TEST PAPER
PRE-REGIONAL MATHEMATICAL OLYMPIAD, 2017

1. How many positive integer less than 1000 have the property that the sum of the digits of each such number is divisible by 7 and the number itself is divisible by 3 ?

2. Suppose a, b are positive real numbers such that $a\sqrt{a}+b\sqrt{b}=183 \cdot a\sqrt{b}+b\sqrt{a}=182$. Find $\frac{9}{5}(a+b)$.

3. A contractor has two team of workers: team A and team B. Team A can complete a job in 12 days and team B can do the same job in 12 days and team B can do the same job in 36 days. Team A starts working on the job and team B joins team A after four days. The team A withdraws after two more days. For how many more days should team B work to complete the job ?

4. Let a, b be integers such that all the roots of the equation $(x^2+ax+20)(x^2+17x+b) = 0$ are negative integers. What is the smallest possible value of a + b ?

5. Let u, v, w be real numbers in geometric progression such that u > v > w. Suppose $u^{40} = v^n = w^{60}$. Find the value of n.

6. Let the sum $\sum_{n=1}^{9}\frac{1}{n(n+1)(n+2)}$ written in its lowest terms be $\frac{p}{q}$. Find the value of q–p.

7. Find the number of positive integers n, such that $\sqrt{n}+\sqrt{n+1}<11$.

8. A pen costs ₹ 11 and a notebook costs ₹ 13. Find the number of ways in which a person can spend exactly ₹ 1000 to buy pens and notebooks.

9. There are five cities A, B, C, D, E, on a certain island. Each city is connected to every other city by road. In how many ways can a person starting from city A come back to A after visiting some cities without visiting a city more than once and without taking the same road more than once? (The order in which he visits the cities also matters: e.g. the routes $A \to B \to C \to A$ and $A \to C \to B \to A$ are different.)

10. There are eight rooms on the first floor of a hotel. with four rooms on each side of the corridor. symmetrically situated (that is each room is exactly opposite to one other room). Four guests have to be accommodated in four of the eight rooms (that is one in each) such that no two guests are in adjacent room. In how ways can the guests be accommodated?

11. Let f(x) = $\sin\frac{x}{3}+\cos\frac{3x}{10}$ for all real x. Find the least natural number n such that $f(n\pi+x) = f(x)$ for all real x.

12. In a class, the total numbers of boys and girls are in the ratio 4:3. On one day it was found that 8 boys and 14 girls were absent from the class. and that the number of boys was the square of the number of girls. What is the total number of students in the class ?

13. In a rectangle ABCD, E is the midpoint of AB; F is a point on AC such that BF is perpendicular to AC; and FE perpendicular to BD. Suppose BC = $8\sqrt{3}$. Find AB.

14. Suppose x is a positive real number such that {x}, {x} and x are in a geometric progression. Find the least positive integer n such that $x^n > 100$. (Here [x] denotes the integer part of x and {x} = x–[x].)

15. Integers 1,2,3......n, where n > 2, are written on a board. Two numbers m,k such that 1 < m < n, 1 < k < n are removed and the average of the remaining numbers is found to be 17. What is the maximum sum of the two removed numbers ?

16. Five distinct 2-digit numbers are in a geometric progression. Find the middle term.

17. Suppose the altitudes of a triangle are 10, 12 and 15. What is its semi-perimeter ?

18. If the real numbers x,y,z are such that $x^2+4y^2+16z^2 = 48$ and $xy+4yz+2zx = 24$, What is the value of $x^2+y^2+z^2$?

19. Suppose 1,2,3 are the roots of the equation $x^4+ax^2+bx = c$. Find the value of c.

20. What is the number of triples (a,b,c) of positive integers such that (i) a < b < c < 10 and (ii) a,b,c,10 form the sides of a quadrilateral ?

21. Find the number of ordered triples (a,b,c) of positive intergers such that abc=108.

22. Suppose in the plane 10 pairwise nonparallel lines intersect one another. What is the maximum possible number of polygons (with finite areas) that can be formed ?

23. Suppose an integer x, a natural number n and a prime number p satisfy the equation $7x^2-44x+12 = p^n$. Find the largest value of p.

