CLASS XII DATE : 27-06-2010

PAPER - II

ANSWER KEY WITH SOLUTION

MATHEMATICS

SECTION – A

1.	B	**2.**	B	**3.**	C	**4.**	A	**5.**	C	**6.**	C	**7.**	A
8.	C												

SECTION – B

1. A–R ; B–R ; C– R **2.** A–P, Q, R ; B–S, T ; C–T ; D–T

SECTION – C

1. 0010 **2.** 0006 **3.** 0180 **4.** 0001 **5.** 0005 **6.** 0003

PHYSICS

SECTION - A

1.	A	**2.**	D	**3.**	C	**4.**	A	**5.**	A	**6.**	A	**7.**	A	**8.**	B

SECTION - B

1. (A) $\rightarrow$ P ; (B) $\rightarrow$ R ; (C) $\rightarrow$ S **2.** (A) $\rightarrow$ P,Q,S; (B) $\rightarrow$ Q,R ; (C) $\rightarrow$ P, Q, S ; (D) $\rightarrow$ P, Q, S, T

SECTION - C

1. 0002 **2.** 0004 **3.** 0002 **4.** 0006 **5.** 0080 **6.** 0001

CHEMISTRY

SECTION – A

1.	B	**2.**	A	**3.**	A	**4.**	C	**5.**	B	**6.**	C	**7.**	A
8.	A												

SECTION B

1. A-P, B-R, C-Q **2.** A-P B-Q,R C-P, D-Q,R

SECTION C

1. 0035 **2.** 0153 **3.** 0001 **4.** 0006 **5.** 0005 **6.** 0010

SOLUTIONS

MATHEMATICS

SECTION – A

1. **B**

$y = f(x) \Rightarrow f(1) = 5 \Rightarrow y = 5$ when $x = 1$

$x = g(y)$ $\quad$ $[g(y) \rightarrow$ inverse of $f(x)]$

$1 = g'(y) \,.\, f'(x) \quad \Rightarrow g''(y) = -\dfrac{f''(x)}{[f'(x)]^3}$

$\Rightarrow g''(5) = \dfrac{-4}{(2)^3} = \dfrac{-1}{2}$ (put $x = 1, y = 5$)

2. **B**

Let three terms of G.P. be $\dfrac{a}{r}$, a, ar

$a^3 = -8 \Rightarrow a = -2$

also $-\dfrac{2}{r} - 2 - 2r = \dfrac{21}{10} \quad \Rightarrow r = -\dfrac{4}{5}$ or $-\dfrac{5}{4}$ (reject)

Now $S_\infty = \dfrac{a/r}{1-r} = \dfrac{5/2}{1+4/5} = \dfrac{25}{18} = \dfrac{p}{q}$

$\therefore \quad p - q = 7$

3. **C**

Only possibility is

$\sin^{-1} x_1 = \sin^{-1} x_2 = \ldots\ldots\ldots = \sin^{-1} x_n = -\pi/2$

$\Rightarrow x_1 = x_2 = \ldots\ldots\ldots = x_n = -1$

4. **A**

5. **C**

$f'(x) \,.\, g(x) + f(x) \,.\, g'(x) = 0 \Rightarrow \dfrac{f'(x)}{f(x)} = -\dfrac{g'(x)}{g(x)}$ ......(1)

$f''(x) \,.\, g(x) + 2f'(x) \,.\, g'(x) + f(x) \,.\, g''(x) = 0$ ......(2)

$f'''(x) \,.\, g(x) + 3f''(x) \,.\, g'(x) + 3f'(x) \,.\, g''(x) + f(x) \,.\, g'''(x) = 0$ ......(3)

Now $\dfrac{g(x)}{g'(x)}\left[\dfrac{f'''(x)}{f'(x)} - \dfrac{g'''(x)}{g'(x)}\right]$

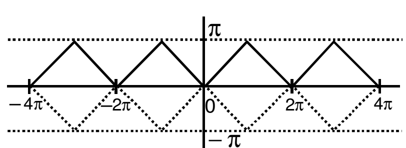
$= -3\left[\dfrac{f''(x)}{f(x)} - \dfrac{g''(x)}{g(x)}\right]\dfrac{f(x)}{f'(x)} = 3\left[\dfrac{f''(x)}{f(x)} - \dfrac{g''(x)}{g(x)}\right]$

