TARGET IIT-JEE
HINT & SOLUTION

CLASS XII **DATE : 27-06-2010**

PAPER - I

ANSWER KEY WITH SOLUTION

MATHEMATICS

SECTION – A

1.	B	**2.**	C	**3.**	C	**4.**	B	**5.**	B	**6.**	A	**7.**	D
8.	D	**9.**	C	**10.**	B	**11.**	A,B,C	**12.**	B,C	**13.**	A,C	**14.**	A,B,C
15.	A,C	**16.**	A,D	**17.**	B	**18.**	A,B,C,D						

SECTION – C

1.	0002	**2.**	0004

PHYSICS

SECTION - A

1.	B	**2.**	C	**3.**	C	**4.**	A	**5.**	B	**6.**	A	**7.**	B
8.	C	**9.**	A	**10.**	A	**11.**	B,C,D	**12.**	B	**13.**	A,D	**14.**	A,B
15.	A,D	**16.**	B,C	**17.**	A	**18.**	A						

SECTION - C

1.	0100	**2.**	0240

CHEMISTRY

SECTION-A

1.	B	**2.**	B	**3.**	D	**4.**	D	**5.**	C	**6.**	C	**7.**	A
8.	A	**9.**	C	**10.**	D	**11.**	BD	**12.**	AD	**13.**	AB	**14.**	ABCD
15.	C	**16**.	C	**17.**	C	**18.**	ABCD						

SECTION-C

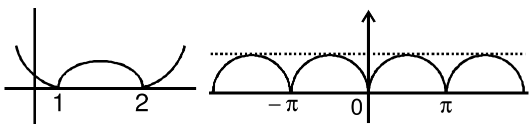
1.	0008	**2.**	0032

SOLUTIONS

MATHEMATICS

SECTION – A

1. **B**



Not differentiable at 0, 1, and 2.

2. **C**

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} \to 1^-$ and $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} \to 1^+$

$= \lim_{\theta\to 0}\left(\left[n\frac{\sin\theta}{\theta}\right]+\left[n\frac{\tan\theta}{\theta}\right]\right)$

$= [n^-] + [n^+] = n - 1 + n = 2n - 1$

3. **C**

Let $\alpha + \beta = -2|a|$, $\alpha\beta = 4$

$\alpha, \beta \in I$ so (α, β) can be (–1, –4) or (–2, –2)

for 'a' to be minimum $|a| \to$ maximum

$|\alpha + \beta| \to$ Maximum

so roots are –1 & –4. $\Rightarrow a = -5/2$

4. **B**

$\sin 1 < 1, \cos 1 < 1 \Rightarrow \sin 1 + \cos 1 < 2$

$\sqrt{\sin 1} > \sin 1$, $\sqrt{\cos 1} > \cos 1$

$\Rightarrow \sqrt{\sin 1} + \sqrt{\cos 1} > \sin 1 + \cos 1$

$\sin 2 < 1,\ 1 + \sin 2 < 2 \Rightarrow \sqrt{1+\sin 2} < 2$

$\therefore \sqrt{\sin 1} + \sqrt{\cos 1} > \sqrt{1+\sin 2}$

5. **B**

$\sin x = \sin 1 \Rightarrow x = 1$ or $\pi - 1$(but 1 is not possible)

6. **A**

$\tan\left[\frac{\cos^{-1}(\sin(\cos^{-1}x)) + \sin^{-1}(\cos(\sin^{-1}x))}{2}\right]$

$= \tan\left[\frac{\cos^{-1}\left(\sin\left(\frac{\pi}{2}-\sin^{-1}x\right)\right) + \sin^{-1}(\cos(\sin^{-1}x))}{2}\right]$

$= \tan\left[\frac{\cos^{-1}(\cos(\sin^{-1}x)) + \sin^{-1}(\cos(\sin^{-1}x))}{2}\right]$

$= \tan\left[\frac{\pi/2}{2}\right] = \tan\frac{\pi}{4} = 1$

7. **D**

$2.\log\left(\frac{3b}{5c}\right) = \log\left(\frac{5c}{a}.\frac{a}{3b}\right)$ $\therefore$ given number in A.P.

Solving $3b = 5c$ ......(1)

Also given $b^2 = ac$ ......(2)

$\therefore\quad a = \frac{25c}{9}$ & $b = \frac{5c}{3}$

Let $a = \frac{25}{9}\lambda$, $b = \frac{5}{3}\lambda$, & $c = \lambda$

$\Rightarrow\quad b + c < a$ $\therefore$ Triangle not possible.