24. Let P be an interior point of a triangle ABC whose side lengths are 26. 65. 78. The line through P parallel BC meets AB in K and AC in L. The line through P parallel to CA meets BC in M and BA in N. The line through P parallel to AB meets CA in S and CB in T. If KL, MN, ST are of equal lengths. find this common length.

25. Let ABCD be a rectangle and let E and F be points on CD and BC respectively such that area (ADE) = 16, area(CEF) = 9 and area (ABF) = 25. What is the area of triangle AEF ?

26. Let AB and CD be two parallel chords in a circle with radius 5 such that the centre O lies between these chords. Suppose AB = 6, CD = 8. Suppose further that the area of the part of the circle lying between the chords AB and CD is $(m\pi+n)/k$, where m, n, k are positive integers with gcd(m, n, k) = 1 What is the value of m+n+k ?

27. Let Ω be a circle with centre O and let AB be a diameter of Ω_1. Let P be a point on the segment OB different from O. Suppose another circle Ω_2 with centre P lies in the interior of Ω_1. Tangents are drawn from A and B to the circle Ω_2 intersecting Ω_1 again at A_1 and B_1 respectively such that A_1 and B_1 are on the opposite sides of AB. Given that $A_1B = 5$, $AB_1 = 15$ and OP = 10, Find the radius of Ω_1.

28. Let p,q be prime numbers such that $n^{3pq} - n$ is a multiple of 3pq for all positive integers n. Find the least possible value of p+q.

29. For each positive integer n, consider the highest common factor h_n of the two numbers n!+1 and (n+1)!. For n < 100. Find the largest value of h_n.

30. Consider the areas of the four triangles obtained by drawing the diagonals AC and ED of a trapezium ABCD. The product olthese areas, taken two at time. are computed. If among the six products so obtained two products are 1296 and 576, determine the square root of the maximum possible area of the trapezium to the nearest integer.

ANSWER KEY
PRE-REGIONAL MATHEMATICAL OLYMPIAD, 2017

1.	28	**2.**	73	**3.**	16	**4.**	25
5.	48	**6.**	83	**7.**	29	**8.**	7
9.	60	**10.**	48	**11.**	60	**12.**	42
13.	24	**14.**	10	**15.**	51	**16.**	36
17.	Bonus	**18.**	21	**19.**	36	**20.**	73
21.	60	**22.**	46	**23.**	47	**24.**	Bonus
25.	30	**26.**	75	**27.**	20	**28.**	28
29.	97	**30.**	13				

SOLUTION
PRE-REGIONAL MATHEMATICAL OLYMPIAD, 2017

1. a + b + c = 3k

a + b + c = $7k_2$

$\therefore$ a + b + c = $21k_1K_2$

(i) When digits are 3/9/9 = $\frac{3!}{2!} = \frac{6}{2} = 3$

(ii) When digits are 4/8/9 = 3! = 6

(iii) When digits are 5/7/9 = 3! = 6

8 8 = $\frac{3!}{2!} = 3$

(iv) When digits are 6/6/9 = $\frac{3!}{2!} = 3$

6/7/8 = 3! = 6

(v) When digits are 7/7/7 = 1

Total = 28

2. Let $a = A^2$, $b = B^2$

$A^3 + B^3 = 183$ ....(i)

$A^2B + B^2A = 182$ ....(ii)

(i) + (ii)

$A^3 + B^3 + 3AB(A+B) = 183 + 3 \times 182$

$(A+B)^3 = 183 + 546$

$(A+B)^3 = 729$

$\Rightarrow A + B = 9$

from (ii) $\rightarrow$ AB (A+B) = 182

$AB = \frac{182}{9}$

$a + b = A^2 + B^2 = (A+B)^2 - 2AB$

$= 81 - \frac{364}{9} = \frac{365}{9}$

$\frac{9}{5} \times \frac{365}{9} = 73$

3. Work done by A in one day = $\frac{1}{12}$

Work den by B in on day = $\frac{1}{36}$

And, let x be number of days in which work is completed by B.

