)}{(x - 1) + \sin(x - 1)}\right\}^{1 + \sqrt{x}} = \frac{1}{4}$

$\lim_{x \to 1}\left\{\frac{-a + \frac{\sin(x - 1)}{x - 1}}{1 + \frac{\sin(x - 1)}{(x - 1)}}\right\}^{1 + \sqrt{x}} = \frac{1}{4}$

$\left\{\frac{-a + 1}{1 + 1}\right\}^2 = \frac{1}{4}$

$(-a + 1)^2 = 1$

$-a + 1 = 1$ or $-a + 1 = -1$

$a = 0$ or $a = 2$

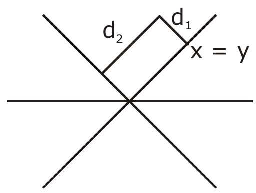
Max value of a = 2

59. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$ is

Sol. **0006**

$x - y = 0$ & $x + y = 0$

$$d_1 = \left|\frac{h-k}{\sqrt{2}}\right|, \; d_2 = \left|\frac{h+k}{\sqrt{2}}\right|$$



$2 \le d_1 + d_2 \le 4$

$$2 \le \left|\frac{h-k}{\sqrt{2}}\right| + \left|\frac{h+k}{\sqrt{2}}\right| \le 4$$

$$2\sqrt{2} \le |h - k| + |h + k| \le 4\sqrt{2}$$

(i) $\dfrac{\text{only for 1st quad}}{h>0,\ k \ge 0}$

(A) $h \ge k$ $\quad 2\sqrt{2} \le h - k + h + k \le 4\sqrt{2}$

$\sqrt{2} \le h \le 2\sqrt{2}$

(B) $h < k$ $\quad 2\sqrt{2} \le k - h + h + k \le 4\sqrt{2}$

$$\frac{\sqrt{2} \le k \le 2\sqrt{2}}{A = (2\sqrt{2})^2 - (\sqrt{2})^2}$$

$A = 8 - 2 = 6.$

60. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Sol. **0007**

Only 7 possible combination and each combination can be arranged in only one way

1, 2, 3, 4 10
1, 2, 3, 5 9
1, 2, 3, 6 8
1, 2, 4, 5 8
1, 2, 4, 6 7
1, 3, 4, 5 7
2, 3, 4, 5 6
∴ Ans = 7