NMTC_2017

(NATIONAL MATHEMATICS TALENT CONTEST)

SUB JUNIOR LEVEL - VII & VIII STANDARDS

PART - A

1. The fraction $\frac{4}{37}$ is written in the decimal form $0.a_1a_2a_3$.......The Value of a_{2017} is :

(A) 8 (B) 0 (C) 1 (D) 5

Sol. C

$\frac{4}{37} = |\underline{08}|\underline{08}|\underline{08} = 0.a_1a_2a_3$ ______

So the value of $a_{2017} = \underbrace{a_1, a_2, a_3}_{108}\ \underbrace{a_4, a_5, a_6}_{108}$ —— $\underbrace{a_{2014}, a_{2015}, a_{2016}}_{108}\ \underbrace{a_{2017}}_{1}$

$a_{2017} = 1$

2. The number of integers x satisfying the equation $(x^2-3x+1)^{x+1} = 1$ is :

(A) 2 (B) 3 (C) 4 (D) 5

Sol. B

$(x^2 - 3x + 1)^{x+1} = 1$

$x^2 - 3x + 1 = 1$

$x^2 - 3x = 0$

$x(x - 3) = 0$

$x = 0, x = 3$

So 2 values

For x

3. The number of two digit numbers ab such that the number ab–ba is a prime number is :

(A) 0 (B) 1 (C) 2 (D) 3

Sol. A

(ab – ba) is always divisible by 9

So, there is no prime number for ab – ba

4. If $A = \frac{5425}{1444} - \frac{2987}{3045} - \frac{493}{4284}$, then

(A) $1 < A < 2$ (B) $2 < A < 3$ (C) $3 < A < 4$ (D) $A < 1$

Sol. B

$A \approx 2.661$

So $2 < A < 3$

5. What is the 2017th letter in ABRACADABRAABRACADABRA....., where the word ABRACADABRA is repeatedly writen ?

(A) A (B) B (C) C (D) R

Sol. A

ABRACDDABRA ________

So 2017 ÷ 11 =

Rem. is 4

& 4th letter is 'A'

6. How many of the following statements are true ?

(A) A 10% increase followed by another 5% increase is equivalent to a 15% increase.

(B) If the radius of a circle is doubled then the ratio of the area of the circle to the circumference is doubled.

(C) If a positive fraction is subtracted from 1 and the resulting fraction is again subtracted from 1 we get the original fraction.

(A) 0 (B) 1 (C) 2 (D) 3

Sol. C

(a) Let total = 100

$100 \times \frac{10}{100} = 10$

$90 \times \frac{5}{100} = 4.5$

Total = 10 + 4.5 = 14.5

But, $100 \times \frac{15}{100} = 15\%$

Not equal

(b) Let radius = r

Ratio = $\frac{\pi r^2}{2\pi r} = \frac{r}{2}$

Now Radius = 2r

Ratio = $\frac{\pi(2r)^2}{2\pi(2r)} = \frac{4\pi r^2}{4\pi r} = \frac{r}{1}$

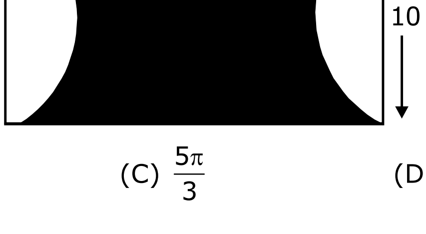
So, It's true

(c) Let fraction is $\frac{3}{4}$

$1 - \frac{3}{4} = \frac{1}{4}$

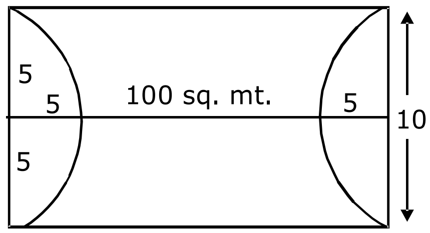
again $1 - \frac{1}{4} = \frac{3}{4}$ True

7. In the adjoining figure the breadth of the rectangle is 10 units. Two semicircles are drawn on the breadth as diameter. The area of the shaded region is 100 sq units. The shortest distance between the semicircles is :



(A) $\frac{5\pi}{2}$ (B) 5π (C) $\frac{5\pi}{3}$ (D) $\frac{3\pi}{4}$

Sol. A



Area of semicircles = $2 \times \frac{\pi(5)^2}{2} = 25\pi$

Total area of rectangle = (25π + 100)

Area = $\ell \times b$

$\ell = \frac{25\pi + 100}{10} = \frac{5\pi}{2} + 10$

Now shortest distance

$= \left(\frac{5\pi}{2} + 10\right) - (10) = \frac{5\pi}{2}$

8. When you arrange the following in descending order :

(A) 15% of 30 (B) 8% of 15 (C) 20% of 20 (D) 26% of 10

(E) 9% of 25.