$\therefore \quad \frac{6}{12} + \frac{2+x}{36} = 1$

$\frac{20+x}{36} = 1$ $\boxed{x = 16 \text{ days}}$

4. $(x^2+ax+20)\ (x^2+17x+b) = 0$

z^- z^- z^- z^-

so a > 0 and b > 0 since sum of roots < 0 and product > 0

(since 20 = (1×20) × (2×10) or (4×5)

min a = 9

$-17 = \alpha+\beta \Rightarrow (\alpha\beta) = (-1, -16), (-2, -15), (-8, -9)$

min b = 16

$(a+b)_{min} = a_{min} + b_{min} = 9+16 = 25$

5. $u = ar$, $v = a$, $w = \frac{a}{r}$

$u^{40} = v^n = w^{60}$

$(ar)^{40} = (a)^n = \left(\frac{a}{r}\right)^{60} = K$

$ar = k^{1/40}$ ....(i)

$a = k^{1/n}$ ....(ii)

$\frac{a}{r} = k^{1/60}$ ....(iii)

$\frac{ar}{r} = \frac{k^{1/40}}{k^{1/n}}$ $\frac{a \times r}{a} = k^{1/n - 1/60}$

$r = k^{1/40 - 1/n}$ r = $r = k^{1/n - 1/60}$

$\frac{1}{40} - \frac{1}{n} = \frac{1}{n} - \frac{1}{60}$

$\frac{1}{40} + \frac{1}{n} = \frac{1}{n} + \frac{1}{n}$

$\frac{3+2}{120} = \frac{2}{n}$

$\frac{5}{120} = \frac{2}{n}$ $\boxed{n = 48}$

6. $\frac{1}{2} \sum_{n=1} \frac{2}{n(n+1)\ (n+2)}$

$\frac{1}{2} \sum_{n=1} \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}$

$\frac{1}{2}\left[\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{2.3} - \frac{1}{3.4} + ... + \frac{1}{9.10} - \frac{1}{10.11}\right]$

$\frac{1}{2}\left[\frac{1}{1.2} - \frac{1}{10.11}\right] = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{100}\right)$

$\frac{1}{2}\left[\frac{55-1}{110}\right]$

$\frac{1}{2}\left[\frac{54}{110}\right] = \frac{27}{110}$

7. $\sqrt{n} + \sqrt{n+1} < 11$

$\sqrt{n+1} < 11 - \sqrt{n}$

$\left(\sqrt{n+1}\right)^2 < \left(11 - \sqrt{n}\right)^2$

$n + 1 < 11^2 - 2 \times 11\ \sqrt{n}\ + n$

$1 < 121 - 22\sqrt{n}$

$-120 < -22\ \sqrt{n}$

$120 > 22\sqrt{n}$

$\sqrt{n} < \frac{120}{22}$

$\sqrt{n} < \frac{60}{11}$

$n < \frac{60}{11} \times \frac{60}{11}$

$n < \frac{3600}{121}$

$n < 29.75$

no of +ve integers $\boxed{n = 29}$

8. cost of a pen in ₹ 11

cost of a notebook is ₹ 13

$y = \frac{1000 - 11x}{13} \in I$

$\frac{12 - 11x}{13} \in 1 \Rightarrow \frac{(13\text{-}1)\text{-}(13x\text{-}2x)}{13} \in 1$

$\frac{2x - 1}{13} \in 1 \Rightarrow \frac{12x\text{-}6}{13} \in 1$

$\frac{13x - (x - 6)}{13} \in 1 \Rightarrow \frac{x\text{-}6}{13} \in 1$

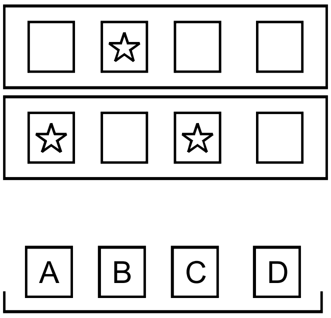
$x = 13\lambda - 6 \quad (1 \le x \le 90)$

$x \in \{7, 20, ...85\}\ \{\lambda = \{1, 2...., 2\}\}$

9. A, B, C, D, E, A

No. of arrangement = $\underline{4}$ = 24

10.