6. **C** $\quad$ **7.** **A** $\quad$ **8.** **C**

SECTION – B

1. (A)–R ; (B)–R ; (C)– R

(A)



(B) $g'(y) = \dfrac{1}{f'(x)}$

$(x = g(y) \Rightarrow 2 = g(a)$ when $y = a, x = 2)$

$g'(a) = \dfrac{1}{f'(2)} = 4$

(C) $L = \lim\limits_{x \to 0} \dfrac{(e^{2x}+1)(e^x - x - 1)}{e^x (e^x - 1) \cdot x}$

$= \lim\limits_{x \to 0} \dfrac{(e^{2x}+1)(e^x - x - 1)}{e^x \left(\dfrac{e^x - 1}{x}\right) x^2} = 1$

2. **(A)–P, Q, R ; (B)–P, Q, R, S, T ; (C)–T ; (D)–T**

(B) $\quad g(x) = x(x - 1)(x + 1)$

$h(x) = gof(x) = \text{sgn}\, x(\text{sgn}\, x + 1)(\text{sgn}\, x - 1)$

$h(x) = \begin{cases} 0 \;, & x > 0 \\ 0 \;, & x = 0 \\ 0 \;, & x < 0 \end{cases}$

$h(x) = 0$ for all $x \in R$

(C) $L = \lim\limits_{n \to \infty} \dfrac{\left\{(n+1)\left(n+\frac{1}{2}\right)\ldots\ldots\left(n+\frac{1}{2^{n-1}}\right)\right\}^n}{n^{n^2}}$

$L = \lim\limits_{n \to \infty} \left\{\dfrac{(n+1)\left(n+\frac{1}{2}\right)\ldots\ldots\left(n+\frac{1}{2^{n-1}}\right)}{n^n}\right\}^n$

$L = \left\{\left(1+\dfrac{1}{n}\right)\left(1+\dfrac{1}{2n}\right)\ldots\ldots\left(1+\dfrac{1}{2^{n-1}n}\right)\right\}^n$

$\ell n(L) = \lim\limits_{n \to \infty} n\left\{\ell n\left(1+\dfrac{1}{n}\right) + \ell n\left(1+\dfrac{1}{2n}\right) + .. + \ell n\left(1+\dfrac{1}{2^{n-1}n}\right)\right\}$

$= \lim\limits_{n \to \infty}\left\{\dfrac{\ell n\left(1+\frac{1}{n}\right)}{1/n} + \dfrac{\ell n\left(1+\frac{1}{2n}\right)}{2.(1/2n)} + \ldots + \dfrac{\ell n\left(1+\frac{1}{2^{n-1}n}\right)}{2^{n-1}.(1/n2^{n-1})}\right\}$

$= 1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \ldots\ldots + \dfrac{1}{2^{n-1}}$

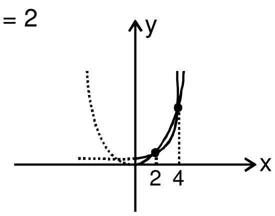
$= \left(\dfrac{1-\frac{1}{2^n}}{1-\frac{1}{2}}\right) = 2\left(1-\dfrac{1}{2^n}\right) = 2$

(D) $\quad 2\,\ell n\, x = x\,\ell n\, 2$

$\Rightarrow x^2 = 2^x$

$\Rightarrow x = 2, 4$

$\therefore$ two solution



SECTION – C

1. **0010**

$x \in [-1, 0]$

$x + \dfrac{1+x^2}{2} = -2x \;\Rightarrow x^2 + 6x + 1 = 0$

$\Rightarrow x = \dfrac{-6 \pm \sqrt{36-4}}{2} = -3 \pm 2\sqrt{2}$

$\therefore\; x = 2\sqrt{2} - 3 \Rightarrow |10a| = |20\sqrt{2} - 30|$ & $x \in [0, 1]$

$x + \dfrac{1+x^2}{2} = 2x \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$

$\therefore\; x = 1 \quad \Rightarrow |10a| = 10$

$|10a| = 10, (20\sqrt{2} - 30) \Rightarrow [|10a|] = 1, 10$

2. **0007**

$f(x) = \dfrac{[x].\sin\dfrac{x}{[x+1]}}{1+[x]} \qquad (\because \sin \pi[x+1] = 0)$

put $x = I + h$ where $I \to$ integers

$\lim\limits_{h\to 0} f(I+h) = \lim\limits_{h\to 0} \dfrac{I.\sin\dfrac{I+h}{I+1}}{I+1} = \dfrac{I.\sin\dfrac{I}{I+1}}{I+1}$

$\lim\limits_{h\to 0} f(I-h) = \dfrac{(I-1)\sin\dfrac{I-h}{I}}{I} \quad \dfrac{(I-1)\sin 1}{I}$

$\therefore$ Discontinuous function at all integers.