8. **D**

$f : R \to R$ & $g : R \to R$ are both one-one onto functions s.t. f & g are mirror images of each other about y = 0.

It means one is –ve of the other i.e. $f(x) = -g(x)$

$\Rightarrow h(x) = f(x) + g(x)$ becomes zero i.e. $h(x) = 0$

h(x) is not onto as well as not one-one

9. **C**

when $f(x) \ge 0$

$x + f(x) = 2f(x) \Rightarrow f(x) = x \quad \therefore \quad x \ge 0$

when $f(x) < 0$

$x - f(x) = 2f(x) \Rightarrow f(x) = x/3 \quad \therefore \quad x < 0$

10. **B**

$\frac{d}{dx}\left\{\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)\right\} = \frac{d}{dx}\left\{\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$

$\therefore\ 0 < x < \frac{\pi}{4} \Rightarrow \frac{\pi}{4} < \frac{\pi}{4} + \frac{x}{2} < \frac{3\pi}{8} \Rightarrow \frac{d}{dx}\left(\frac{\pi}{4}+\frac{x}{2}\right) = \frac{1}{2}$

& $\frac{d}{dx}\left\{\cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)\right\} = \frac{d}{dx}\left\{\cot^{-1}\left(\cot\frac{x}{2}\right)\right\}$

$\therefore\ 0 < x < \frac{\pi}{4} \Rightarrow 0 < \frac{\pi}{2} < \frac{\pi}{8} \Rightarrow \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$

11. **A,B,C**

$f(0^-) = 0, f(0^+) = 1, f(0) = 1$

$f(1^-) = 1, f(1^+) = 0, f(1) = 1$

12. **B,C**

$A_n = \frac{(n+1)+(n+2)+.......+(n+n)}{n} = \frac{n^2+\frac{n(n+1)}{2}}{n}$

$\Rightarrow \frac{A_n}{n} = \frac{n^2+\frac{n(n+1)}{2}}{n^2} = 1 + \frac{1}{2} + \frac{1}{2n}$ $\therefore \lim_{n\to\infty}\frac{A_n}{n} = \frac{3}{2}$

$G_n = ((n + 1)(n + 2)...............(n + n))^{1/n}$

$\Rightarrow \frac{G_n}{n} = \left(\frac{(n+1)(n+2)........(n+n)}{n^n}\right)^{1/n}$

$\Rightarrow \frac{G_n}{n} = \left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).........\left(1+\frac{n}{n}\right)\right)^{1/n}$

$\Rightarrow \lim_{n\to\infty}\frac{G_n}{n} = \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).........\left(1+\frac{n}{n}\right)\right)^{1/n}$

$\Rightarrow \ell n\, L = \lim_{n\to\infty}\frac{1}{n}\left[\ell n\left(1+\frac{1}{n}\right)+\ell n\left(1+\frac{2}{n}\right)+......+\ell n\left(1+\frac{n}{n}\right)\right]$

$\Rightarrow \ell n\, L = \lim_{n\to\infty}\frac{1}{n}\left[\frac{n(n+1)}{2n}\right]$

$\Rightarrow \ell n\, L = \frac{1}{2} \quad \Rightarrow L = e^{1/2}$ $\therefore \lim_{n\to\infty}\frac{G_n}{n} = e^{1/2}$

13. **A,C**

14. **A,B,C**

(A) f (x) is non-negative, $\forall\, x \in R$

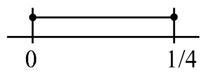
$\Rightarrow$ f (x) ≥ 0, $\forall\; x \in R$

$\therefore$ $D \leq 0$

$\Rightarrow$ $64k^2 - 16k \leq 0$

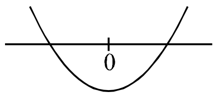
$4k^2 - k \leq 0$

$k\,(4k - 1) \leq 0$



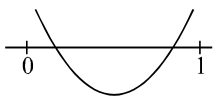
$\therefore$ integral value of k = 0

(B) f (0) < 0

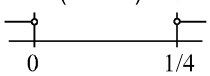


k < 0

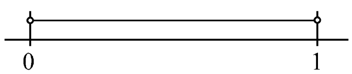
(C) for distinct roots in (0, 1)



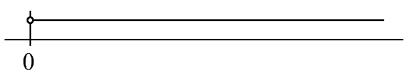
$D > 0 \;\Rightarrow\; k\,(4k - 1) > 0$ ....(1)



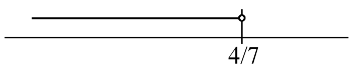
$0 < -\dfrac{b}{2a} < 1 \;\Rightarrow\; 0 < k < 1$ ....(2)