$\underline{|4} \times 2 = 24\times 2 = 48$

11. $f(x) = \sin\frac{x}{3} + \cos\frac{3x}{10}$

$f(n\pi + x) = f(x)$

$f(x+T) = f(x)$

T is fundamental period

$T = \text{LCM}\ \frac{2\pi}{1/3}, \frac{2\pi}{3/10}$

$= \text{LCM}\ \frac{6\pi}{1}, \frac{10\pi}{3}$

$T = 60\pi$

$n\pi = 60\pi$

$\boxed{n = 60}$

12. Total = (4x+3x)

$(4x-8) = (3x-18)^2$

$4x-8 = 9x+196 - 84x$

$9x^2-88x + 204 = 0$

$\frac{88 + \sqrt{7744-7344}}{18}$

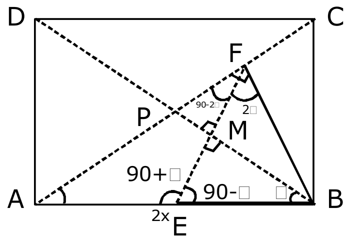
$\frac{88 \pm \sqrt{400}}{18}$

$x = \frac{88 \pm 20}{18}$

$\boxed{x = 6}$

Total = 4×6 + 3×6 = 42

13. $\sin\theta = \frac{8\sqrt{3}}{2x} = \frac{4\sqrt{3}}{x}$



$FA = 2x\cos\theta$

$FB = 2x\sin\theta$

$MB = FB x \sin 2\theta = 2x \sin\theta \sin 2\theta$

$ME^2 + MB^2 = BE^2 = x^2$

$x^2\sin^2\theta + 4x^2\sin^2\theta\sin^2 2\theta = x^2$

$\sin^2\theta + 4\sin^2\theta\sin^2 2\theta = 1$

$4\sin^2\theta\sin^2 2\theta = \cos^2\theta$

$16\sin^4\theta = 1$

$\sin^2\theta = \frac{1}{4} \Rightarrow \sin\theta = \frac{1}{2}$

$\frac{4\sqrt{3}}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = 12 \Rightarrow 2x = AB = 24$

14. $[x]^2 = x\{x\}$

$\{x\} = a$

$[x] = ar$

$x = ar^2$

$a + ar = ar^2$

$r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{1+\sqrt{5}}{2}$

$ar = I$

$a = \frac{2I}{(1+\sqrt{5})}$

$a = \frac{I(\sqrt{5}-1)}{2}$

$0 < a < 1$

$$0<\frac{I(\sqrt{5}-1)}{2}<1$$

$$0<I<\frac{2}{\sqrt{5}-1}$$

$$0<I<\frac{\sqrt{5}+1}{2}$$

$I = 1$

$$ar = 1 \Rightarrow a = \frac{2}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{2}$$

$$x = ar^2 = r = \frac{\sqrt{5}+1}{2}$$

$$\left(\frac{\sqrt{5}+1}{2}\right)^N > 100 \Rightarrow N \log_{10}\left(\frac{\sqrt{5}+1}{2}\right) > 2$$

$N > 9.5 \Rightarrow N_{min} = 10$

15. $$\frac{\frac{n(n+1)}{2}-(2n-1)}{n-2} < 17 < \frac{\frac{n(n+1)}{2}-3}{n-2}$$

$$\frac{n^2+n-4n+2}{2(n-2)} < 17 < \frac{n^2+n-6}{2(n-2)}$$

$$\frac{n^2+3n+2}{2(n-2)} < 17 < \frac{(n+3)(n-2)}{2(n-2)}$$

$$\frac{n-1}{2} < 17 < \frac{n+3}{2}$$

$n < 35$ and $n > 31$

$n = 32, 33, 34$

case–1, $n = 32$

$$\frac{\frac{n(n+1)}{2}-p}{(n-2)} = 17 \Rightarrow \frac{n(n+1)}{2} - 17(n-2) = p$$