3. **0180**

$$f(x) = \begin{cases} ax^3 + b & , \; 0 \le x \le 1 \\ 2\cos \pi x + \tan^{-1} x & , \; 1 < x \le 2 \end{cases}$$

$$f'(x) = \begin{cases} 3ax^2 & , \; 0 < x < 1 \\ -2\pi \sin \pi x + \dfrac{1}{1+x^2} & , \; 1 < x < 2 \end{cases}$$

As the function is differentiable in [0, 2]

$\Rightarrow$ function is differentiable at x = 1

$\therefore\; f'(1^-) = f'(1^+)$

$\Rightarrow\; 3a = \dfrac{1}{2} \qquad \Rightarrow a = \dfrac{1}{6}$

Function will also be continuous at x = 1

$\therefore\; \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\Rightarrow\; a + b = -2 + \dfrac{\pi}{4}$

$\therefore\; b = -2 - \dfrac{1}{6} + \dfrac{\pi}{4} = \dfrac{\pi}{4} - \dfrac{13}{6}$

$\Rightarrow\; k_1 = 6$ and $k_2 = 12 \;\Rightarrow k_1^2 + k_2^2 = 180$

4. **0001**

$L_1 = \lim\limits_{x\to 0^-} (\cos x - \sin x)^{\text{cosec } x} = e^{\ell}$

$\ell = \lim\limits_{x\to 0^-}\left(\dfrac{\cos x - \sin x - 1}{\sin x}\right)$

$= \lim\limits_{x\to 0^-}\left(\dfrac{1 - 2\sin^2\frac{x}{2} - 2\sin\frac{x}{2}.\cos\frac{x}{2} - 1}{2\sin\frac{x}{2}.\cos\frac{x}{2}}\right)$

$= -\lim\limits_{x\to 0^-}\left(\dfrac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\cos\frac{x}{2}}\right) = -1$

$\Rightarrow L_1 = e^{-1} \;\therefore\; a = 1/e$ (as function is continuous)

$L_2 = \lim\limits_{x\to 0^+} \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{a\cdot e^{2/x} + b\cdot e^{3/x}}$

Divided N^r & D^r by $e^{3/x}$

$= \lim\limits_{x\to 0^+} \dfrac{e^{-2/x} + e^{-1/x} + 1}{a\cdot e^{-1/x} + b} = \dfrac{1}{b}$

$\Rightarrow L_2 = \dfrac{1}{b}$ by definition $\dfrac{1}{b} = \dfrac{1}{e} \;\Rightarrow\; b = e$

$\therefore\; a\cdot b = 1$

5. **0005**

$y = e^{(\sin^2 x + \sin^4 x + \ldots\ldots\ldots.\infty)\log_e 2} = e^{\frac{\sin^2 x}{1-\sin^2 x}\log_e 2}$

$= e^{(\tan^2 x).\log_e 2} = 2^{\tan^2 x}$

roots of $x^2 - 9x + 8 = 0$ are 8 and 1

$\Rightarrow\; 2^{\tan^2 x} = 2^3 \quad \Rightarrow\; \tan x = \sqrt{3}$

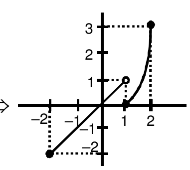
and $2^{\tan^2 x} = 2^0 \quad \Rightarrow \tan x = 0$ (reject)

$\therefore\; \dfrac{\sin x + \cos x}{\sin x - \cos x} = 2 + \sqrt{3} \;\Rightarrow\; a + b = 5$

6. **0003**

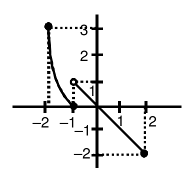
$g(x) = |f(x)| - f(-x)$

$f(x) = \begin{cases} x & ; \; -2 \le x < 1 \\ x^2 - 1 & ; \; 1 \le x \le 2 \end{cases} \Rightarrow$