The middle one is

(A) 15% of 30 (B) 8% of 15 (C) 20% of 20 (D) 26% of 10

Sol. D

(A) $30 \times \frac{15}{100} = 4.5$

(B) $15 \times \frac{8}{100} = 1.2$

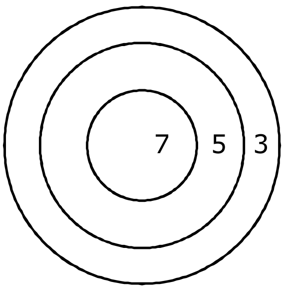
(C) $20 \times \frac{20}{100} = 4$

(D) $10 \times \frac{26}{100} = 2.6$

(E) $25 \times \frac{9}{100} = 2.25$

A > C > D > E > B

9. A boy aims a target shown in the figure. When he hits the center circle he gets 7 points, first annular region 5 points and second annular region 3 points. He shoots six times. Which one of the following is a possible score ?



(A) 16 (B) 26 (C) 19 (D) 41

Sol. B

3 × 3 = 9

2 × 5 = 10

1 × 7 = 7

Total = 26

10. After simplifying the fraction

$$\left[\frac{a+\frac{b-a}{1+ab}}{1-\frac{a(b-a)}{1+ab}}\right]\left[\frac{\frac{x+y}{1-xy}-y}{1+\frac{y(x+y)}{1-xy}}\right]$$

We get a term independent of

(A) a, y (B) b, x (C) a, b (D) x, y

Sol. A

$$\left\{\frac{a+\frac{b-a}{1+ab}}{1-\frac{a(b-a)}{1+ab}}\right\}\left\{\frac{\frac{x+y}{1-xy}-y}{1+\frac{y(x+y)}{1-xy}}\right\}$$

$$\left\{\frac{a+a^2b+b-a}{1+ab-ab+a^2}\right\}\times\left\{\frac{x+y-y+xy^2}{1-xy+yx+y^2}\right\}$$

$$\frac{a^2b+b}{1+a^2}\times\frac{x+xy^2}{1+y^2}$$

$$\frac{b(a^2+1)}{(a^2+1)}\times\frac{x(1+y^2)}{(1+y^2)}$$

bx

So It's independent of a, y

11. If 7 Rasagullas are distributed to each boy of a group, 10 rasagullas would be left. If 8 are given to each boy then 5 rasagullas would be left. So the person who distributes the rasagullas brought 15 more rasagullas and distributed the same number (x) rasagullas to each. There is no rasagulla left. Then x is:

(A) 10 (B) 11 (C) 12 (D) 14

Sol. C

Let total rusgullas = Z

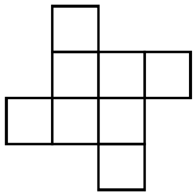
Total boys = Y

$$\begin{array}{r} 7y + 10 = Z \\ \underline{-8y \pm 5 = Z} \\ y = 5 \\ Z = 45 \end{array}$$

Now 15 more then total = 60

So, $x = \frac{60}{5}$, x = 12

12. In the adjoining diagram all squares are of the same size. The total area of the figure is 288 square cms. The perimeter of the figure (in cm) is :



(A) 86 (B) 96 (C) 106 (D) 92

Sol. B

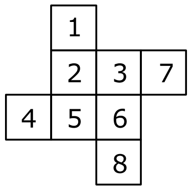
Side = a

Total Area = $8a^2 = 288$

$a^2 = 36$

a = 6

Perimeter = 16a = 16 × 6 = 96 cm



13. When Newton was a primary school student he had to multiply a number by 5. But by mistake he divided the number by 5. The percentage error he committed is :

(A) 95% (B) 96% (C) 50% (D) 75%

Sol. B

Let the number = 100

So, original was = 500

But by mistake = $\frac{100}{5} = 20$

% error = $\frac{480}{500} \times 100 = 96\%$

14. ABC is an isoceles triangle with sides AB = AC = $3x - 4 = \frac{3}{4}x + 32$. The area of the equilateral triangle with side length x is :

(A) $32\sqrt{3}$ (B) $36\sqrt{3}$ (C) $54\sqrt{3}$ (D) $64\sqrt{3}$

Sol. D

$AB = AC = 3x - 4 = \frac{3}{4}x + 32$

$3x - \frac{3}{4}x = 36$

$\frac{9}{4}x = 36$

x = 16

Area = $\frac{\sqrt{3}}{4} \times 16 \times 16$

Area = $64\sqrt{3}$

15. Two distinct number a and b are selected from 1,2,3,......60. The maximum value of $\frac{a \times b}{a - b}$ is :

(A) 6750 (B) 5270 (C) 4850 (D) 3540

Sol. D

1,2,3............60

$\frac{a \times b}{a - b} = \frac{60 \times 59}{60 - 59} = 3540$

PART - B

16. Two cogged wheels of which one has 16 cogs and the other 27 cogs, mesh into each other. If the latter turns 80 times in three quarters of a minute, the number of turns made by the other in 8 seconds is_______.