$p = 18$

case-2, $n = 33 \Rightarrow p = 34$

case-3, $n = 34 \Rightarrow p = 51$

Maximum sum = 51

16. Let five numbers are $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

het $r = \frac{3}{2}$ and $a = 36$

GP will be 16, 24, 36, 54, 81

middle term = 36

17. $px = qy = rz = 2A$

$$x = \frac{2A}{p}, \quad y = \frac{2A}{q}, \quad z = \frac{2A}{r}$$

$$A = \sqrt{\frac{A}{4}\left(\frac{A}{4}-\frac{2A}{10}\right)\left(\frac{A}{4}-\frac{2A}{12}\right)\left(\frac{A}{4}-\frac{2A}{15}\right)}$$

$$A = \sqrt{\frac{A}{4}\left(\frac{5A-4A}{20}\right)\left(\frac{3A-2A}{12}\right)\left(\frac{15A-8A}{60}\right)}$$

$$A = \sqrt{\frac{A}{4}\left(\frac{A}{20}\right)\left(\frac{A}{12}\right)\left(\frac{7A}{60}\right)}$$

$$A = \sqrt{\frac{7A^4}{57600}}$$

$$A = \frac{A^2\sqrt{7}}{240}$$

$$240A = \sqrt{7}A^2$$

$$A = \frac{240}{\sqrt{7}}$$

$$P = \frac{48}{\sqrt{7}} + \frac{40}{\sqrt{7}} + \frac{32}{\sqrt{7}} = \frac{120}{\sqrt{7}}$$

$$= \frac{60}{\sqrt{7}}$$

18. $x^2 + 4y^2 + 16z^2 = 48$ ....(i)

$xy + 4yz + 2zx = 24$ ....(ii)

$x^2 + 4y^2 + 16z^2 = 2(xy + 4zy + 2zx)$

$2x^2 + 8y^2 + 32z^2 - 4xy - 16zy - 8zx = 0$

$x^2 + 4y^2 - 4xy + 4y^2 + 16z^2 - 16yz + x^2 + 16z^2$
$-82x = 0$

$(x-2y)^2 + (2y-4z)^2 + (x-4z)^2 = 0$

$x^2 + y^2 + z^2 = 0$

i.e. $x = 0$, $y = 0$, $z = 0$

$x-2y = 0$, $2y-4z = 0$, $x-45 = 0$

$x = 2y$, $y = 2z$, $x = 4z$

$x = 2y = 4z$

$\frac{x}{4} = \frac{y}{2} = \frac{z}{1} = \lambda$

$x = 4\lambda$, $y = 2\lambda$, $z = \lambda$

$x^2 + 4y^2 + 16^2 = 48$

$16\lambda^2 + 16\lambda^2 + 16\lambda^2 = 48 \Rightarrow 48\lambda^2 = 48$

$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$

$x^2 + y^2 + z^2$

$= 16 + 4 + 1 = 21$

19. $1 + a + b = c$ ....(i)

$16 + 4a + 2b = c$ ....(ii)

$81 + 9a + 3b = c$ ....(iii)

by (i) & (ii)

$16+4a+2b = c$

$2 + 2a + 2b = 2c$

$- \quad - \quad - \quad -$

$14+2a = -c$ ....(iv)

now (ii) & (iii)

$162+18a+6b = 2c$

$48 + 12a+6b = 3c$

$- \quad - \quad - \quad -$

$114 + 6a = -c$ ....(v)

by (iii) & (v)

$114 + 6a = c$

$42 + 6a = 2c$

$- \quad - \quad +$

$72 = 2c$

$c = \frac{72}{2}$ $\boxed{c = 36}$

20. $a + b + c > 10$

$\therefore$ (a,b,c) can be

a	b	c
1	2	8,9
1	3	7,8,9
1	4	6,7,8,9
1	5	6,7,8,9
1	6	7,8,9
1	7	8,9
1	8	9
2	3	6,7,8,9
2	4	5,6,7,8,9
2	5	6,7,8,9
2	6	7,8,9
2	7	8,9
2	8	9
3	4	5,6,7,8,9
3	5	6,7,8,9
3	6	7,8,9
3	7	8,9
3	8	9
4	5	6,7,8,9
4	6	7,8,9
4	7	8,9
4	8	9
5	6	7,8,9
5	7	8,9
5	8	9
6	7	8,9
6	8	9
7	8	9