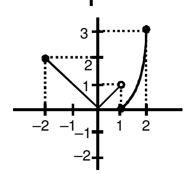


$f(-x) = \begin{cases} -x & ; \; -2 \le -x < 1 \\ x^2 - 1 & ; \; 1 \le -x \le 2 \end{cases}$

$= \begin{cases} -x & ; \; -1 < x \le 2 \\ x^2 - 1 & ; \; -2 \le x \le -1 \end{cases} \Rightarrow$



$|f(x)| = \begin{cases} -x & ; \; -2 \le x < 0 \\ x & ; \; 0 \le x < 1 \\ x^2 - 1 & ; \; 1 \le x \le 2 \end{cases} \Rightarrow$



$$g(x) = \begin{cases} -x - x^2 + 1 & ; \; -2 \le x \le -1 \\ 0 & ; \; -1 \le x < 0 \\ 2x & ; \; 0 \le x < 1 \\ x^2 + x - 1 & ; \; 1 \le x < 2 \end{cases}$$

Not differentiable at –1, 0, 1.

SECTION - (A)

1. **A**

$A < C \Rightarrow \sin 60° < \sin C$

$$\frac{\sqrt{3}}{2} < \frac{1}{\mu} \Rightarrow \mu < \frac{2}{\sqrt{3}}$$

2. **C**

$$(\Delta x)_p = 2y = \frac{\lambda}{2} \Rightarrow y = \frac{\lambda}{4}$$

$$(\Delta x)_Q = 2(y - d) = \frac{3\lambda}{2}$$

$$(\Delta x)_p = 2(y - 2d) = \frac{5\lambda}{2}$$

$$\left|2\left(\frac{\lambda}{4} - d\right)\right| = \frac{3\lambda}{2}$$

$2d = \lambda$

$v = f\lambda \Rightarrow v = 2df$

3. **C**

4. **A**

5. **A**

6. **A**

E will be zero at distance a, only when $Q_1 > Q_2$

7. **A**

$$\frac{kQ_1}{(a+l)^2} = \frac{kQ_2}{a^2} \Rightarrow \left|\frac{Q_1}{Q_2}\right| = \left(\frac{a+l}{a}\right)^2$$

8. **B**

E_{net} at b distance

$$E_b = \frac{kQ_1}{(l+b)^2} - \frac{kQ_2}{b^2}$$

$$\frac{dE_b}{db} = 0 \Rightarrow b = \frac{l}{\left(\frac{Q_1}{Q_2}\right)^{1/3} - 1}$$

SECTION - (B)

1. **(A) → P ; (B) → R ; (C) → S**

(A) **Case - I :** when $mg \sin\theta < \mu mg \cos\theta$

$f = mg \sin\theta$ As $\theta \uparrow f \uparrow$

Static friction is working

Case - II When $mg \sin\theta > \mu mg \sin\theta$

kinetic friction will act

and $\theta \uparrow f \uparrow$

(B) $N = mg \cos\theta$

$\theta = 0, N = mg$

$\theta \uparrow, N \downarrow$

(C) **Case - I**

$$f_c = \sqrt{N^2 + (f_r)^2} = \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2} = mg$$

Case - II

$$f = \sqrt{(mg\cos\theta)^2 + \mu(mg\cos\theta)^2} = mg\cos\theta\sqrt{1+\mu^2}$$

2. (A) → P,Q,S; (B) → R ; (C) → P, Q, S ; (D) → P, Q, S, T

SECTION - (C)