$f(0) > 0 \Rightarrow k > 0$ ....(3)



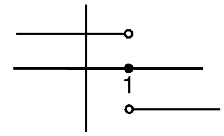
$f(1) > 0 \Rightarrow 4 - 7k > 0 \;\Rightarrow\; k < 4/7$ ....(4)



$\therefore\; (1) \cap (2) \cap (3) \cap (4) \;\Rightarrow\; k \in (1/4,\; 4/7)$

15. **A,C**

$$f(x) = \lim_{n\to\infty} \frac{1 - x^n}{1 + x^n} = \begin{cases} -1, & x > 1 \\ 0, & x = 1 \\ 1, & x < 1 \end{cases}$$



16. **A,D** 17. **B** 18. **A,B,C,D**

SECTION – C

1. **0002**

$f\{g(x)\} = x$

$f'\{g(x)\}\,.\,g'(x) = 1$ (Put $x = a^2 + b^2$)

$f'\{g(a^2 + b^2)\}\,.\,g'(a^2 + b^2) = 1$

$\Rightarrow f'(c)\,.\,2 = 1 \;\Rightarrow\; f'(c) = \dfrac{1}{2}$ $\therefore$ N = 2

2. **0004**

$$y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2 \Rightarrow y_1 = \frac{2\sin^{-1} x}{\sqrt{1 - x^2}} - \frac{2\cos^{-1} x}{\sqrt{1 - x^2}}$$

$$\sqrt{1 - x^2}\;\, y_1 = 2(\sin^{-1} x - \cos^{-1} x)$$

$$(1 - x^2)y_1^{\,2} = 4(y - 2\sin^{-1}x \cos^{-1}x)$$

$$2(1 - x^2)\, y_1 y_2 - 2xy_1^{\,2} = 4y_1 + 8\left[\frac{\sin^{-1} x}{\sqrt{1 - x^2}} - \frac{\cos^{-1} x}{\sqrt{1 - x^2}}\right] = 8y_1$$

$$(1 - x^2)y_2^{\,2} - xy_1 = 4$$

$$(1 - x^2)y_3 - 3xy_2 - y_1 = 0$$

PHYSICS

SECTION - A

1. **B**

Net force on the system is zero

2. **C**

f = mg/2

Tension at centre = mg – mg/2 = mg/2

3. **C**

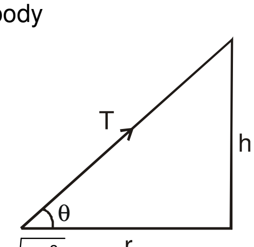
Weightless condition only occur when only gravitational force act on the body

4. **A**

$\tan\theta = \dfrac{h}{r}$

$T\cos\theta = \dfrac{mv^2}{r}$

$T\sin\theta = mg$



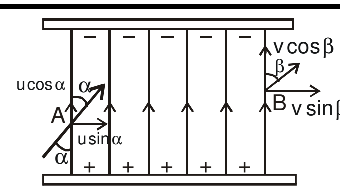
$\tan\theta = \dfrac{gr}{v^2} = \dfrac{h}{r} \Rightarrow v = \sqrt{\dfrac{gr^2}{h}}$

$T = \dfrac{2\pi r}{v} = 2\pi\sqrt{\dfrac{h}{g}} \;\Rightarrow\; f = \dfrac{1}{T}$

5. **B**

$u \sin\alpha = v \sin\beta$

$\dfrac{u}{v} = \dfrac{\sin\beta}{\sin\alpha}$



6. **A**

Angular fringe width $= \dfrac{\beta}{D} = \dfrac{\lambda}{d} \geq \dfrac{\pi}{180} \times \dfrac{1}{60}$

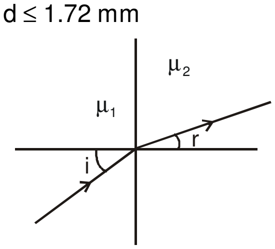
$\Rightarrow d \leq \dfrac{180 \times 60 \times \lambda}{\pi} \Rightarrow d \leq 1.72$ mm

7. **B**

$\mu_1 \sin i = \mu_2 \sin r$

$\Rightarrow$ $\sin r = \dfrac{\mu_1}{\mu_2}\sin i$

slope $= \dfrac{\mu_1}{\mu_2} = \tan 30°$



$\Rightarrow \mu_2 = \sqrt{3}\,\mu_1$

Medium (2) is denser no TIR will take place when incidence is in medium (1)