Total 73 cases

21. The number of possible ordered pair

(i) $1\times54\times2 = 6$

(ii) $1\times4\times27 = 6$

(iii) $1\times12\times9 = 6$

(iv) $1\times3\times36 = 6$

(v) $2\times18\times3 = 6$

(vi) $3\times4\times9 = 6$

(vii) $2\times6\times9 = 6$

(viii) $1\times18\times6 = 6$

(ix) $6\times3\times6 = 3$

(x) $2\times2\times27 = 3$

(xi) $1\times1\times108 = 3$

(xii) $3\times3\times12 = 3$

Total No. of ordered pair = 60

22. Number of non-overlapping polygons = 56 - 20 = 46

1 line divide plane into 2 regions

2 line divide plane into 4 regions

3 line divide plane into 7 regions

4 line divide plane into 11 regions

5 line divide plane into 16 regions

6 line divide plane into 22 regions

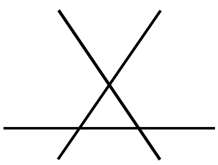
7 line divide plane into 29 regions

8 line divide plane into 37 regions

9 line divide plane into 46 regions

10 line divide plane into 56 regions

Now open regions for 3 lines are 6



Similarly for 10 lines are 20

23. $7x^2 - 44x + 12 = p^n$

We know that prime number is always positive then we put x = 7

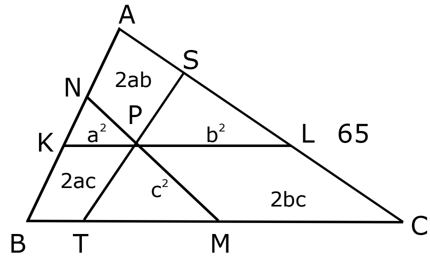
$7(7)^2 - 44\times7 + 12 = p^n$

$343-308+12 = p^n$

$47 = p^n$

$p = 47$

24.



Let MN = ST = KL = ℓ

$$\frac{\ell}{26} = \sqrt{\frac{(b+c)^2}{(a+b+c)^2}}$$

$$\frac{\ell}{26} = \frac{b+c}{a+b+c}$$

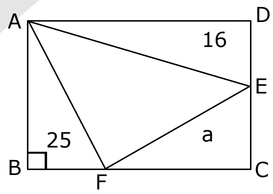
$$\frac{\ell}{65} = \frac{a+c}{a+b+c}$$

$$\frac{\ell}{78} = \frac{\ell}{26} = 2$$

$$\frac{\ell}{26} + \frac{\ell}{65} + \frac{\ell}{78} = 2$$

ℓ = 30 which is not possible as ℓ has to be less than 26

25.



area of

Δ ABF $\frac{1}{2}\times BF\times AB = 25 \Rightarrow \frac{1}{2}\times BF\times(DE+EC) = 25$

area of

ΔAED $\frac{1}{2}\times DE\times AD = 16 \Rightarrow \frac{1}{2}\times DE\times(BF+FC) = 16$

area of

Δ CEF $\frac{1}{2}\times EC\times FC = 9 \Rightarrow \frac{1}{2}\times EC\times FC = 9$

$\frac{1}{2} \times BF \times CD = 25$ ....(i)

$\frac{1}{2} \times DE \times BC = 16$ ....(ii)

$\Rightarrow \quad \frac{1}{2} \times (BC-BF)\,(CD-DE) = 9$

$\frac{1}{2}\; BC\times CD - BC\times DE - BF\times CD + BF\times DE = 9$

..........(iii)

$\frac{1}{2}[BF\times CD + DE\times BC + BC\times CD - BC\times DE - BF\times CD$