1. **2**

$$y = [3\sin(kx + \omega t) + 3\sin(kx - \omega t)] + 2\sin(kx - \omega t)$$

↓ Standing wave ↓ Traveling wave

2. **4**

$$n.\lambda = 2\mu_2 t \cos r + \frac{\lambda}{2}$$

3. **2**

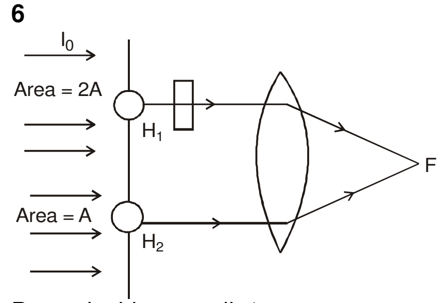
Since $T_0 = 2\pi\sqrt{\frac{I_0}{C}}$

and $T = 2\pi\sqrt{\frac{I_0}{C}}$, where $I = I_0 + I_m$

We have $I_m = \frac{C}{4\pi^2}(T^2 - T_0^2)$

$$= \frac{12.5}{4 \times 9.8}[(3.2)^2 - 2^2] = 2 \text{ kg m}^2$$

4. **6**



Power incident on slit 1

$P_{s_1} = I_0 2A$

Power incident on slit 2

$P_{s_2} = I_0 A$

Total incident power = $P_1 = I_0 2A + I_0 A = 3 I_0 A$

Path difference = $(\mu - 1)\, t = \frac{\lambda}{8}$

Power goes in original direction

$P_{s_1}' = 0.2 I_0 A$

$P_{s_2}' = 0.1 I_0 A$

$A_{net}{}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$

$P = kA^2$

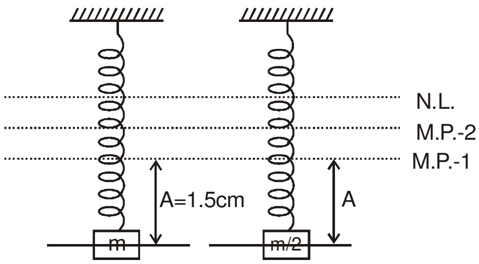
$\Rightarrow \quad P_{net} = P_{s_1}' + P_{s_2}' + 2\sqrt{P_{s_1}' + P_{s_2}'}\cos\left(\frac{\lambda}{8}\times\frac{2\pi}{\lambda}\right)$

$P_2 = P_{net} = 0.2\ I_0 A + 0.1\ I_0 A + \sqrt{2}\ +\ \sqrt{.02} I_0\ A$

$P_2 = 0.5\ I_0\ A$

$\Rightarrow \quad \frac{P_1}{P_2} = \frac{3I_0A}{0.5I_0A} = 6$

5. 80



$A_f = A + \frac{mg}{2k} = \left[1.5\times10^{-2} + \frac{5\times10}{2\times1000}\right] = 0.04\ m$

$\Rightarrow$ maximum speed $= A_f\,\omega$

$= \left[\left(\sqrt{\frac{1000\times2}{5}}\right)\times0.04\right] = 80\ cm/sec$

6. 1

At maximum streached condition

$(QE)\ x = \frac{1}{2}kx^2$

$x = \frac{2QE}{K} = \frac{2\times50\times10^{-6}\times10^6}{100} = 1\ m$

CHEMISTRY

SECTION – A

1. **B**

N_2 will not be oxidised

$CO + NH_4 + N_2$

x y 20 – x–y

contraction = x + y + volume of O_2 – (vol. of CO_2 + vol. of unreacted O_2)

= x + y + vol. of reacted O_2 – 14

$\Rightarrow$ volume if reacted O_2 = 27 – x – y

$CO + CH_4 + O_2 \rightarrow CO_2 + H_2O$

x y 27–x–y 14

P_{OAC} for C

x + y = 14 ....(1)

for H

4y = 2x moles of H_2O

$\Rightarrow$ moles of H_2O = 2y

P_{OAC} for O

x + 2(27–x–y) = 2 ×14 + moles of H_2O

x + 54 – 2x – 2y = 28 + 2y

x + 4y = 26 ....(ii)

from (i) & (ii)

y = 4 , x = 10

vol. of N_2 = 20 – x – y = 6

2. **A**

The combustion reaction is

$C_7H_{16} + 11O_2 \rightarrow 7CO_2 + 8H_2O$

$\Rightarrow \quad -4853.40 = -7\times393.5 - 8\times285.9 - \Delta H^o_f$

$\Delta H^o_f =$ **– 188.3 kJ**

3. **A**

Given, $\frac{p}{V} = K$ (constant) Also, $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

$\Rightarrow \quad T_2 - T_1\left(\frac{p_2V_2}{p_1V_1}\right) = T_1\left(\frac{p_2}{p_1}\right)^2 = 4T_1$

$\Rightarrow \quad \Delta U = C_V\Delta t = 3C_VT_1$

$\Rightarrow \quad -dw = pdV = kVdV$

$\Rightarrow \quad -w = \frac{k}{2}\left(V_2^2 - V_1^2\right)$: Also $\frac{p}{V} = k = \frac{RT}{V^2}$