Sol. $80 \times \frac{27}{16} \times \frac{8}{45}$ = 24 turns

17. If n is a positive integer such that $a^{2n} = 2$, then $2a^{6n} - 16$ is________.

Sol. $a^{2n} = 2$

$2a^{6n} - 16 = ?$

$2(a^{2n})^3 - 16$

$2 \times 2^3 - 16$

$2 \times 8 - 16 = 0$

18. The least number of children in a family such that every child has at least one sister and one brother is______.

Sol. Total Children = 4

19. A water tank is $\frac{4}{5}$ full. When 40 liters of water is removed, its becomes $\frac{3}{4}$ full. The capacity of the tank in liters is_______.

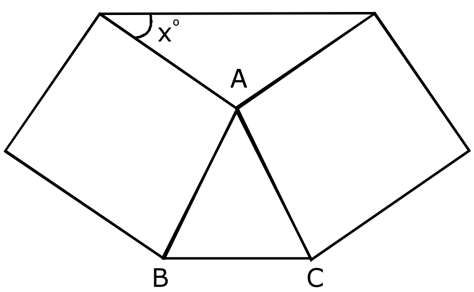
Sol. Let total water = x lit

$$\frac{4}{5}x - 40 = \frac{3}{4}x$$

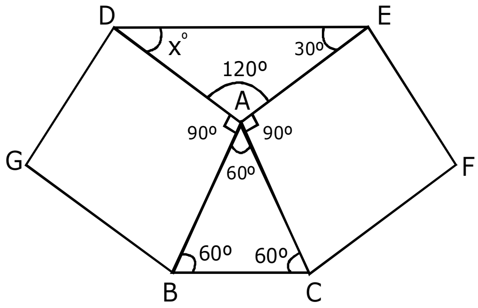
$$\frac{4}{5}x - \frac{3}{4}x = 40$$

$$\frac{16x - 15x}{20} = 40 \qquad \boxed{x = 800\,\text{litre}}$$

20. ABC is an equilateral triangle. Squares are described on the sides AB and AC as shown. The value of x is________.



Sol.



$\angle DAE = 360 - (90 + 90 + 60)$

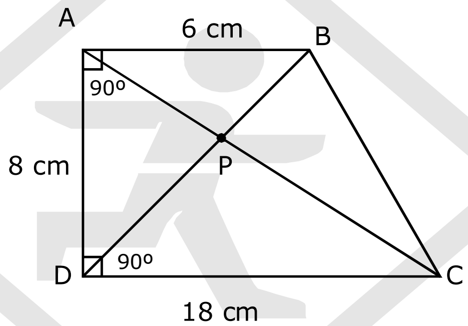
$\angle DAE = 120°$

$\angle D + \angle E = 60°$

$\angle D = \angle E$

So, $\angle D = x = 30°$

21. ABCD is trapezium with AB = 6cm, AD = 8 cm and CD=18 cms. The sides AB and CD are parallel and AD is perpendicular to AB. P is the point of intersection of AC and BD. The difference between the areas of the triangles PCD and PAB in square cms is__________.



Sol. Area of $\Delta BAD = \frac{1}{2} \times 6 \times 8 = 24cm^2$

Area of $\Delta ADC = \frac{1}{2} \times 8 \times 18 = 72cm^2$

New area of ΔADC = Area of ΔPCD + Area of ΔAPD = 72 cm^2 ....(i)

Area of ΔBAD = Area of ΔPAB + Area of ΔAPD = 24 cm^2 ....(ii)

Eq.(i) – (ii)

Area of ΔPCD – Area of ΔPAB = 72–24 = 48 cm^2

22. The price of cooking oil has increased by 25%. The percentage of reduction that a family should effet in the use of oil so as not to increase the expenditure is__________.