$+ BF \times DE] = 50$

$BC\times CD + BF\times DE = 100$

$(AB\times AB)^2 - 100AB\times AD + 800 = 0$

$x^2 - 100x + 1600 = 0$

$x^2 - 80x - 20x + 1600 = 0$

$x(x-80) - 20(x-80) = 0$

$(x-20)\,(x-80) = 0$

$x = 20, \qquad x = 80$

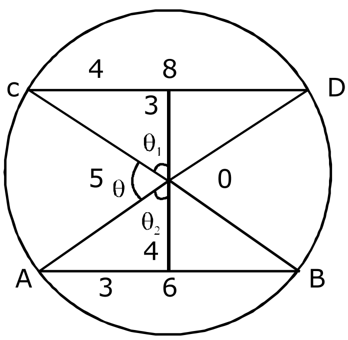
$AB \times AD = 20, \qquad AB\times AD = 80$

Area of $\Delta AFE = 80-(25+16+9)$

$= 80 - 50$

Area of $\Delta AFE = 30$

26.



$A = 2\left[\frac{1}{2}\times 25\times\theta\right] + \frac{1}{2}\times 3\times 8 + \frac{1}{2}\times 4\times 6$

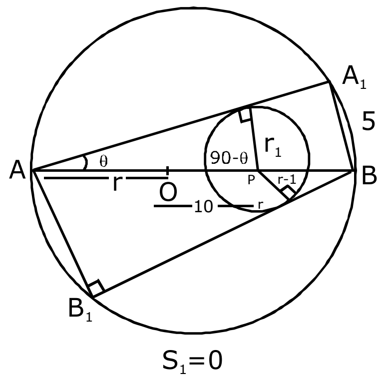
$\theta = \left[\pi - (\theta_1 + \theta_2\right] = \pi - \left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{3}{4}\right)$

$\theta = \frac{\pi}{2}$

$A = 24 + \frac{25\pi}{2} \Rightarrow A = \frac{48+25\pi}{2}$

$(m + n + k) = (48+2+25) = 75$

27.



$S_1=0$

$\frac{r_1}{r+10} = \frac{5}{2r}$ ....(i)

$\frac{r_1}{r-10} = \frac{15}{2r}$ ....(ii)

$\frac{r-10}{r+10} = \frac{1}{3}$

$3r - 30 = r + 10$

$2r = 40$

$r = 20$

29. n! + 1 is not divisible by 1,2.......,n

(n+1)! is divisible by 1,2.......,n

so HCF $\geq$ n + 1

also (n+1)! is not divisible by n + 2, n+3....

so HCF can be n+1 only

Let us start by taking n = 99

$\Rightarrow$ 99! + 1 and 100!

HCF = 100 is not possible as 100 divides 99!

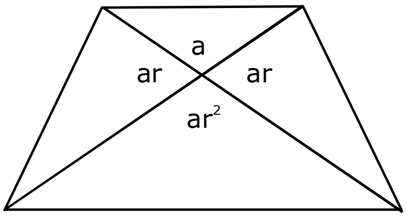
composite number will not be able to make it

so let us take prime i.e. n = 97

now 96! + 1 and 97! are both divisible by 97

so HCF = 97

(by Wilson's theorem (p-1)! + 1) is divisible by p)

30. 

case-1 $a^2r^2 = 1296$

$ar^2 - 576$

$r = \left(\frac{36}{24}\right)^2 = \frac{9}{4}$

$a^2 = 24 \times 24 \times \frac{4}{9} \Rightarrow a = 24\times \frac{2}{3}$

$a = 16$

case-2 $a^2r = 576$

$a^2r^3 = 1296$

$r^2 = \frac{1296}{576}$

$r^2 = \left(\frac{3}{2}\right)^2 \Rightarrow r = \frac{3}{2}$

$a^2 = 576 \times \frac{2}{3}$

$a^2 = 192 \times 2$

$a^2 = 384$

case-3 $a^2r^3 = 1296$

$a^2r^2 = 576$

$r^2 = \frac{1296}{576}$

$r = \frac{9}{4}$

$a^2 = \frac{576 \times 16}{81}$

$a = \frac{32}{3}$

area = $a(r+1)^2$

case 1 : area = $16\left(1+\frac{9}{4}\right)^2 = 169 \Rightarrow$ square root is 13

case 2 : area = $\left(1+\frac{3}{2}\right)^2 = 122.47$

case 3 : area = $\frac{32}{3}\left(1+\frac{9}{4}\right)^2$ so maximum area is 13