$V^2 = \frac{RT}{K}$

$\Rightarrow \quad V_2^2 - V_1^2 = \frac{R}{k}(T_2 - T_1) = \frac{3T_1R}{K}$

$\Rightarrow \quad w = -\frac{K}{2}\times\frac{3T_1R}{K} = -\frac{3RT_1}{2}$

$\Rightarrow \quad \frac{\Delta U}{w} = -\frac{2C_V}{R} = -3$

4. **C**

The change occur in following sequence:

$$Ice\,(-20^\circ C) \xrightarrow[q_1]{} Ice(0^\circ C) \xrightarrow[q_2]{} H_2O(l)(0^\circ C) \xrightarrow[q_3]{} H_2O(l)(0^\circ C)$$

$q_1 = 25 \times 2.03 \times 20 = 1015$ J

$q_2 = 333 \times 25 = 8325$ J

$q_3 = 15000 - 1015 - 8325 = 5660$

$\Rightarrow$ $5660 = 25 \times 4.184 \times \Delta T$

$\Rightarrow$ $\Delta T = 54 \Rightarrow$ Final temperature $= 54^\circ C$

5. **B**

Number of moles of $O_2 = \frac{3}{4}$

No of moles of $H_2O_2 = 1.5$

$$\%\ w/v = \frac{1.5 \times 34}{500} \times 100 = 10.2\ \%$$

6. **C**

For AB

$$\frac{P_0}{T_0} = \frac{2P_0}{T_B} \Rightarrow T_B = 2T_0$$

Also, $\frac{V_0}{2T_0} = \frac{2V_0}{T_C} \Rightarrow T_C = 4T_0$

For AC (Isothermal Process)

$$\frac{P_0}{V_0} = \frac{2P_0}{V_C}$$

$V_C = 2V_0$

7. **A**

Total work done = Area under curve

$$= -\frac{1}{2} \times P_0 \times V_0 = -\frac{1}{2} P_0V_0 \text{(clockwise)}$$

Now $(\Delta U)_{cycle} = nC_V\,\Delta T = 0 = w + q_T$

$$q_T = \frac{1}{2} P_0V_0$$

$$q_{AB} = \Delta U_{AB} = n\,C_V\Delta T = \frac{P_0V_0}{RT_0} \times \frac{5}{2} R\,(2T_0 - T_0)$$

$$= \frac{5}{2} P_0V_0$$

$q_{BC} = \Delta U_{BC} - w_{BC} = nC_V\Delta T + 2P_0V_0$

$$= \frac{P_0V_0}{RT_0} \times \frac{5}{2} R\,(4T_0 - 2T_0) + 2P_0V_0$$

$= 5P_0V_0 + 2P_0V_0 = 7P_0V_0$

$$q_{CA} = nC_V\Delta T - w_{CA} = 0 - \frac{3}{2}P_0V_0 = -\frac{3}{2}P_0V_0$$

$$q_{AB} + q_{BC} = \left(\frac{5}{2} + 7\right)P_0V_0 = \frac{19}{2}\ P_0V_0$$

8. **A**

$$\eta = \frac{|\,work\,|}{\text{Heat added}} = \frac{\frac{1}{2}P_0V_0}{\frac{19P_0V_0}{2}} = \frac{100}{19}\,\%$$

SECTION B

1. **A-P, B-R, C-S**

2. **A-Q,T B-Q,R C-P, D-Q,T**

SECTION C

1. **35**

2. **153**

3. **1M**

No. of moles of $AgNO_3 = \frac{100 \times 8 \times 17}{100 \times 170} = 0.8$ mole

$NaCl \rightarrow Na^+ + Cl^-$

0.6 0.6

$AgNO_3 \rightarrow Ag^+ + NO_3^-$

0.8 0.8

	Cl^- +	Ag^+ →	$AgCl \downarrow$
t = 0	0.6	0.8	
After reaction	0	0.2	0.6

$$[M]_{Ag^+} = \frac{0.2}{200 \times 10^{-3}} = 1M$$

4. **6 litre**

22.4 l of C_4H_{10} produces to 2800 kJ

1.5 .... $\frac{2800}{22.4} \times 1.5$

22.4 l of CH_4 produces to 700 kJ

x .... $\frac{700}{22.4} \times x$

$$\Rightarrow \frac{2800}{22.4} \times 1.5 = \frac{700}{22.4} \times x$$

$\Rightarrow x = 6\ l$

5. **5**

$$\Delta U = \int nC_V dt = \int_{300}^{400} 2.5 \times (16.5 + 10^{-2}T)\,dT$$

$= 5000$ J $= 5$ kJ

6. **11**