Sol. % Reduction = $\frac{25}{125} \times 100 = 20\%$

23. The number of natural numbers between 99 and 999 which contains exactly one zero is_______.

Sol. 99............999

$101 + 110 \longrightarrow 10$

⋮

$910 \text{ to } 910 \longrightarrow 10$

$9\times10 = 90$

and

120,130,..........190 = 8

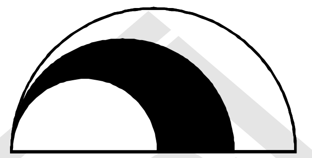
⋮

920, 930.........990 = 8

$9\times8 = 72$

Total Zero = 90 + 72 = 162

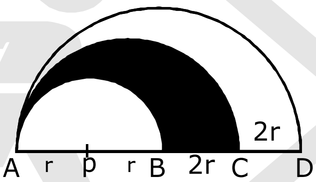
24. In the adjoining figure we have semicircles and AB = BC = CD. The ratio of the unshaded area to the shaded area is__________.



Sol. Let AP = r

AB = BC = CD

$\dfrac{\text{unshaded area}}{\text{shaded area}} =$



Area of smallest semicircle = $\dfrac{\pi r^2}{2}$

Area of middle semicircle = $\dfrac{\pi(2r)^2}{2} = 2\pi r^2$

Area of shadded protion = $2\pi r^2 - \dfrac{\pi r^2}{2} = \dfrac{3\pi r^2}{2}$

Area of larger semicircle = $\dfrac{\pi(3r)^2}{2} = \dfrac{9\pi r^2}{2}$

Area of unshaded portion in larger semicircle = $\dfrac{9\pi r^2}{2} - 2\pi r^2 = \dfrac{5\pi r^2}{2}$

Total unshaded area = $\dfrac{5\pi r^2}{2} + \dfrac{\pi r^2}{2} = 3\pi r^2$

Ratio = $\dfrac{3\pi r^2}{\dfrac{3\pi r^2}{2}} = 2:1$

25. Gold is 19 times as heavy as water and copper is 9 times as heavy as water. The ratio in which these two metals be mixed so that the mixture is 15 times as heavy as water is________.

Sol. $G = 19 \times W$

$C = 19 \times W$

$A\times G + B\times C = 15\times W \times (A+B)$

$A\times(19W) + B\times(9W) = 15W \times (A+B)$

$19A + 9B = 15A + 15B$

$4A = 6B$

$\boxed{\frac{A}{B} = \frac{3}{2}}$

26. Five angles of a heptagon (seven sided polygon) are 160º, 135º, 185º, 145º and 125º. If the other two angles are both equal to xº, then x is_______.

Sol. $160 + 135 + 185 + 145 + 125 + 2x = (n-2) \times 180^\circ$

$\boxed{n = 7}$

$2x + 750 = 5\times 180$

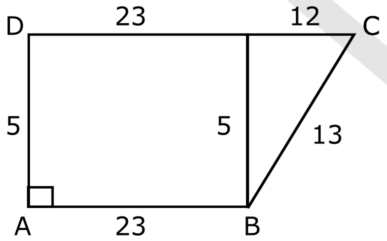
$2x = 900-750$

$2x = 150$

$\boxed{x^\circ = 75^\circ}$

27. ABCD is trapezium with AB parallel to CD and AD perpendicular to AB. If AB = 23cm, CD = 35 cm, and AD = 5 cm. The perimeter of the given trapezium in cms is___________.

Sol.



$BC = \sqrt{144 + 25}$

$BC = 13$

Perimeter of ABCD $= 23 + 13 + 35 + 5$

Perimeter of ABCD = 76 cm

28. The number of three digit numbers which are multiples of 11 is_______.

Sol. 110, 121,.............., 990

$T_n = a + (n-1)d$

$990 = 110 + (n-1)\ 11$

$\frac{880}{11} = (n-1)$

$80 = n-1$ $\boxed{n = 81}$

29. If a,b are digits, ab denotes the number 10a+b. Similarly, when a,b,c are digits, abc denotes the number 100a + 10b + c. If X,Y,Z are digits such that XX+YY+ZZ = XYZ, then XX × YY × ZZ is_____.

Sol.

$$\begin{array}{r} xx \\ yy \\ \underline{zz} \\ \underline{xyz} \end{array} \qquad \begin{array}{r} 11 \\ 99 \\ \underline{88} \\ \underline{198} \end{array}$$

Values of x=1, y = 9, z = 8

xx × yy × zz = 11 × 99 × 88 = 95,832

30. The positive integer n has 2,5 and 6 as its factors and the positive integer m has 4,8,12 as its factors. The smallest value of m + n is________.

Sol. $n \longrightarrow 2,5,6 = 30$

$m \longrightarrow 4,8,12 = 24$

$m + n = 30 + 24 = 54$