$\dfrac{v_2}{v_1} = \dfrac{\mu_1}{\mu_2} = \dfrac{1}{\sqrt{3}}$

8. C **9.** A **10.** A **11.** B,C,D

12. B

To increase linear charge density

13. A,B,D

at A $a_A = g\sin\theta = 8\ m/s^2$ at B $a_B = \frac{v^2}{l}$

from E.C between A & B

$mg\,l\,(1-\cos\theta) = \frac{1}{2}mv^2$

$v^2 = 2lg\,(1-\cos\theta) \Rightarrow a_B = \frac{2lg(1-\cos\theta)}{l} = 1\ m/s^2$

14. A,B

15. A,D

$F = q\,\lambda\,l\,E$

$a = \frac{\lambda l e}{\mu l} \Rightarrow a = \frac{\lambda E}{\mu}$

speed $v = \sqrt{\frac{2\lambda E d}{\mu}}$

force act on each charge is same and in the same direction so net torque is zero.

16. B,C

Luminous flux per unit area × area of hole

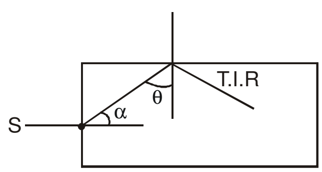
$= \frac{\phi}{4\pi r^2}ds = \frac{\phi}{4\pi r^2}\times\pi\left(\frac{d}{2}\right)^2 = \frac{\phi d^2}{16r^2}$

17. A

$\sin i = \mu \sin r$

$\sin\frac{\pi}{2} = n\sin\alpha$

$\Rightarrow \sin\alpha = \frac{1}{n}$



18. A

For TIR $\alpha > c$

$90 - \alpha > c \Rightarrow \cos\alpha > \sin c$

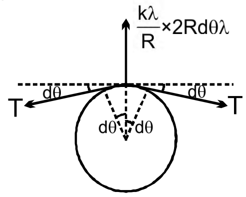
$\frac{\sqrt{n^2-1}}{n} > \frac{1}{n} \Rightarrow n > \sqrt{2}$

SECTION - C

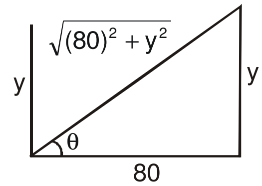
1. 0100

$2T\sin d\theta = \frac{k\lambda}{R}\times 2d\theta\lambda$

$\Rightarrow T = k\lambda^2\frac{d\theta}{\sin d\theta} \cong \frac{\lambda^2}{4\pi\varepsilon_0}$

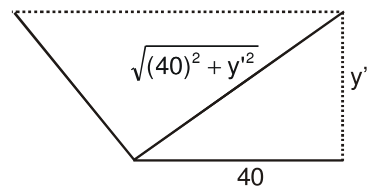


2. 240



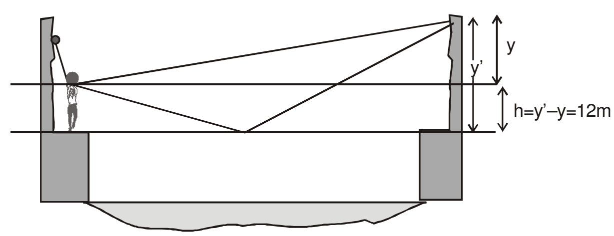
$\sqrt{(80)^2 + y^2} = 100$

y = 18cm



$\sqrt{(40)^2 + y'^2} = 100$

y′ = 30 m



so h = 12 m

$mgh = \frac{1}{2}mv^2 \Rightarrow v^2 = 2gh$

$= 2 \times 10 \times 12 = 240$

CHEMISTRY

SECTION - A

1. B **2.** B

3. D

$PE = \frac{-Ze^2}{4\pi\varepsilon_0 r}$ ($Z = 3$ for Li^{2+})

4. D

∵ 106 gm of new sample requires 12 gm of H_2O

∴ 100 gm of new sample requires $= \frac{12}{106}\times 100 = 11.32$ gm

∴ % labelling = 100 + 11.32 gm = 111.32

5. C **6.** C **7.** A

8. A **9.** C **10.** D

11. BD **12.** AD **13.** AB

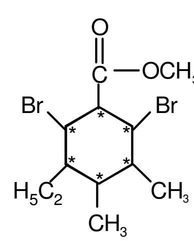
14. ABCD **15.** C **16.** C

17. C **18.** ABCD

SECTION C

1. 0008

2. 0064



Total isomers = $2^n = 2^6 =$ **64**